

Unification of dynamical decoupling and the quantum Zeno effect

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We unify the quantum Zeno effect (QZE) and the “bang-bang” (BB) decoupling method for suppressing decoherence in open quantum systems: in both cases strong coupling to an external system or apparatus induces a dynamical superselection rule that partitions the open system’s Hilbert space into quantum Zeno subspaces. Our unification makes use of von Neumann’s ergodic theorem and avoids making any of the symmetry assumptions usually made in discussions of BB. Thus we are able to generalize the BB to arbitrary fast and strong pulse sequences, requiring no symmetry, and to show the existence of two alternatives to a pulsed BB: continuous decoupling and pulsed measurements. Our unified treatment enables us to derive limits on the efficacy of the BB method: we explicitly show that the inverse QZE implies that the BB can in some cases accelerate, rather than inhibit, decoherence.

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I. INTRODUCTION

Recent years have witnessed a surge of interest in ways to protect quantum coherence, driven mostly by developments in the theory of quantum information processing [1]. A number of promising strategies for combating decoherence have been conceived and in some cases experimentally tested, including quantum error correcting codes and topological codes (for a review see [2]), decoherence free subspaces and (noiseless) subsystems (for a review see [3]), and “bang-bang” (BB) decoupling [4–6] (for an overview see [7]). Two recent papers have shown that these various methods can be unified under a general algebraic framework [8]. Here, using a very different approach, we continue this development for BB decoupling and the quantum Zeno effect (QZE).

The idea behind the BB is that application of sufficiently strong and fast pulses, with appropriate symmetry (notions we make precise later), when applied to a system, can decouple it from its decohering environment. The notion of a strong and fast interaction with a quantum system is also the key idea behind the QZE [9] (for reviews see [10,11]). The standard view of the QZE is that by performing frequent projective measurements one can freeze the evolution of a quantum state (“a watched pot cannot boil”). However, recently it has become clear that this view of the QZE is too narrow, in two main respects: (i) The projective measurements can be replaced by another quantum system interacting strongly with the principal system [11,12]. (ii) The states of the principal system need not be frozen: instead, the general situation is one of dynamically generated quantum Zeno subspaces, in which nontrivial coherent evolution can take place [13]. It is therefore not only physically reasonable, but also logically appealing to view the QZE as a dynamical effect: in this broader context, both BB decoupling and the QZE can be understood as arising from the same physical considerations and hence can be unified under the same conceptual and formal framework. Furthermore, they appear as particular cases of a more general dynamics in which the

system of interest is “strongly” coupled to an external system that (loosely speaking) plays the role of a measuring apparatus.

We use these insights to (i) generalize the BB method to pulse sequences with no symmetry, (ii) to point out that the BB pulses can have the opposite from the desired effect (a situation well known from the QZE literature as the “inverse” or “anti” Zeno effect) [14,15], and (iii) to show that alternatives to the unitary pulse control scheme are available to suppress the system-environment interaction: namely, (a) *continuous* unitary interaction and (b) pulsed *measurements*.

II. SIMPLEST BB CYCLE

Consider the “BB evolution” induced by the two-element control set (not necessarily a group) $\{I, U_1\}$, where I is the identity operator, in which the controlled system Q alternately undergoes N “kicks” U_1 (*instantaneous* unitary transformations) and free evolutions in a time interval t :

$$U_N(t) = [U_1 U(t/N)]^N. \quad (1)$$

We take $U = \exp(-iHt)$, with H the (time-independent) Hamiltonian of Q , its environment, and their interaction, and will sometimes abbreviate $U(t/N)$ by U . We present a new derivation of this BB evolution that allows for a transparent connection to the formulation of the QZE.

In the large- N limit, the dominant contribution to $U_N(t)$ is U_1^N . We therefore consider the sequence of unitary operators

$$V_N(t) = U_1^{\dagger N} U_N(t). \quad (2)$$

Observe that $V_N(0) = I$ for any N and

$$\begin{aligned}
i\frac{d}{dt}V_N(t) &= U_1^{\dagger N} \sum_{k=0}^{N-1} (U_1 U)^k \left(U_1 i \frac{dU}{dt} \right) (U_1 U)^{N-k-1} \\
&= U_1^{\dagger N} \frac{1}{N} \sum_{k=0}^{N-1} (U_1 U)^k U_1 H U_1^{\dagger} (U_1 U)^{\dagger k} (U_1 U)^N \\
&= H_N(t) V_N(t) \quad [V_N(0) = I],
\end{aligned} \tag{3}$$

with

$$H_N(t) = \frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger N} (U_1 U)^k U_1 H U_1^{\dagger} (U_1 U)^{\dagger k} U_1^N. \tag{4}$$

The limiting evolution operator

$$\mathcal{U}(t) \equiv \lim_{N \rightarrow \infty} V_N(t) \tag{5}$$

satisfies the equation

$$i\frac{d}{dt}\mathcal{U}(t) = H_Z \mathcal{U}(t) \quad [\mathcal{U}(0) = 1], \tag{6}$$

with the “Zeno” Hamiltonian

$$H_Z \equiv \lim_{N \rightarrow \infty} H_N(t). \tag{7}$$

Therefore $\mathcal{U}(t) = \exp(-iH_Z t)$. In order to study the behavior of the limiting operator we first observe that for $N \rightarrow \infty$ we can neglect the free evolution $U(t/N)$ in Eq. (4) and so

$$H_N \sim \frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger N} U_1^{k+1} H U_1^{\dagger k+1} U_1^N = \frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger k} H U_1^k. \tag{8}$$

Next, we will show that for any bounded H and any U_1 with a pure point spectrum—namely,

$$U_1 = \sum_{\mu} e^{-i\lambda_{\mu}} P_{\mu} \tag{9}$$

$[\lambda_{\mu} \neq \lambda_{\nu} \pmod{2\pi}$ for $\mu \neq \nu$, $P_{\mu} P_{\nu} = \delta_{\mu\nu} P_{\mu}]$ —one gets

$$H_Z \equiv \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger k} H U_1^k = \sum_{\mu} P_{\mu} H P_{\mu} \equiv \Pi_{U_1}(H), \tag{10}$$

where the map Π_{U_1} is the projection onto the centralizer (or commutant) of U_1 ,

$$\mathcal{Z}(U_1) = \{X \mid [X, U_1] = 0\}. \tag{11}$$

First, we show that the (strong) limit H_Z in Eq. (10) is a bounded operator which satisfies the intertwining property

$$H_Z P_{\mu} = P_{\mu} H P_{\mu} = P_{\mu} H_Z \tag{12}$$

for any eigenprojection P_{μ} of U_1 , with eigenvalue $e^{-i\lambda_{\mu}}$. Equation (10) follows whenever U_1 admits the spectral decomposition (9). Here is the proof. For any vector ψ in the Hilbert space \mathcal{H} , we get, using Eq. (9),

$$\frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger k} H U_1^k P_{\mu} \psi = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{U}^k \phi, \tag{13}$$

where $\tilde{U} = (U_1 e^{i\lambda_{\mu}})^{\dagger}$ is a unitary operator whose eigenprojection P_{μ} has eigenvalue 1 and $\phi = H P_{\mu} \psi \in \mathcal{H}$. Recall now an ergodic theorem due to von Neumann ([16], p. 57) that states that if \tilde{U} is a unitary operator on the Hilbert space \mathcal{H} and P_{μ} its eigenprojection with eigenvalue 1 ($\tilde{U} P_{\mu} = P_{\mu}$), then, for any $\phi \in \mathcal{H}$,

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=0}^{N-1} \tilde{U}^k \phi = P_{\mu} \phi. \tag{14}$$

As a consequence, by taking the limit of Eq. (13), we get Eq. (12).

Notice that the intertwining property (12) holds also for an unbounded H whose domain D contains the range of P_{μ} —namely, $P_{\mu} \mathcal{H} \subset D(H)$. For a generic unbounded Hamiltonian, we can still formally consider Eq. (10) as the limiting evolution, but the meaning of $P_{\mu} H P_{\mu}$ and its domain of self-adjointness should be properly analyzed.

In conclusion,

$$\mathcal{U}(t) = \exp(-iH_Z t) = \exp\left[-i \sum_{\mu} P_{\mu} H P_{\mu} t\right] \tag{15}$$

and, due to Eqs. (2) and (5),

$$\begin{aligned}
U_N(t) &\sim U_1^N \mathcal{U} \\
&= U_1^N \exp(-iH_Z t) \\
&= \exp\left[-i \sum_{\mu} (N\lambda_{\mu} P_{\mu} + P_{\mu} H P_{\mu} t)\right].
\end{aligned} \tag{16}$$

This proves that the BB evolution (1) yields a Zeno effect and a partitioning of the Hilbert space into “Zeno subspaces,” in the sense of [13].

We emphasize that no cyclic group properties are required for pulse sequences. This extends previous studies, in which “symmetrization” was thought to play an important role in order to obtain decoupling and suppression of decoherence.¹ Indeed the dynamics (1) is different from the dynamics $[U_1^{\dagger} U(t/2N) U_1 U(t/2N)]^N$, originally proposed in [4], because it is only constructed with a single “bang” U_1 , without the second “bang” U_1^{\dagger} which would close the group. We will further elaborate on this issue in Sec. IV.

By taking H to be a system-bath interaction Hamiltonian, we see that the effect of the U_1 “kicks” is to project the decohering evolution into disjoint subspaces defined by the spectral resolution of U_1 . A proper choice of U_1 can either eliminate this evolution or make it proceed in some desired fashion. To give the simplest possible example, suppose

¹Though apparently this point is well appreciated in the practice of high-resolution NMR; i.e., there are many sequences—e.g., WAHUA—achieving the intended averaging effect without averaging over a subgroup. Nevertheless, averaging still results from symmetry arguments in these cases [24].

$$H = \sigma_x \otimes B, \quad U_1 = \sigma_z. \quad (17)$$

H generates “bit flips” and the projection operators are

$$P_{\pm} = \frac{1}{2}(I \pm \sigma_z), \quad (18)$$

with eigenvalues $\lambda_{\pm} = \pm 1$. Thus

$$H_Z = \sum_{\mu=\pm} P_{\mu} H P_{\mu} = \sum_{\mu=\pm} P_{\mu} \sigma_x P_{\mu} \otimes B = 0, \quad (19)$$

so the decohering evolution is completely canceled.

The physical mechanism giving rise to the Zeno subspaces in the $N \rightarrow \infty$ limit can be understood by considering the case of a finite dimensional Hilbert space. Then the limit (10) reads

$$\frac{1}{N} \sum_{k=0}^{N-1} U_1^{\dagger k} H U_1^k = \sum_{\mu, \nu} P_{\mu} H P_{\nu} \frac{1}{N} \sum_{k=0}^{N-1} e^{ik(\lambda_{\mu} - \lambda_{\nu})} \quad (20)$$

and one sees that the last sum is 1 for $\mu = \nu$ and vanishes as $O(1/N)$ otherwise [remember that $\lambda_{\mu} \neq \lambda_{\nu} \pmod{2\pi}$ for $\mu \neq \nu$ in Eq. (9)]. The appearance of the Zeno subspaces in thus a direct consequence of the fast oscillating phases between different eigenspaces of the kick. This is equivalent to a procedure of phase randomization, and is analogous to the case of strong continuous coupling [13].

III. IMPLICATIONS OF THE INVERSE ZENO EFFECT

The above conclusions are correct in the (mathematical) limit of large N . However, it is known that, if N is not too large, the form factors of the interaction play a primary role and can provoke an *inverse Zeno effect* (IZE), by which the decohering evolution is accelerated, rather than suppressed [14,15]. Reconsider the example (17), with B coupling Q to a generic bath with a thermal spectral density

$$\kappa(\omega) = \int dt \exp(i\omega t) \langle B(t)B \rangle, \quad (21)$$

where $B(t) = e^{iH_B t} B e^{-iH_B t}$ is the interaction-picture-evolved bath operator, H_B the free bath Hamiltonian, and $\langle \cdots \rangle$ the average over the bath state. For instance, one can consider the linear coupling $B = \int d\omega f(\omega) [a(\omega) + a^{\dagger}(\omega)]$, where $[a(\omega), a^{\dagger}(\omega')] = \delta(\omega - \omega')$ are boson operators and $f(\omega)$ a form factor, while $H_B = \int d\omega \omega a^{\dagger}(\omega) a(\omega)$. The form factor of the interaction (together with the bath state) determines the spectral density (21). For instance, for an Ohmic bath,

$$\kappa(\omega) \propto \frac{\omega}{[1 + (\omega/\omega_c)^2]^n} \coth\left(\frac{\omega}{2T}\right), \quad (22)$$

where ω_c is the frequency cutoff, T the temperature of the bath (Boltzmann's constant $k=1$), and n an integer $n \geq 2$ ($n=2$ is typical of quantum dots [17]). The free decay rate is

$$\gamma = 2\pi\kappa(\omega_0), \quad (23)$$

ω_0 being the energy difference between the two qubit states (Fermi golden rule). The modified decay rate can be shown to read [4,18]

$$\gamma(\tau) = \lim_{t \rightarrow \infty} \int_{-\infty}^{\infty} d\omega \kappa(\omega) \text{sinc}^2\left(\frac{\omega - \omega_0}{2} t\right) \tan^2\left(\frac{\omega - \omega_0}{4} \tau\right), \quad (24)$$

where $\tau = t/N$ is the period between kicks and $\text{sinc}(x) \equiv x^{-1} \sin x$. By expanding for large values of N one gets [19]

$$\gamma(\tau) \sim \frac{8}{\pi} \kappa\left(\frac{2\pi}{\tau}\right), \quad \tau \rightarrow 0. \quad (25)$$

Notice that, according to Eq. (25), for small values of τ the modified decay rate $\gamma(\tau)$ is proportional to the “tail” of the spectral density $\kappa(\omega)$. By defining a characteristic *transition time* τ^* as the solution of the equation

$$\kappa\left(\frac{2\pi}{\tau^*}\right) \approx \frac{\pi}{8} \gamma = \frac{\pi^2}{4} \kappa(\omega_0), \quad (26)$$

one obtains

$$\begin{aligned} \gamma(\tau) &< \gamma & \text{for } \tau < \tau^*, \\ \gamma(\tau) &> \gamma & \text{for } \tau > \tau^*. \end{aligned} \quad (27)$$

Decoherence is suppressed in the former case, but it is *enhanced* in the latter situation (which is analogous to what one calls the IZE in the case of projective measurements). This shows that an “inverse Zeno regime” is a serious drawback also in the case of dynamical decoupling. Since the limit $\tau < \tau^*$ can be very difficult to attain, for a *bona fide* dissipative system, the efficacy of the BB as a method for decoherence suppression must be carefully analyzed. For instance, in the Ohmic case (22) at low temperature $T \ll \omega_0 \ll \omega_c$, one easily gets, from Eq. (25)

$$\tau^* \approx 2\pi\omega_c^{-1} \left(\frac{\pi^2 \omega_0}{4 \omega_c}\right)^{1/(2n-1)} \ll 2\pi\omega_c^{-1}, \quad (28)$$

a condition that may be difficult to achieve in practice. In fact, we see here that the relevant time scale is not simply the inverse bandwidth ω_c^{-1} , but can be much shorter if $\omega_0 \ll \omega_c$, as is typically the case. It has already been observed that the Ohmic bath is a particularly demanding setting for the BB and that spin-boson baths with decaying spectral density $I(\omega)$ [not to be confused with the thermal spectral density $\kappa(\omega)$], such as $1/f$, are more amenable to successful BB decoupling [18]. We will reconsider this issue from the point of view of the IZE in [19].

IV. BB CYCLE OF SEVERAL PULSES

We now generalize the previous result to the situation where each cycle consists of g kicks. This will allow us to show how the procedure of “decoupling by symmetrization” [6]—i.e., the standard view of the BB effect—arises as a special case of such cycles and is related to the QZE. We consider N cycles of g instantaneous kicks U_1, \dots, U_g in a time interval t :

$$U_N(t) = \left[U_g U \left(\frac{t}{gN} \right) \cdots U_2 U \left(\frac{t}{gN} \right) U_1 U \left(\frac{t}{gN} \right) \right]^N. \quad (29)$$

We use the same notation as above, sometimes abbreviating $U(t/gN)$ by U , unless confusion may arise. Similarly to the single-kick case, in the $N \rightarrow \infty$ limit, the dominant contribution is $(U_g \cdots U_2 U_1)^N$ and it is convenient to consider the sequence of unitary operators

$$V_N(t) = (U_g \cdots U_1)^{\dagger N} U_N(t). \quad (30)$$

The differential equation is again

$$i \frac{d}{dt} V_N(t) = H_N(t) V_N(t) \quad [V_N(0) = I], \quad (31)$$

where

$$H_N(t) = \frac{1}{N} \sum_{k=0}^{N-1} (U_g \cdots U_1)^{\dagger k} (U_g U \cdots U_1 U)^k \times \bar{H}_N (U_g U \cdots U_1 U)^{\dagger k} (U_g \cdots U_1)^N, \quad (32)$$

with

$$\begin{aligned} \bar{H}_N = & \frac{1}{g} [U_g H U_g^\dagger + (U_g U U_{g-1}) H (U_g U U_{g-1})^\dagger + \cdots \\ & + (U_g U U_{g-1} \cdots U_2 U U_1) H (U_g U U_{g-1} \cdots U_2 U U_1)^\dagger]. \end{aligned} \quad (33)$$

We can now follow through the same calculations as in the single-kick case, substituting $U_1 H U_1^\dagger$ everywhere by \bar{H}_N and U_1 by $U_g \cdots U_1$. It is then straightforward to verify that in the $N \rightarrow \infty$ limit we get

$$\mathcal{U}(t) \equiv \lim_{N \rightarrow \infty} V_N(t), \quad (34)$$

which again satisfies Eq. (6), with the Zeno Hamiltonian

$$H_Z = \Pi_{U_g \cdots U_1}(\bar{H}) = \sum_{\mu} P_{\mu} \bar{H} P_{\mu}, \quad (35)$$

where

$$U_g \cdots U_2 U_1 = \sum_{\mu} P_{\mu} e^{-i\lambda_{\mu}}, \quad (36)$$

$$\begin{aligned} \bar{H} = & \frac{1}{g} [H + \cdots + (U_{g-2} \cdots U_1)^\dagger H (U_{g-2} \cdots U_1) \\ & + (U_{g-1} \cdots U_1)^\dagger H (U_{g-1} \cdots U_1)]. \end{aligned} \quad (37)$$

In conclusion,

$$\begin{aligned} U_N(t) & \sim (U_g \cdots U_1)^N \mathcal{U}(t) \\ & = (U_g \cdots U_1)^N \exp(-iH_Z t) \\ & = \exp\left(-i \sum_{\mu} (N\lambda_{\mu} P_{\mu} + P_{\mu} \bar{H} P_{\mu} t)\right). \end{aligned} \quad (38)$$

It is clear that also in this case we get a QZE, with relevant Zeno subspaces [13]. The only difference from the single-kick case is that the Hamiltonian \bar{H} [Eq. (37)] and the product of the cycle $U_g \cdots U_2 U_1$ [Eq. (36)] take the place of H and U_1 , respectively.

It is important to observe again that *no symmetry* or group structure is required from the “kick” sequence (29): the above formulas are of general validity, as they rely on the von Neumann ergodic theorem. They reduce to the usual expression in the case of a finite closed group of unitaries \mathcal{G} with elements V_r , $r=1, \dots, g$ and $V_1=I$. Indeed, decoupling by symmetrization [6] is recovered as a particular case by considering the unitary operators

$$U_r = V_{r+1} V_r^\dagger \quad (r=1, \dots, g-1), \quad U_g = V_g^\dagger. \quad (39)$$

A single cycle yields

$$U_{\text{cycle}}(t) = V_g^\dagger U \left(\frac{t}{gN} \right) V_g \cdots V_1^\dagger U \left(\frac{t}{gN} \right) V_1, \quad (40)$$

while

$$U_g \cdots U_1 = V_g^\dagger V_g V_{g-1}^\dagger \cdots V_2^\dagger V_2 = I. \quad (41)$$

We therefore reobtain, as a special case of the QZE, the well-known BB result [6]

$$U_N(t) = V_N(t) \sim \exp(-iH_{\text{eff}} t), \quad (42)$$

where $H_{\text{eff}} = H_Z$ and

$$H_Z = \Pi_I(\bar{H}) = \bar{H} = \frac{1}{g} \sum_{r=1}^g V_r^\dagger H V_r = \Pi_{\mathcal{G}}(H). \quad (43)$$

V. ORIGIN OF THE EQUIVALENCE BETWEEN CONTINUOUS AND PULSED FORMULATIONS

The equivalence between the ways in which the QZE can be generated via observation and via Hamiltonian interaction have been discussed in [13]. We now explain the equivalence between the continuous and pulsed Hamiltonian interaction pictures in generating the Zeno subspaces. In fact, the two procedures differ only in the order in which two limits are computed. We recall that the continuous case deals with the strong-coupling limit [13]

$$H_{\text{tot}} = H + K H_1, \quad K \rightarrow \infty, \quad (44)$$

and the Zeno subspaces are the eigenspaces of H_1 . On the other hand, the kicked dynamics entails the limit $N \rightarrow \infty$ in Eq. (1) and the Zeno subspaces are the eigenspaces of U_1 . This evolution is generated by the Hamiltonian

$$H_{\text{tot}} = H + \tau_1 H_1 \sum_n \delta(t - n\tau_2), \quad \tau_2 \rightarrow 0, \quad (45)$$

where τ_2 is the period between two kicks and the unitary evolution during a kick is $U_1 = \exp(-i\tau_1 H_1)$. The limit $N \rightarrow \infty$ in Eq. (1) corresponds to $\tau_2 \rightarrow 0$. The two dynamics (44) and (45) are both limiting cases of

$$H_{\text{tot}} = H + KH_1 \sum_n g\left(\frac{t - n(\tau_2 + \tau_1/K)}{\tau_1/K}\right), \quad (46)$$

where the function g has the properties

$$\sum_n g(x - n) = 1, \quad (47)$$

$$\lim_{K \rightarrow \infty} Kg(Kx) = \delta(x). \quad (48)$$

For example, we can consider $g(x) = \chi_{[-1/2, 1/2]}(x)$, where χ_I is the characteristic function of the set I . In Eq. (46) the period between two kicks is $\tau_1/K + \tau_2$, while the kick lasts for a time τ_1/K . By taking the limit $\tau_2 \rightarrow 0$ in Eq. (46)—i.e., a sequence of pulses of finite duration τ_1/K without any idle time among them—and using property (47), one recovers the continuous case (44). Then, by taking the strong-coupling limit $K \rightarrow \infty$ one gets the Zeno subspaces. On the other hand, by taking the $K \rightarrow \infty$ limit—i.e., the limit of shorter pulses [but with the same global (integral) effect]—and using property (48) and the identity $\delta(t/\tau_1) = \tau_1 \delta(t)$, one obtains the kicked case (45). Then, by taking the vanishing idle time limit $\tau_2 \rightarrow 0$ one gets again the Zeno subspaces. In short, the mathematical equivalence between the two approaches is expressed by the relation

$$\lim_{K \rightarrow \infty} \lim_{\tau_2 \rightarrow 0} H_{\text{tot}} = \lim_{\tau_2 \rightarrow 0} \lim_{K \rightarrow \infty} H_{\text{tot}} \quad (49)$$

(for almost all τ_1), with the left (right) side expressing the continuous (pulsed) case. Note that this formal equivalence must physically be checked on a case-by-case basis and it is legitimate only if the inverse Zeno regime is avoided and the role of the form factors clearly spelled out. That is, physically the relevant time scales play a crucial role, and in practice there certainly can be a difference between kicked dynamics and continuous coupling, in spite of their equivalence in the above mathematical limit.

Another key issue of physical relevance, in particular if one is interested in possible applications, is played by the physical meaning of “strong” when one talks of the strong-coupling regime. We showed that strong coupling is equivalent to large N (number of interruptions) and, since experiments with large N have been performed, proving both the quantum Zeno and inverse quantum Zeno effects [15], the strong-coupling regime is attainable in real physical systems.

VI. CONCLUSIONS

In this work we have shown the formal equivalence of the quantum Zeno effect, which has been known since von Neumann laid down the mathematical foundations of quantum mechanics ([20], p. 366) and has been the subject of intense investigations since the seminal paper [9], to the recently introduced [4–6] “bang-bang” decoupling method for reducing decoherence in quantum information processing.² The QZE is traditionally derived by considering a series of rapid, *pulsed observations* [9]. This became almost a dogma and motivated interesting seminal experiments [15,21]. Later formulations emphasized that the QZE can also be generated by *continuous Hamiltonian interaction* [12,13,22]. The BB method, on the other hand, employs a series of rapid *pulsed interactions*. Here we have shown that both the QZE (in its continuous-interaction formulation) and the BB method can be understood as limits of a single Hamiltonian, Eq. (46), giving rise to either pulsed or continuous dynamics, with a resulting partitioning of the controlled system’s Hilbert space into quantum Zeno subspaces, defined by Eqs. (9) and (10). This unified view not only offers the advantage of conceptual simplicity, but also has significant practical consequences: it shows that the scope of all methods analyzed here (QZE, BB, and continuous interaction) are wider than previously suspected, leading to greater flexibility in their implementation. In particular, since all formulations of the QZE are physically equivalent and the BB is equivalent to the kicked unitary formulation of the QZE, it is clear that the BB can also be formulated in terms of a *continuous interaction* and *pulsed measurements*. The continuous-interaction version of the BB avoids the frequently criticized off-resonant transitions associated with the large bandwidth pulses required in the pulsed BB implementation [23]. We have not studied the practical advantages or drawbacks of the pulsed measurement formulation of the BB.

We emphasize that our conclusions about greater flexibility in the practical implementation of the BB method are supported by the fact that experiments with large N have been performed, proving both the quantum Zeno [15,21] and the inverse quantum Zeno [15] effects and showing that the strong-coupling regime is attainable in real physical systems.

Another consequence of our work is that the Zeno-subspace dynamics, in its pulsed formulation, can be generated by a sequence of *arbitrary* (fast and strong) pulses, without any (symmetry) assumptions about the relation between pulses. This generalizes all previously published formulations of the BB method, which assumed such relations.

Finally, owing perhaps to its longer history, the QZE has been more thoroughly studied than the BB method, and it has

²In fact the original BB paper [4] recognized the mathematical connection to the QZE, in particular the features of Cook’s method for the inhibition of a stimulated two-level transition by pulsed measurements [21], but stated that “the analogy stops from a more physical point of view.”

been recognized that in physically relevant limits an inverse QZE can arise. We have shown that the same conclusion applies to the BB method, with the important implication that in some cases the BB can actually enhance, rather than reduce, decoherence. This issue will be the subject of further investigations [19].

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