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Quantum dephasing by chaos

Hiroimichi Nakazato^a, Mikio Namiki^a, Saverio Pascazio^{b,c}, Yoshiya Yamanaka^a

^a Department of Physics, Waseda University, Tokyo 169, Japan

^b Dipartimento di Fisica, Università di Bari, Bari, Italy

^c Istituto Nazionale di Fisica Nucleare, Sezione di Bari I-70126 Bari, Italy

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Abstract

We examine whether the chaotic behavior of classical systems with a limited number of degrees of freedom can produce quantum dephasing, against the conventional idea that dephasing takes place only in large systems with a huge number of constituents and complicated internal interactions. On the basis of this analysis, we briefly discuss the possibility of defining quantum chaos and of inventing a “chaos detector”.

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Quantum dephasing is a central issue in quantum measurements. Many physicists used to think that quantum dephasing takes place as a result of the interaction with large systems endowed with a huge number of elementary constituents and complicated internal interactions. Usually, some randomness associated with large systems is considered to play an important role in this context. See the discussions on this matter within the framework of the many-Hilbert space theory [1].

It is now known that a class of classical systems with *a few degrees of freedom* behaves chaotically due to nonlinear dynamics [2]. This fact leads us to expect that the interactions between a quantum particle and a classical system with a limited number of degrees of freedom, if the latter is in chaotic motion, may give rise to quantum dephasing on the former. In this note, we shall investigate the possibility of such a kind of mechanism (yielding quantum dephasing by chaos arising from a classical system with a few degrees of freedom), and shall see from simple model calculations that this can indeed happen. We also discuss the issue of quantum chaos as a natural extension of this kind of argument, and look for a way to invent a new type of “chaos” detector.

Let us start by introducing the notion of dephasing from the measurement-theoretical point of view. Consider a typical yes–no experiment of the Stern–Gerlach type as schematically shown in Fig. 1. The whole measurement process is usually decomposed in two steps: the first being the spectral decomposition and the second the detection. Each incoming particle emitted by E and represented by a wave packet ψ_0 is separated by V into two branch waves ψ_A and ψ_B , running in channels A and B, respectively, corresponding to mutually-exclusive measurement propositions \mathcal{A} and \mathcal{B} . This is nothing but the spectral-decomposition step in which the phase

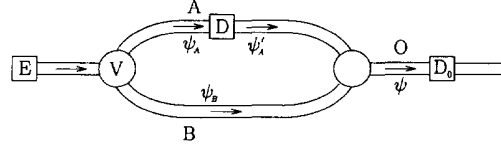


Fig. 1. Typical yes-no experimental setup of Stern-Gerlach type.

correlation between the two branch waves is kept. This is followed by the detection step at detector D placed in channel A. Suppose that ψ_A is changed to ψ'_A by passing through D. If we have a coincidence (an anti-coincidence) signal between E and D, we get an affirmative (negative) answer to \mathcal{A} . Here we have considered D to be a perfect detector, in the sense that it completely destroys the phase correlation between the two branch waves ψ_A and ψ_B , that is, we have perfect dephasing between them.

Suppose that D is simply an instrument, but not necessarily a perfect detector. In this case, in general, we have a semi-coherent and semi-mixed case, as will presently be seen. If the two branch wave packets are guided into the final channel O to make a superposed state $\psi = \psi'_A + \psi_B$ (for simplicity, we have used the same notation for channel O as in channels A and B), the probability of observing the particle by a perfect detector D_0 , placed in channel O, reads

$$P = \int_{\Omega} |\psi|^2 dx = (\psi'_A, \psi'_A) + (\psi_B, \psi_B) + 2 \text{Re}(\psi_B, \psi'_A), \quad (1)$$

where Ω is the nonvanishing support of the wave packets. Usually, we perform a quantum-mechanical observation by sending many incoming particles (their total number, in an experimental run, being N_p) through a steady (and very weak) incident beam into the target (D in this case), and by accumulating many results obtained in such a run. If D is a perfect detector, that is, if it gives perfect dephasing between ψ'_A and ψ_B , the third term (the interference term) of (1) disappears for the accumulated distribution. If, on the other hand, D fully keeps the phase correlation between the branch waves, D_0 will read a perfect interference pattern. Otherwise, we obtain an imperfect measurement, yielding a semi-coherent and semi-mixed case. This way one can see, by making use of D_0 , whether D works well or not as a quantum detector.

In a conventional measurement process, each incoming particle will meet, particle by particle, a different local system. In other words, the l th particle will interact with the l th local system and, correspondingly, get a transmission coefficient $T_{(l)}$ when it passes through D. Furthermore, suppose that we can consider the measurement process as a one-dimensional collision process, so that we can safely set $\psi_B \simeq e^{ikx}$ and $\psi'_{A(l)} \simeq T_{(l)}e^{ikx}$, as good approximations. (Notice that e^{ikx} symbolically represents a wave packet close to a plane wave.) In this approximation scheme the accumulated distribution of results obtained by D_0 is given by

$$\overline{P} = 1 + t + 2\sqrt{t(1-\epsilon)} \cos(\arg \overline{T}), \quad (2)$$

where t and ϵ are, respectively, the transmission probability and the *decoherence* parameter, defined by

$$t \equiv |\overline{T}|^2, \quad \epsilon \equiv 1 - \frac{|\overline{T}|^2}{|\overline{T}|^2}, \quad (3)$$

with

$$\overline{\dots} \equiv \frac{1}{N_p} \sum_{l=1}^{N_p} (\dots)_{(l)}. \quad (4)$$

Here, of course, ϵ is a positive number between 0 and 1, and $\epsilon = 1$ corresponds to a perfect measurement (dephasing), $\epsilon = 0$ to perfect interference (coherence), and its intermediate values to imperfect measurements

(partially coherent and partially mixed states). Thus the value of ϵ gives us a criterion to judge how well the instrument D can work as a measuring apparatus [1]. In particular, the notion of dephasing can be expressed by

$$\text{Re}[\overline{\psi'_A \psi_B^*}] = 0, \quad \text{or } \overline{T} = 0, \quad \text{or equivalently } \epsilon = 1, \quad (5)$$

provided that $|\overline{T}|^2 \neq 0$.

Let us now turn to a different situation from the conventional setup of the measurement problem, in which the instrument D consists of *only one* particle subject to classical dynamics. We examine whether this instrument D causes quantum dephasing or not, and, if yes, under which circumstances. For simplicity, we consider that this classical particle (to be called D-particle) interacts with an incoming quantum particle via a potential, its position being the center of the potential. Furthermore, all recoil effects and internal structures of the D-particle are neglected. In such a case, the motion of the l th incoming particle is described by the potential $V_{(l)} \equiv V(\mathbf{r} - \mathbf{r}_{(l)}(t))$ where $\mathbf{r}_{(l)}$ is the position of the D-particle when the l th incoming particle meets the D-particle. Our crucial assumption is that the D-particle moves *chaotically*. Because of the l -dependence of the chaotic potential center, we have to add the subscript (l) to P and to ψ'_A in (1). As the scatterer consists of a single particle and is not expected to be large in size, it is not appropriate to treat our present problem approximately as a one-dimensional collision process. This means that the above formula (2) in terms of T no longer holds, but we will be able to formulate the *decoherence* parameter in this case along a similar line of thought, as will be shown later. This new *decoherence* parameter preserves its original interpretation: If dephasing takes place, the *decoherence* parameter is equal to unity, and then the above instrument D is considered to work well as a quantum detector.

In what follows, for simplicity, we shall suppress the subscript A of the branch wave. Of course, we shall keep the subscript (l) for the branch wave of the l th incoming particle in channel A. We can write down the Schrödinger equation for the l th wave packet running in channel A as

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r} - \mathbf{r}_{(l)}(t)) \right) \psi_{(l)}(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi_{(l)}(\mathbf{r}, t). \quad (6)$$

Note again that $\mathbf{r}_{(l)}(t)$ stands for the center of the potential at time t . It is easy to see that the translation operator $\exp[(i/\hbar)\hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}(t)]$ reduces the above Eq. (6) to

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + i\hbar \mathbf{v}_{(l)} \cdot \nabla \right) \overline{\psi}_{(l)}(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \overline{\psi}_{(l)}(\mathbf{r}, t), \quad (7)$$

where $\mathbf{v}_{(l)} = \dot{\mathbf{r}}_{(l)}$ and

$$\overline{\psi}_{(l)}(\mathbf{r}, t) = \exp[(i/\hbar)\hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}(t)] \psi_{(l)}(\mathbf{r}, t). \quad (8)$$

In this note, we shall consider only two extreme cases: (i) adiabatic change, and (ii) rapid change.

Adiabatic change case. If the center of the potential moves very slowly during the passage of each incoming wave packet, we can safely consider the center fixed in each scattering process. In this case we can put $\mathbf{v}_{(l)} = 0$, so that

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \overline{\psi}_{(l)}(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \overline{\psi}_{(l)}(\mathbf{r}, t). \quad (9)$$

Here $\overline{\psi}_{(l)}$ simply becomes a wave function for the scattering process by a fixed potential $V_0 = V(\mathbf{r})$, whose center is located at the origin, so that we can omit the subscript (l) . (Recall that we are excluding the case in

which the inner motion of the scatterer may give rise to an additional l -dependence.) If we deal with a wave packet very close to a plane wave, we can put

$$\psi_{(l)}(\mathbf{r}, t) \simeq \exp\left(-\frac{i}{\hbar} E_k t\right) u_{\mathbf{k}(l)}^{(+)}(\mathbf{r}), \quad (10)$$

$$\bar{\psi}_{(l)}(\mathbf{r}, t) \simeq \exp\left(-\frac{i}{\hbar} E_k t\right) \bar{u}_{\mathbf{k}(l)}^{(+)}(\mathbf{r}), \quad (11)$$

during the passage of the wave packet, where $E_k = \hbar^2 k^2 / 2m$ and

$$u_{\mathbf{k}(l)}^{(+)}(\mathbf{r}) = \exp\left(-\frac{i}{\hbar} \hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}\right) \bar{u}_{\mathbf{k}(l)}^{(+)}(\mathbf{r}). \quad (12)$$

Here $u_{\mathbf{k}(l)}^{(+)}(\mathbf{r})$ and $\bar{u}_{\mathbf{k}(l)}^{(+)}(\mathbf{r})$ are, respectively, the outgoing solutions of

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r} - \mathbf{r}_{(l)})\right) u_{\mathbf{k}(l)}^{(+)}(\mathbf{r}) = E_k u_{\mathbf{k}(l)}^{(+)}(\mathbf{r}) \quad (13)$$

and

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})\right) \bar{u}_{\mathbf{k}(l)}^{(+)}(\mathbf{r}) = E_k \bar{u}_{\mathbf{k}(l)}^{(+)}(\mathbf{r}). \quad (14)$$

Note that $\mathbf{r}_{(l)}$ is independent of time and that $\bar{u}_{\mathbf{k}(l)}^{(+)}$ has an $\mathbf{r}_{(l)}$ -dependent constant phase in order to match the boundary conditions for $u_{\mathbf{k}(l)}^{(+)}$ and $\bar{u}_{\mathbf{k}(l)}^{(+)}$, both of which are subject to the plane-wave normalization. Taking into account that $u_{\mathbf{k}(l)}^{(+)} = W_{(l)} u_{\mathbf{k}(l)}^{(0)}$ and $T_{(l)} = V_{(l)} W_{(l)}$ ($W_{(l)}$ and $T_{(l)}$ being the W - and T -matrices for the potential $V_{(l)}$, respectively), we can write down the scattering amplitude as

$$F_{(l)}(\mathbf{k}', \mathbf{k}) = -\frac{4\pi^2 m}{\hbar^2} (u_{\mathbf{k}'}^{(0)}, T_{(l)} u_{\mathbf{k}}^{(0)}) = -\frac{4\pi^2 m}{\hbar^2} \exp(-i\mathbf{K} \cdot \mathbf{r}_{(l)}) (u_{\mathbf{k}'}^{(0)}, T_0 u_{\mathbf{k}}^{(0)}), \quad (15)$$

where $\mathbf{K} = \mathbf{k}' - \mathbf{k}$ stands for the momentum transfer and T_0 for the T -matrix corresponding to the potential V_0 , because

$$V_{(l)} = V(\mathbf{r} - \mathbf{r}_{(l)}) = \exp\left(-\frac{i}{\hbar} \hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}\right) V_0 \exp\left(\frac{i}{\hbar} \hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}\right), \quad (16)$$

$$T_{(l)} = \exp\left(-\frac{i}{\hbar} \hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}\right) T_0 \exp\left(\frac{i}{\hbar} \hat{\mathbf{p}} \cdot \mathbf{r}_{(l)}\right). \quad (17)$$

These formulas are easily understood on the basis of the Born approximation

$$F_{(l)}(\mathbf{k}', \mathbf{k}) \simeq -\frac{m}{2\pi\hbar^2} \int d^3\mathbf{r} e^{-i\mathbf{K} \cdot \mathbf{r}} V(\mathbf{r} - \mathbf{r}_{(l)}) = -\frac{4\pi^2 m}{\hbar^2} e^{-i\mathbf{K} \cdot \mathbf{r}_{(l)}} (u_{\mathbf{k}'}^{(0)}, V_0 u_{\mathbf{k}}^{(0)}), \quad (18)$$

and its generalization

$$V_0 \rightarrow T_0 = V_0 + V_0 \frac{1}{E_k - \hat{H} + i\epsilon} V_0, \quad (19)$$

with the total Hamiltonian $\hat{H} = \hat{\mathbf{p}}^2 / 2m + V_0$.

If we deal with low energy scattering by a short-distance force, we can put

$$F_{(l)}(\mathbf{k}', \mathbf{k}) = -\exp(-i\mathbf{K} \cdot \mathbf{r}_{(l)}) kb, \quad (20)$$

where b is the scattering length.

Along the general line of thought given in Ref. [1], we introduce the *decoherence* parameter in a three-dimensional scattering process

$$\epsilon \equiv 1 - \frac{|\int_{\Delta\omega} d\omega \overline{F}|^2}{\int_{\Delta\omega} d\omega |F|^2} = 1 - \left| \frac{\int_{\Delta\omega} d\omega \overline{\exp(-i\mathbf{K} \cdot \mathbf{r}_{(l)})}}{\int_{\Delta\omega} d\omega} \right|^2, \quad (21)$$

where $\Delta\omega = (\Delta\theta, \Delta\varphi)$ stands for the solid angle around $\omega_0 = (\theta_0, \varphi_0)$ under which the scatterer sees the detector. Clearly this ϵ serves as a quantitative measure of the degree of quantum dephasing as in the one-dimensional case. Notice that this ϵ depends on θ_0 and φ_0 in general.

In order to estimate $\exp(-i\mathbf{K} \cdot \mathbf{r}_{(l)})$, consider that $\mathbf{r}_{(l)}$ is the resultant point of a random walk, $\mathbf{r}_{(1)} \rightarrow \mathbf{r}_{(2)} \rightarrow \dots$, according to the theory of classical chaos [2]. Therefore, if the incident beam is steady and very weak, we can use the Gaussian law with characteristic length ΔL for the distribution of $\mathbf{r}_{(l)}$, or, in other words, we can replace the above bar-averaged quantity $\exp(-i\mathbf{K} \cdot \mathbf{r}_{(l)})$ with

$$\exp[-N_p \frac{1}{2} (\Delta L)^2 K^2] = \exp[-N_p (\Delta L)^2 k^2 (1 - \cos \theta)], \quad (22)$$

where $K = 2k \sin(\theta/2)$.

The angle-integrals are computed in the following way,

$$\int_{\Delta\omega} d\omega = \Delta\varphi [\cos \theta_0 - \cos(\theta_0 + \Delta\theta)] \simeq \Delta\varphi [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0], \quad (23)$$

$$\begin{aligned} \int_{\Delta\omega} \exp[-N_p (\Delta L)^2 k^2 (1 - \cos \theta)] d\omega &= \Delta\varphi \exp[-N_p (\Delta L)^2 k^2] \int_{\cos(\theta_0 + \Delta\theta)}^{\cos \theta_0} \exp[N_p (\Delta L)^2 k^2 \xi] d\xi \\ &= \frac{\Delta\varphi \exp[-N_p (\Delta L)^2 k^2 (1 - \cos \theta_0)]}{N_p (\Delta L)^2 k^2} (1 - \exp\{-N_p (\Delta L)^2 k^2 [\cos \theta_0 - \cos(\theta_0 + \Delta\theta)]\}) \\ &\simeq \frac{\Delta\varphi \exp[-N_p (\Delta L)^2 k^2 (1 - \cos \theta_0)]}{N_p (\Delta L)^2 k^2} (1 - \exp\{-N_p (\Delta L)^2 k^2 [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0]\}). \end{aligned} \quad (24)$$

Thus we obtain

$$\epsilon \simeq 1 - \left(\frac{\exp[-N_p (\Delta L)^2 k^2 (1 - \cos \theta_0)] (1 - \exp\{-N_p (\Delta L)^2 k^2 [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0]\})}{N_p (\Delta L)^2 k^2 [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0]} \right)^2, \quad (25)$$

from which we conclude that

$$\epsilon \simeq 0 \quad \text{for } N_p (\Delta L)^2 k^2 [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0] \ll 1, \quad (26)$$

and

$$\epsilon \simeq 1 \quad \text{for } N_p (\Delta L)^2 k^2 [\Delta\theta \sin \theta_0 + \frac{1}{2} (\Delta\theta)^2 \cos \theta_0] \gg 1. \quad (27)$$

We get coherence in the case (26), and dephasing in the case (27).

Consequently, we arrive at the conclusion that the chaotic motion of a classical system can generate dephasing, for sufficiently large ΔL , even though it has very few degrees of freedom, or, alternatively, when the classical system has reached a well-developed (“aged”) stage, i.e., $N_p \gg 1$ so that the replacement (22) becomes quite reasonable, after the interaction with many incoming particles. This suggests the possibility of inventing a new type of quantum detector, by making use, for instance, of a “randomly oscillating mirror”. On the contrary, we

observe coherence for very small ΔL , or when the system is in the developing stage, before “aging” ($N_p \simeq 1$), even though in the latter case we have no sound reasoning of the replacement (22).

The dependence of the decoherence parameter on N_p , the number of particles in an experimental run, may be an interesting feature in our opinion. Such a possibility was previously envisaged within the framework of the many-Hilbert-space approach (see the last paper in Ref. [1]), but only in the trivial case of complete dephasing. On the contrary, Eq. (25) displays a nontrivial N_p -dependence. Such a feature is not known in other theories of measurement, and in particular is absent in the “naive” Copenhagen interpretation. Notice that (25) is a rather general expression, in that it does not imply any dependence on details of the interaction (the potential V). Indeed, such a dependence simplifies out in (21).

Rapid change case. Let us consider the case in which $\mathbf{r}_{(l)}(t)$ in (6) changes very rapidly, during the passage of the l th incoming wave packet. In this case, we may first replace the potential term with

$$\langle V(\mathbf{r} - \mathbf{r}_{(l)}(t)) \rangle \equiv \frac{1}{\tau} \int_t^{t+\tau} dt V(\mathbf{r} - \mathbf{r}_{(l)}(t)) = \int d^3\mathbf{r}' V(\mathbf{r} - \mathbf{r}') w_{(l)}(\mathbf{r}', t), \quad (28)$$

$$w_{(l)}(\mathbf{r}', t) \equiv \frac{1}{\tau} \int_t^{t+\tau} dt \delta(\mathbf{r}' - \mathbf{r}_{(l)}(t)), \quad (29)$$

where τ is the passage time of the wave packet. Furthermore, let us restrict ourselves to the situation in which $w_{(l)}(\mathbf{r}', t)$ can be replaced with a statistical distribution $\mathcal{W}_{(l)}(\mathbf{r}' - \mathbf{R}_{(l)})$ which has width $\sim \Delta \mathbf{R}_{(l)}$ around $\mathbf{R}_{(l)}$ and no time dependence except that through l . Under these circumstances, we are allowed to reduce our scattering problem effectively to that of an average potential given by

$$\mathcal{V}_{(l)}(\mathbf{r} - \mathbf{R}_{(l)}) = \int V(\mathbf{r} - \mathbf{r}') \mathcal{W}_{(l)}(\mathbf{r}' - \mathbf{R}_{(l)}) d^3\mathbf{r}', \quad (30)$$

leading to the effective Schrödinger equation

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + \mathcal{V}_{(l)}(\mathbf{r} - \mathbf{R}_{(l)}) \right) \psi_{(l)}(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi_{(l)}(\mathbf{r}, t). \quad (31)$$

It is remarked that in using this equation we are neglecting some sort of higher-order fluctuation effects on the Schrödinger wave function.

For the particular case $V(\mathbf{r}) = (2\pi\hbar^2/m)b\delta(\mathbf{r})$ (i.e. Yang’s approximation, b being the scattering length), we further reduce (30) to

$$\mathcal{V}_{(l)}(\mathbf{r} - \mathbf{R}_{(l)}) = \frac{2\pi\hbar^2}{m} b \mathcal{W}_{(l)}(\mathbf{r} - \mathbf{R}_{(l)}) \quad (32)$$

as a good approximation, because the force range of V is much shorter than $|\Delta \mathbf{R}_{(l)}|$.

We now find a parallelism between the adiabatic change case and the rapid case in the above approximation, with correspondence between $\mathbf{r}_{(l)}$ and $V(\mathbf{r} - \mathbf{r}_{(l)})$ (see (6)) in the former and $\mathbf{R}_{(l)}$ and $\mathcal{V}_{(l)}(\mathbf{r} - \mathbf{R}_{(l)})$ in the latter. Note that $\mathbf{R}_{(l)}$ and $\mathbf{r}_{(l)}$ are assumed constant in both cases. Therefore, we can extend the arguments on the conditions for quantum dephasing in the adiabatic change case to the present one as well.

It should be remarked that $\mathcal{W}_{(l)}$, in many practical cases, describes a very dilute and broad distribution, in which we can regard (32) as a constant potential with strength $(2\pi\hbar^2/m)b\mathcal{W}_{(l)}(0)$ ($\mathcal{W}_{(l)}(0) \simeq |\Delta \mathbf{R}_{(l)}|^{-3}$) over a spatial region of a wide spread $|\Delta \mathbf{R}_{(l)}|$. In this case, we can easily estimate the scattering phase shift χ by

$$\chi \simeq -\frac{\lambda b}{|\Delta \mathbf{R}_{(l)}|^2}. \quad (33)$$

Here λ is the particle wavelength and $\rho \simeq |\Delta \mathbf{R}_{(t)}|^{-3}$ stands for the scatterer density. For very large $|\Delta \mathbf{R}_{(t)}|$, this phase shift becomes very small. This means that we can hardly observe quantum dephasing in this case. In conclusion, this type of instrument is nothing but a phase shifter, which can never yield quantum dephasing.

We have so far discarded possible effects caused by the recoil of the scatterer. In order to take these recoil effects into account, we just have to reformulate the scattering amplitude in the above discussion, in an appropriate way, according to the quantum theory of scattering. In this way, we can discuss the following two possibilities.

Quantum chaos. Consider the case in which both incoming and target particles are quantum-mechanical. (Note that the target particle has been treated as a classical particle in the above discussion.) The formalism described above still holds if we use the quantum mechanical scattering amplitude for the collision between incoming and target particles. Within this framework, we may be able to reach the notion of “quantum chaos”, for the *target particle state*, via the observation of “quantum dephasing” of the scattering amplitude in the above sense. On the other hand, Saito [3] suggested that quantum chaos can arise from possible random phases of the quantum-mechanical scattering amplitude in the path-integral representation. This idea may be realized by replacing the T -matrix in our formula (15) with a related one in the path-integral representation, in particular with those in the WKB approximation. This, as a natural extension of the present approach, is a promising means to open a doorway into quantum chaos.

Chaos detector. If we can detect the above-mentioned recoil effect as a signal, we have the possibility of making detectors that contain only a few constituents, for example, by means of a randomly moving mirror. Detectors of this kind are quite new; conventional detectors have a huge number of constituents. Even though we know that the generation and detection of such a signal poses difficult problems, one such possibilities would be to utilize the Fourier analysis of the response functions in momentum space. If the detector D is characterized by a large value of the decoherence parameter ϵ , we can catch the signal information by observing the Fourier spectrum of the correlation of the wave functions, defined by $(\psi_B, I(k)\psi'_A)(t)$, where $I(k)$ stands for a spectral function yielding the momentum components around k . We expect such a correlation function to depend strongly on ϵ , in particular when its values are close to unity (dephasing).

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