20 March 1995

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 199 (1995) 27-32

On the quantum Zeno effect

Hiromichi Nakazato [a], Mikio Namiki [b], Saverio Pascazio [c], Helmut Rauch [d]

[a] Department of Physics, University of the Ryukyus, Okinawa 903-01, Japan
[b] Department of Physics, Waseda University, Tokyo 169, Japan
[c] Dipartimento di Fisica, Università di Bari, and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, I-70126 Bari, Italy
[d] Atominstitut der Österreichischen Universitäten, A-1020 Vienna, Austria

Received 3 January 1995; accepted for publication 13 January 1995
Communicated by P.R. Holland

Abstract

The limit of infinitely frequent measurements (continuous observation), yielding the quantum Zeno paradox, is critically analyzed and shown to be unphysical. A specific example involving neutron spin is considered and some practical estimates are given.

A quantum mechanical system that is initially prepared in an eigenstate of the unperturbed Hamiltonian undergoes a temporal evolution that can be roughly divided into three parts: A Gaussian-like behavior at short times, a Breit–Wigner exponential decay at intermediate times, and a power law at long times [1–4].

The Gaussian short-time behavior is particularly significant because it leads, under general physical conditions, to the inhibition of the decay of unstable quantum mechanical systems [3–5]. This phenomenon was named *quantum Zeno paradox* by Misra and Sudarshan [5] [1].

This effect was not easily amenable to experimental test until Cook [7] proposed using atomic transitions in two-level atoms. On the basis of this idea, Itano and his group [8] recently carried out an interesting experiment that provoked a lively debate [9,10].

Nowadays most physicists tend to view this phenomenon as a purely dynamical process and refer to it as *quantum Zeno effect* (QZE) [9,10], rather than paradox. As the whole class of phenomena hinging upon the notion of "wave function collapse", the QZE is very interesting from the point of view of the quantum measurement theory [11–16] and is closely related to the partial beam detection experiments in neutron interferometry [17].

The purpose of the present Letter is just to investigate the QZE in *physical*, rather than mathematical, terms. We shall show that the uncertainty principle and the unavoidable losses that are always present in any physical apparatus impose remarkable limitations on the mathematical limit involved in the QZE. Consequently, the so-called limit of "continuous observation", namely the very possibility of performing infinitely frequent measurements on a quantum system, turns out to be an abstract idealization, void of physical meaning.

In this sense, we can say that the quantum Zeno *effect* becomes a real *paradox* when the limit of continuous observation is considered. In some sense, the main purpose of this Letter is to bring dreams down to earth, by clarifying in which sense and up to which approximations one can speak of QZE. Notice that some criticisms

[1] A recent account, in which one can find other references, is given by in Ref. [6]

SSDI 0375-9601(95)00037-2

against the physical meaning and realizability of the above-mentioned limit were put forward some years ago [4]. Although different from those to be considered in this Letter, those criticisms were based on the time–energy uncertainty relations.

In this context, it is useful to stress a very interesting (and peculiar) feature of the QZE [10]: The inhibition of the transition to states that are different from the initial one can be derived by assuming either that the evolution is dynamical (namely unitary) or that the collapse à la von Neumann takes place by observation. This remark is a confirmation, simplification and generalization of Prigogine et al.'s objection [9] against the claim that the observation of the QZE itself is an experimental confirmation of the projection rule.

The above-mentioned feature has been put forward in a particular example with neutron spin (to be shortly reviewed in the present Letter) and has been proven in full generality [10]. In some sense, the above observation makes the issues related to the QZE even more fascinating, because no conclusion can be drawn, at present, in support of any particular quantum measurement theory.

Let us start our analysis by briefly reviewing the essential ingredients necessary to obtain the QZE. Let Q be a quantum system, whose states are vectors in the separable Hilbert space $\mathscr{H}$ and whose evolution is described by the unitary operator $U(t)=\exp(-\mathrm{i}Ht)$, where H is a semi-bounded Hamiltonian.

We prepare Q in an unstable initial state at time 0 and exploit the Gaussian-like short-time behavior by performing a series of observations at times T/N, $2T/N$, ..., $(N-1)T/N$, T, in order to check whether Q is still undecayed. The measurements are idealized to be *instantaneous*.

Let $P^{(N)}(T)$ be the probability that the system is found undecayed after N measurements. In such a situation, we shall say that a quantum Zeno effect occurs if

$$P^{(N)}(T)>P^{(N')}(T) \quad \text{for } N>N' . \tag{1}$$

The above definition is meaningful only if the phase correlation among different branch waves is completely lost at each step ("dephasing" or "decoherence" due to a quantum measurement). This is thoroughly discussed in Ref. [10]. If, on the other hand, one performs only a "spectral decomposition" [13,14], in which the phase correlation is (partially) kept, the possibility of probability retrieval along one direction of the Hilbert space cannot be ruled out. In such a case, in order that the above definition make sense, one must make sure that two or more branch waves cannot be recombined. Notice also that, in practice, we can observe the "quantum Zeno effect" only if N is not too large (we shall discuss *how* large in the following).

In the $N\to\infty$ limit (continuous observation), it is possible to prove that the mathematical limit yields

$$\mathscr{P}(T)\equiv \lim_{N\to\infty} P^{(N)}(T)=1 . \tag{2}$$

This is the "quantum Zeno paradox": If Q is continuously observed (to check whether it decays or not), it is "frozen" in its initial state and will never be found to decay! Notice that T is kept finite in taking the above limit.

It is important to observe that the observations (measurements) are supposed to be *instantaneous*. This is a general characteristics of many descriptions of the measurement process: The Q system *instantaneously* collapses by measurement. Such a picture is very useful for computational purposes, but is rather misleading in the present context: Indeed a measurement process, as a physical process, takes place during a *very long time* on a *microscopic* scale, although we can regard it as if it happened *instantaneously* on a *macroscopic* scale. This problem will be thoroughly analyzed in this Letter.

We stress that in any conceivable experiment, only the QZE, with N finite (and rather small), can be observed. Our aim is to show that the $N\to\infty$ limit is physically unattainable *as a matter of principle*, and is rather to be regarded as a mathematical limit (although a very interesting one). In this sense, we shall say that the quantum Zeno *effect*, with N finite, becomes a quantum Zeno *paradox* when $N\to\infty$.

Let us now sketch an example that involves neutron spin and yields, in virtue of its simplicity, rich physical insight [10]. An analogous situation was outlined by Peres [18] with photons and has recently been experimentally realized by Zeilinger's group in Innsbruck [19].

We shall consider two different experiments, sketched in Fig. 1. In the first one, shown in Fig. 1a, the neutron interacts with several identical regions in which there is a static magnetic field B, oriented along

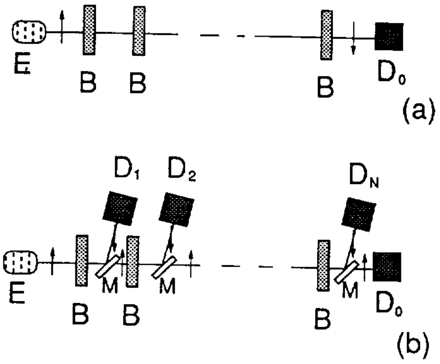

Fig. 1. (a) "Free" evolution of the neutron spin under the action of a magnetic field. An emitter E sends a spin-up neutron through several regions where a magnetic field B is present. Detector D_0 detects a spin-down neutron. (b) Quantum Zeno effect: the neutron spin is "monitored" at every step by selecting and detecting the spin-down component. D_0 detects a spin-up neutron.

the x-direction. We describe this interaction by the Hamiltonian $H=\mu B\sigma_1$, μ being the (modulus of the) neutron magnetic moment, and σ_1 the first Pauli matrix.

Let the initial neutron state be $\rho_0=\rho_{\uparrow\uparrow}\equiv|\uparrow\rangle\langle\uparrow|$, where $|\uparrow\rangle$ is the spin-up state along the z-axis. After crossing the region containing the magnetic field, the state of the neutron is

$$\rho(t)=\cos^2(\tfrac{1}{2}\omega t)\rho_{\uparrow\uparrow}+\sin^2(\tfrac{1}{2}\omega t)\rho_{\downarrow\downarrow}$$
$$-\mathrm{i}\cos(\tfrac{1}{2}\omega t)\sin(\tfrac{1}{2}\omega t)\rho_{\uparrow\downarrow}+\text{h.c.}, \qquad (3)$$

where $\omega=2\mu B/\hbar$, $t=l/v$ (l is the length of the region where B is present and v the neutron speed) and the other notation is obvious. We call this a "free" evolution, during which the system evolves under the sole influence of H.

If the neutron crosses the N regions in Fig. 1a, the final density matrix at time $T=Nt$ is still given by (3), but with T in place of t. (Notice that we are neglecting the time spent between adjacent regions containing B.) We design the experimental setup so that $\cos(\frac{1}{2}\omega T)=0$ (this "matching" condition is similar to the one experimentally realized by Itano et al. [8]),

$$Nt=T=(2m+1)\frac{\pi}{\omega}, \quad m\in\mathbb{N}. \qquad (4)$$

In such a case, the density matrix and the probability that the neutron spin is down at time T read respectively

$$\rho(T)=\rho_{\downarrow\downarrow}, \qquad (5)$$

$$P_\downarrow(T)=1. \qquad (6)$$

In our example, if the system is initially prepared in the up state, it will evolve to the down state after time T, just as if there were a compact field of length Nl, which causes spin reversal in the sense of a routinely used spin flipper [20].

Let us now modify the experiment just described as shown in Fig. 1b, by inserting at every step a device able to select and detect the down component of the neutron spin. Every magnetic mirror M acts as a "spectral decomposer" [13,14], by splitting a neutron wave with arbitrary spin (a superposed state of up and down spin) into two branch waves each of which is in a definite spin state (up *or* down) along the z-axis. The down state is then forwarded to detector D. Notice that the magnetic mirror does *not* destroy the coherence between the two branch waves, which can be brought back to interfere [21].

We choose the same initial state for Q as in the previous experiment, and follow the evolution of the particle only along the horizontal direction. The calculation is straightforward [10]: The density matrix and the probability that the neutron spin is up at time T read respectively

$$\rho^{(N)}(T)=\left(\cos^2\frac{\omega t}{2}\right)^N\rho_{\uparrow\uparrow}$$
$$=\left(\cos^2\frac{\pi}{2N}\right)^N\rho_{\uparrow\uparrow}\xrightarrow{N\to\infty}\tilde{\rho}(T)=\rho_{\uparrow\uparrow}, \qquad (7)$$

$$P_\uparrow^{(N)}(T)=\left(\cos^2\frac{\omega t}{2}\right)^N$$
$$=\left(\cos^2\frac{\pi}{2N}\right)^N\xrightarrow{N\to\infty}\mathscr{P}_\uparrow(T)=1, \qquad (8)$$

where the "matching" condition (4) for $T=Nt$ has been required again. This is an example of QZE: Frequent observations "freeze" the neutron spin in its initial state, by inhibiting ($N\geqslant 2$) and eventually hindering ($N\to\infty$) transitions to other states. Notice the difference with Eqs. (5) and (6): The situation is completely reversed.

We will now show that the $N\to\infty$ limit considered above (continuous observation) is only mathematical, and is *impossible* to realize, *physically*.

First observe that, by setting for simplicity $m=0$ in Eq. (4), the condition $\omega T=\omega N t=\pi$, which is to be met at every step in Fig. 1b, implies

$$Bl=\frac{\pi\hbar v}{2\mu N}=\mathrm{O}(N^{-1}), \tag{9}$$

where all quantities are defined after Eq. (3). Obviously, as N increases in the above formula, the *practical* realization of the experiment becomes increasingly difficult, because any unavoidable practical uncertainties of the experimental setup would suffer an N-fold enhancement, in Eq. (8), so that it becomes extremely important to have high-quality mirrors and very accurate control on the Bl parameter [22].

But close scrutiny of Eq. (8) shows that $P_{\uparrow}^{(N)}(T)$ cannot tend to 1, even *in principle*, in the $N\to\infty$ limit, because of the quantum mechanical uncertainty relations. Indeed, let

$$\phi\equiv\omega t=\frac{\mu Bl}{\hbar v}=\frac{\pi}{2N} \tag{10}$$

be the argument of the cosinus in Eq. (8). Mathematically, the above quantity is of order $\mathrm{O}(N^{-1})$. On the other hand, from a *physical* point of view, it is impossible to avoid uncertainties in the neutron speed Δv and position Δx. As a consequence, ϕ is lower bounded as follows

$$\phi\sim\phi_0\equiv\frac{\mu Bl}{\hbar v_0}>\frac{\mu B\Delta x}{\hbar v_0}>\frac{\mu B}{2mv_0\Delta v}=\frac{1}{4}\frac{\Delta E_m}{\Delta E_k}, \tag{11}$$

where v_0 is the mean velocity of the beam and we defined the magnetic energy gap $\Delta E_m=2\mu B$ and the (kinetic) energy spread of the neutron beam $\Delta E_k=\Delta(\frac{1}{2}mv^2)|_{v=v_0}$. We also assumed that the size l of the interaction region (where the neutron spin undergoes a rotation under the action of the magnetic field) is smaller than the longitudinal spread Δx of the neutron wave packet. The constraint $l>\Delta x$ (or the distances between the fields are larger than Δx) means that each stage acts independently on the quantum system, and not collectively.

The assumption $l>\Delta x$ cannot be motivated on fundamental grounds, however, the following argument shows that the result (11) is correct also in the opposite case: If the magnetic field is well localized in a small region of size l, such that $l<\Delta x$ (we consider here an almost monochromatic neutron beam) then it is reasonable to assume that the "passage" time be given by $t\simeq\Delta x/v$, rather than $t=l/v$, so that instead of (10) we have

$$\phi=\omega t\simeq\frac{\mu B\Delta x}{\hbar v}, \tag{12}$$

and by mimicking the reasoning leading to the lower bound (11), we obtain again

$$\phi>\frac{1}{4}\frac{\Delta E_m}{\Delta E_k}. \tag{13}$$

It appears therefore that the above lower bound on ϕ is independent of the relative magnitude of l and Δx.

It is now straightforward to obtain an expression for the value of the probability that a spin-up neutron is observed at D_0 when N is large

$$\begin{aligned}P_{\uparrow}^{(N)}(T)&\simeq(\cos\phi_0)^{2N}\simeq(1-\tfrac{1}{2}\phi_0^2)^{2N}\\&\simeq\left[1-\frac{1}{32}\left(\frac{\Delta E_m}{\Delta E_k}\right)^2\right]^{2N}.\end{aligned} \tag{14}$$

Notice that not only the above quantity does *not* tend to 1, but it *vanishes* in the $N\to\infty$ limit. In other words, in the experiment outlined in Fig. 1(b), *no* spin-up neutron would be observed at D_0 in the $N\to\infty$ limit, and *no* quantum Zeno paradox would occur!

It is then interesting to set a limit on the maximum value that N can attain in order that the QZE be still observable in the experiment outlined above. Set $P_{\uparrow}^{(N)}(T)\sim\frac{1}{2}$. We get.

$$N\sim\frac{64\ln 2}{(\Delta E_m/\Delta E_k)^2}\simeq10^4, \tag{15}$$

where we assumed reasonable values for the energies of a thermal neutron. In conclusion, N turns out to be large enough in order that the QZE be experimentally observable, at least up to a certain approximation.

Besides the doubts cast by the uncertainty principle on the physical meaning of the $N\to\infty$ limit, there are other interesting limitations arising from the presence of losses due to reflections and rotations. We shall now briefly analyze these problems.

To this end, the interaction between a neutron spin and a magnetic field can be schematized by describing the magnetic field as a potential of strength $V_0=\pm\mu B$ and width l. We shall henceforth neglect the aforementioned problems stemming from the uncertainty principle and consider a very thin potential such that $V_0 l = \mu B l$ is (small but) finite. The reflection probability reads

$$r \simeq \left(\frac{V_0 l}{\hbar v}\right)^2 , \tag{16}$$

and if we assume again that $l > \Delta x$ (see discussion after Eq. (11)), we obtain

$$r > \left(\frac{V_0 \Delta x}{\hbar v}\right)^2 > \left(\frac{V_0}{2\Delta E_k}\right)^2 = \left(\frac{\Delta E_m}{4\Delta E_k}\right)^2 , \tag{17}$$

due to the energy spread ΔE_k of the neutron beam. By estimating N via Eq. (10), one obtains a total loss by reflection of order $Nr \simeq \Delta E_m / \Delta E_k$, which is easily seen to be comparable with that of Eq. (14).

Formulas (11) (14) and (17) show that the loss factors depend on the speed of the neutrons. Thus, slower neutrons suffer more losses than the faster ones, which causes a progressive narrowing of the velocity distribution and this in turn increases Δx (and therefore l) accordingly (see, e.g., Ref. [23]). These effect will be considered in a follow-up study.

We stress that, if $l < \Delta x$, the various rotation stages may act together coherently, like a periodic potential. In general, the reflectivity of such a coherent multiple system is larger than the sum of the individual reflectivities. Therefore, in any practical test of the QZE, one must carefully avoid such resonance situations due to reflections by a lattice-like structure, which may even lead, by Eqs. (10) and (16), to a total reflection loss of order $N^2 r \simeq 1$, corresponding to a multilayer reflection [24].

One can also give a rough estimate of the losses due to spin rotation. After every transmission through the magnetic field, the probability that the spin is down (and therefore that it is reflected upwards by M, in Fig. 1b) is written as $s = \sin^2(\frac{1}{2}\omega t) \simeq \mu B l / \hbar v$. This is of the same order as r in Eq. (16). Therefore analogous considerations apply also in this case.

All the above arguments, outlined for the neutron experiment described in Fig. 1, are of rather general validity and hold true also in the recent experiment performed with photons in Innsbruck [19]. These experiments showed losses which are related to the finite number of stages and to experimental imperfections, but they were not analysed in respect to their quantum limit. The analyses above seem to imply that *absolute* "interaction-free" measurements might be forbidden as a matter of principle, by elementary quantum laws. Remember that the final observation includes an interaction which provokes "dephasing" between the spin-up and spin-down components.

Let us now endeavour to outline how the practical limitation arising in a real experiment can be taken into account. Consider again the situation shown in Fig. 1b, after the neutron has only gone through *one* step (namely, one interaction with the magnetic field B and the mirror M). If the initial neutron spin is up, the evolution is

$$\rho_{\uparrow\uparrow} \to \tilde{\rho}^{(1)}(t) = \sigma \cos^2(\tfrac{1}{2}\omega t) \rho_{\uparrow\uparrow} + \dots , \tag{18}$$

where σ accounts for all the losses and practical limitations and the dots stand for smaller terms. In principle, the value of σ can be calculated theoretically. In practice, it is much simpler to measure it experimentally, by performing a preliminary one-step experiment.

After N steps, the probability to observe a spin-up neutron by the final detector reads

$$\tilde{P}_{\uparrow}^{(N)}(T) = \sigma^N P_{\uparrow}^{(N)}(T) , \tag{19}$$

where $P_{\uparrow}^{(N)}(T)$ is given by Eq. (8) and we supposed that σ is the same at every step.

As was to be expected, $\tilde{P}_{\uparrow}^{(N)}(T)$ does *not* tend to one in a real experiment: Indeed, even though in general σ depends on N, it cannot tend to 1 as $N \to \infty$, even in principle, for the many reasons discussed above. In practice, the factor σ^N will quickly tend to zero, as N increases. A practical realization of an experiment consisting of relatively few steps is certainly feasible, but one should keep in mind that losses are unavoidable from basic argument of quantum mechanics.

SP was partially supported by the Department of Physics of Waseda University, with additional funding from the Administration Council of the University of Bari. He thanks the High Energy Physics group of Waseda University for their kind hospitality.

References

[1] E.J. Hellund, Phys. Rev. 89 (1953) 919.
[2] M. Namiki and N. Mugibayashi, Prog. Theor. Phys. 10 (1953) 474.

[3] L.A. Khalfin, Zh. Eksp. Teor. Fiz. 33 (1958) 1371 [Sov. Phys. JETP 6 (1958) 1053]; Phys. Lett. B 112 (1982) 223; Usp. Fiz. Nauk. 160 (1990) 185 [Sov. Phys. Usp. 33 (1990) 10].
[4] A. DeGasperis, L. Fonda and G.C. Ghirardi, Nuovo Cimento A 21 (1974) 471;
L. Fonda, G.C. Ghirardi and A. Rimini, Rep. Prog. Phys. 41 (1978) 587.
[5] B. Misra and E.C.G. Sudarshan, J. Math. Phys. 18 (1977) 756;
C.B. Chiu, B. Misra and E.C.G. Sudarshan, Phys. Rev. D 16 (1977) 520; Phys. Lett. B 117 (1982) 34.
[6] G.-C. Cho, H. Kasari and Y. Yamaguchi, Prog. Theor. Phys. 90 (1993) 803.
[7] R.J. Cook, Phys. Scr. T 21 (1988) 49.
[8] W.H. Itano, D.J. Heinzen, J.J. Bollinger and D.J. Wineland, Phys. Rev. A 41 (1990) 2295.
[9] T. Petrosky, S. Tasaki and I. Prigogine, Phys. Lett. A 151 (1990) 109; Physica A 170 (1991) 306;
A. Peres and A. Ron, Phys. Rev. A 42 (1990) 5720;
L.E. Ballentine, Phys. Rev. A 43 (1991) 5165;
W.H. Itano, D.J. Heinzen, J.J. Bollinger and D.J. Wineland, Phys. Rev. A 43 (1991) 5168;
S. Inagaki, M. Namiki and T. Tajiri, Phys. Lett. A 166 (1992) 5;
D. Home and M.A.B. Whitaker, Phys. Lett. A 173 (1993) 327.
[10] S. Pascazio, M. Namiki, G. Badurek and H. Rauch, Phys. Lett A 179 (1993) 155;
S. Pascazio and M. Namiki, Dynamical quantum Zeno effect, Phys. Rev. A, in print.
[11] J. von Neumann, Die mathematische grundlagen der quantenmechanik (Springer, Berlin, 1932).
[12] J.A. Wheeler and W.H. Zurek, eds., Quantum theory and measurement (Princeton Univ. Press, Princeton, 1983);
B. d'Espagnat, Conceptual foundations of quantum mechanics (Benjamin, New York, 1971);
P. Busch, P.J. Lahti and P. Mittelstaedt, The quantum theory of measurement (Springer, Berlin, 1991);
M. Namiki and S. Pascazio, Phys. Rep. 232 (1993) 301.
[13] E.P. Wigner, Am. J. Phys. 31 (1963) 6.
[14] S. Machida and M. Namiki, Prog. Theor. Phys. 63 (1980) 1457, 1833;
M. Namiki, S. Pascazio and C. Schiller, Phys. Lett. A 182 (1994) 17.
[15] W.H. Zurek, Phys. Rev. D 24 (1981) 1516; 26 (1982) 1862; Phys. Today 44 (1991) 36.
[16] G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34 (1986) 470; 36 (1987) 3287; Found. Phys. 18 (1988) 1.
[17] J. Summhammer, H. Rauch and D. Tuppinger, Phys. Rev. A 36 (1987) 4447;
H. Rauch and J. Summhammer, Phys. Rev. A 46 (1992) 7284.
[18] A. Peres, Am. J. Phys. 48 (1980) 931.
[19] P. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger and M. Kasevich, to be published in: Proc. Conf. on Fundamental problems in quantum theory, Maryland, 1994.
[20] F. Mezei, Comm. Phys. 1 (1976) 81a; 2 (1977) 41;
G. Badurek, G.P. Westphal and P. Ziegler, Nucl. Instrum. Methods 120 (1974) 351.
[21] J. Summhammer, G. Badurek, H. Rauch, U. Klischko and A. Zeilinger, Phys. Rev. A 27 (1983) 2523;
G. Badurek, H. Rauch and J. Summhammer, Phys. Rev. Lett. 51 (1983) 1015.
[22] G. Badurek, H. Rauch, A. Wilfing, U. Bonse and W. Graeff, J. Appl. Cryst. 12 (1979) 186.
[23] M.M. Agamakyan, G.M. Drabkin and V.I. Sbitner, Phys. Rep. 168 (1988) 265.
[24] M. Born and E. Wolf, Principles of optics (Pergamon, Oxford, 1980) p. 66;
V.F. Sears, Acta Cryst. A 39 (1983) 601.