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Two-Level System with a Noisy Hamiltonian

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We study the dynamical properties of a two-level system in interaction with its environment, whose action on the system is modeled by means of a noise term in the Hamiltonian. We solve the Schrödinger equation, obtain an evolution equation of the Lindblad type for the noise average of the density matrix, and discuss the results in terms of a "decoherence parameter." Finally, we concentrate our attention on the possibility of hindering the transitions between the two levels in two (apparently unrelated) ways: (a) by increasing the strength of the noise; (b) by a series of frequent measurements. There is an interesting relation between these two situations.

KEY WORDS: Decoherence; stochastic; two-level system.

To model the interaction between a quantum mechanical system and its environment is in general a complicated problem. A possible approach is to represent the action of the environment via "noise" terms in the Hamiltonian of the system; however, it is not clear whether there is a general recipe in order to get such terms from the total Hamiltonian (describing the environment + the system) in a rigorous way. As a matter of fact, this program can be carried out only for some solvable models. Interesting examples are the Ford–Kac–Mazur [1] and a whole class of related models [2] that have played an important role in clarifying several aspects related to dissipative phenomena. In these cases, an ensemble of coupled oscillators interacts via a quadratic Hamiltonian and the reduced dynamics of one of these oscillators yields, in an appropriate macroscopic limit, a Langevin equation [3]. The noise terms are therefore "derived" from the total Hamiltonian in some approximation and under some assumptions, by formally solving the equations of motion. Closely related examples are the Caldeira–Leggett [4] and the Feynman–Vernon model [5].

A somewhat more pragmatic approach consists of investigating the effect of noise without endeavoring to clarify its origin. In this case, the action of the environment on the system is schematized from the outset by means of a sort of stochastic operator added to the original Hamiltonian of the system.

The purpose of the present article is to scrutinize a simple model proposed by Blanchard, Bolz, Cini, De Angelis, and Serva (BBCDS) [6] and provide an explicit solution to discuss the short-time dynamics of a quantum system under the influence of a "noisy" environment, in relation to the so-called quantum Zeno phenomena. This Hamiltonian models a two-level system interacting with an environment, whose action on the system is simply schematized by means of a white noise multiplying an operator of the system. Its interest lies in the fact that the model can schematize a superconducting ring enclosing a quantized magnetic flux. Coherent tunneling between the two flux configurations is possible if the system is very well isolated from its environment. Due to the high sensitivity of the properties of such a device to its interaction with the environment, it becomes very important to model and analyze the action of the latter. The same model was also investigated by Berry [7], who concentrated his attention on the relative time scales as well as on the so-called quantum Zeno effect [8].

The plan of the article is as follows. In Section 1 we set up a general framework for our analysis.

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In Section 2 we introduce and analyze the BBCDS model, rederiving some results by an alternative method. We discuss our results in Section 3, by means of a "decoherence parameter." The quantum Zeno effect is discussed in Sections 4 and 5. It is shown that the decoherence caused by the interaction with the environment leads naturally to the exponential decay of the system and such behavior is in general at variance with the occurrence of the Zeno effect, even though the latter can be observed for a particular choice of the initial state. Section 6 is devoted to summary and comments.

1. THE GENERAL FRAMEWORK

We shall describe a system embedded in its environment via the time-dependent Hamiltonian

$$H = H_0 + \eta(t) H_1 \tag{1}$$

where H_0 and H_1 are Hermitian, time-independent operators. The action of the environment on the system is modeled by the stochastic term ηH_1, where η is a white noise, whose stochastic properties read

$$\langle \eta(t) \rangle = 0 \qquad \langle \eta(t)\eta(t') \rangle = \delta(t - t') \tag{2}$$

the brakets denoting an ensemble average over all possible realizations of the noise. The above formulas are recast into a more rigorous form by introducing the Wiener process:

$$\begin{aligned} dW(t) &\equiv W(t + dt) - W(t) = \int_t^{t+dt} \eta(s)\, ds \\ \langle dW(t) \rangle &= 0 \qquad \langle dW(t)\, dW(t) \rangle = dt \end{aligned} \tag{3}$$

The corresponding Schrödinger equation reads, by Ito calculus, ($\hbar = 1$)

$$\begin{aligned} |d\psi\rangle &= -iH_0|\psi\rangle\, dt - iH_1|\psi\rangle \circ dW \\ &= (-iH_0 - \tfrac{1}{2}H_1^2)|\psi\rangle\, dt - iH_1|\psi\rangle\, dW \end{aligned} \tag{4}$$

where $\circ$ denotes the Stratonovich product. Notice that, when the average (2)–(3) (to be denoted with a bar) is computed, the above equation reads

$$\overline{|d\psi\rangle} = (-iH_0 - \tfrac{1}{2}H_1^2)\, \overline{|\psi\rangle}\, dt \tag{5}$$

and probabilities are conserved, since

$$\overline{\||\psi + d\psi\rangle\|^2} = \overline{\||\psi\rangle\|^2} = 1 \tag{6}$$

The evolution of the density matrix is governed by

$$\begin{aligned} d\rho &= \rho + d\rho - \rho \\ &= |\psi + d\psi\rangle\langle\psi + d\psi| - |\psi\rangle\langle\psi| \\ &= -i[H_0, \rho]\, dt - \tfrac{1}{2}\{H_1^2, \rho\}\, dt \\ &\quad - i[H_1, \rho]\, dW + H_1\rho H_1\, dt \end{aligned} \tag{7}$$

where $[\cdot, \cdot]$ is the commutator and $\{\cdot, \cdot\}$ is the anticommutator. This yields, on the average, a Lindblad equation [9],

$$\frac{d}{dt}\bar{\rho} = -i[H_0, \bar{\rho}] - \frac{1}{2}\{H_1^2, \bar{\rho}\} + H_1\bar{\rho}H_1 \tag{8}$$

The operator H_1 is therefore the generator of a Gaussian semigroup, which in turn justifies Eq. (6). The Hamiltonian considered in [6,7] is a particular case of the above.

2. THE BBCDS MODEL AND THE ASSOCIATED LINDBLAD EQUATION

The BBCDS Hamiltonian describes a two-level system interacting with an environment,

$$H = \alpha\sigma_1 + \beta\eta(t)\sigma_3 \tag{9}$$

where α and β are real positive constants and σ_i ($i = 1,2,3$) Pauli matrices. The action of the environment on the system is modeled by the white noise η, and the corresponding Schrödinger equation reads

$$\begin{aligned} |d\psi\rangle &= -i\alpha\sigma_1|\psi\rangle\, dt - i\beta\sigma_3|\psi\rangle \circ dW \\ &= \left(-i\alpha\sigma_1 - \frac{1}{2}\beta^2\right)|\psi\rangle\, dt - i\beta\sigma_3|\psi\rangle\, dW \end{aligned} \tag{10}$$

where $|\psi\rangle = (|\psi_+\rangle, |\psi_-\rangle)^t$ is a two-component spinor. We shall work in the basis of the eigenstates of σ_3. Notice that when $\beta = 0$, the above equation yields coherent (Rabi) oscillations between the two eigenstates of σ_3.

By introducing the polarization (Bloch) vector

$$\mathbf{x}(t) = \langle\psi|\boldsymbol{\sigma}|\psi\rangle \tag{11}$$

one easily derives the stochastic differential equation

$$d\mathbf{x}(t) = A\mathbf{x}(t)\, dt + B\mathbf{x}(t)\, dW(t) \tag{12}$$

where

$$A = \begin{pmatrix} -2\beta^2 & 0 & 0 \\ 0 & -2\beta^2 & -2\alpha \\ 0 & 2\alpha & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & -2\beta & 0 \\ 2\beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{13}$$

The third component $z = \langle\psi_+|\psi_+\rangle - \langle\psi_-|\psi_-\rangle$ of the Bloch vector contains information about the probability of finding the system in one of the eigenstates of σ_3. Observe that the norm of the Bloch vector is preserved,

$$\|\mathbf{x}(t)\|^2 \equiv x^2(t) + y^2(t) + z^2(t) = 1 \qquad \forall t \quad (14)$$

as can be readily derived from the completeness relation $\frac{1}{2}\sum_{\mu=0}^{4}(\sigma_\mu)_{ab}(\sigma_\mu)_{cd} = \delta_{ad}\delta_{bc}$, with $\sigma_0 = 1$.

The density matrix of a two-level system (like the one considered above) can always be expressed in terms of the Bloch vector (11), according to the formula

$$\rho = \frac{1}{2}(\mathbf{1} + \mathbf{x}\cdot\boldsymbol{\sigma}) \quad (15)$$

Notice that ρ is normalized $[\mathrm{Tr}(\rho) = 1]$ and $\mathrm{Tr}(\rho\boldsymbol{\sigma}) = \mathbf{x}$. Pure states are characterized by $\|\mathbf{x}\| = 1$.

The Lindblad equation (8) for the BBCDS model reads

$$\frac{d}{dt}\bar{\rho} = -i[\alpha\sigma_1, \bar{\rho}] - \beta^2(\bar{\rho} - \sigma_3\bar{\rho}\sigma_3) \quad (16)$$

This equation holds for the *averaged* density matrix. By making use of the explicit expression (15), one obtains

$$\frac{d}{dt}\bar{x} = -2\beta^2\bar{x}$$

$$\frac{d}{dt}\bar{y} = -2\alpha\bar{z} - 2\beta^2\bar{y}$$

$$\frac{d}{dt}\bar{z} = 2\alpha\bar{y} \quad (17)$$

which yield the solutions

$$\bar{x}(t) = \bar{x}(0)e^{-2\beta^2 t}$$

$$\bar{y}(t) = e^{-\beta^2 t}[\bar{y}(0)\cos\omega t + c_1\sin\omega t]$$

$$\bar{z}(t) = e^{-\beta^2 t}[\bar{z}(0)\cos\omega t + c_2\sin\omega t] \quad (18)$$

where $c_1 = [-\beta^2\bar{y}(0) - 2\alpha\bar{z}(0)]/\omega$ and $c_2 = [\beta^2\bar{z}(0) + 2\alpha\bar{y}(0)]/\omega$ and $\omega = \sqrt{4\alpha^2 - \beta^4}$ can be real or purely imaginary. Note that if $4\alpha^2 - \beta^4 < 0$ the solution is simply obtained by replacing the trigonometric functions in (18) with the hyperbolic ones: $\cos\omega t \to \cosh\omega t, \sin\omega t \to \sinh\omega t$. All these results are obviously in agreement with those obtained by BBCDS [6] and Berry [7], by making use of different techniques.

3. THE DECOHERENCE PARAMETER

The vector $\mathbf{x}$ is a stochastic process, and is such that $\|\mathbf{x}(t)\| = 1$, $\forall$t. As a consequence, from Eq. (15) one easily derives

$$\rho^2(t) = \rho(t) \qquad \forall t \quad (19)$$

For every realization of the stochastic process, or, in other words, if the quantity $\eta(t)$ in (1) or (9) is given a meaning as an ordinary function of time, the system remains in a pure state. On the other hand, if we are only interested in the expectation values of the system operators, the relevant density matrix is $\bar{\rho}$ (i.e., the one averaged over η), which satisfies

$$\bar{\rho}^2(t) = \tfrac{1}{4}[\mathbf{1} + 2\bar{\mathbf{x}}(t)\cdot\boldsymbol{\sigma} + \bar{\mathbf{x}}^2(t)] \neq \overline{\rho^2} = \bar{\rho} \quad (20)$$

Introduce the parameter

$$\begin{aligned}\varepsilon &\equiv 2\,\mathrm{Tr}(\Delta\rho) \equiv 2\,\mathrm{Tr}(\overline{\rho^2} - \bar{\rho}^2) = 2\,\mathrm{Tr}(\bar{\rho} - \bar{\rho}^2)\\ &= 1 - \bar{\mathbf{x}}^2(t)\end{aligned} \quad (21)$$

Notice that $0 \leq \varepsilon \leq 1$. This quantity measures how far $\bar{\rho}$ is from being idempotent [10], yielding a quantitative estimate of the degree of decoherence of the quantum system: if $\varepsilon = 0$, coherence is preserved; if, on the other hand, $\varepsilon = 1$, coherence is totally lost and the system is totally incoherent. For this reason, this parameter was named the *decoherence parameter* [11]. It is a very useful quantity in the theory of quantum mechanical measurements [12]. It is noteworthy that all the coherence properties of our system can be summarized by a numerical variable like our ε. This is an interesting characteristic of the model considered.

It is easy to check from the explicit expression (18) that $\varepsilon \to 1$ exponentially as $t \to \infty$: the quantum coherence is lost as time goes by, as a consequence of the noise term in the Hamiltonian (9). Such a loss of coherence is irretrievable, as one easily understands from physical considerations as well as from the structure (8) of the dynamical evolution law.

4. HINDERED EVOLUTION AND QUANTUM ZENO EFFECT

The interest of the above model lies in the different dynamical regimes that are obtained by varying the strength β of the coupling with the environment: if β is small, the interaction with the environment is weak and the system undergoes coherent quantum oscillations between its two states. If, on the other hand, β is large, the oscillations are hindered and the system becomes "localized" in one of its two states. BBCDS called this phenomenon "localization stabilized by noise" [6], and Berry emphasized the links with the quantum Zeno effect [7].

The purpose of the present section is to shed some additional light on the latter regime and the quantum Zeno effect. To this end, it is convenient to analyze a particular case: prepare the system in the initial state $\overline{x}(0) = \overline{y}(0) = 0, \overline{z}(0) = 1$ (all particles in state $|\psi_+\rangle$). If the coupling with the environment is large $\beta^2 \gg 2\alpha$, the solution is

$$\overline{\mathbf{x}}(t) = e^{-\beta^2 t} \begin{pmatrix} 0 \\ -\dfrac{2\alpha}{\omega} \sinh \omega t \\ \cosh \omega t + \dfrac{\beta^2}{\omega} \sinh \omega t \end{pmatrix} \overset{\beta^2 \gg 2\alpha; \text{large } t}{\simeq} e^{-(2\alpha^2/\beta^2)t} \begin{pmatrix} 0 \\ O\left(\dfrac{\alpha}{\beta^2}\right) \\ 1 + O\left(\dfrac{\alpha^2}{\beta^4}\right) \end{pmatrix} \tag{22}$$

where "large t" means $t \gg \omega^{-1} \simeq \beta^{-2}$ (observe that β^{-2} is a small quantity, so the above expansion is valid for rather small t). As one can see, when β is large the oscillations are hindered and the system tends to remain in its initial state:

$$\overline{z}(t) \simeq e^{-(2\alpha^2/\beta^2)t} \xrightarrow{\beta \to \infty} 1 \tag{23}$$

for $t < \infty$. This "halting" of the quantum evolution due to strong coupling with the environment is familiar in a variety of physical situations: a remarkable and well-known example is the explanation of the chiral nature of certain molecules [13].

Let us now take a different approach. Assume that the coupling with the environment is small $\beta^2 \ll 2\alpha$, but *frequent* measurements are performed on the system in order to ascertain whether it is localized in one of the eigenstates of σ_3: $|\psi_+\rangle$ or $|\psi_-\rangle$. This is the usual framework of "pulsed" observation, typical of the quantum Zeno effect [8].

Since we are considering small coupling ($\beta^2 \ll 2\alpha$), the solution of the stochastic differential equation is

$$\overline{\mathbf{x}}(t) = e^{-\beta^2 t} \begin{pmatrix} 0 \\ -\dfrac{2\alpha}{\omega} \sin \omega t \\ \cos \omega t + \dfrac{\beta^2}{\omega} \sin \omega t \end{pmatrix} \overset{\beta^2 \ll 2\alpha; \text{small } t}{\simeq} \begin{pmatrix} 0 \\ -2\alpha t \\ 1 - 2\alpha^2 t^2 \end{pmatrix} \tag{24}$$

where "small t" means $t \ll \omega^{-1} \simeq 2\alpha$.

If a σ_3 measurement is performed at time $t = \delta t_1$, $\mathbf{x}(t)$ "collapses" into the state $(0, 0, \lambda)$ with probability $p_\lambda = [1 + \lambda \overline{z}(\delta t_1)]/2$, where $\lambda = \pm 1$ is the result of the measurement. Consequently, the density matrix after one measurement becomes

$$\rho \rightarrow \overline{\rho}^{(1)} = \frac{1}{2} [\mathbf{1} + \overline{z}(\delta t_1)\sigma_3] \tag{25}$$

Notice that we are considering the average $\overline{\cdots}$. After the measurement, the evolution starts anew, with the initial condition $\mathbf{x}(\delta t_1) = (0, 0, \lambda)$, where λ is *either* $+1$ *or* -1, each event occurring with probability p_λ. The density matrix reads

$$\rho(t) = \frac{1 + \overline{z}(\delta t_1)}{2} \rho^{(+)}(t - \delta t_1) + \frac{1 - \overline{z}(\delta t_1)}{2} \rho^{(-)}(t - \delta t_1) \tag{26}$$

where $t - \delta t_1 > 0$ is the time elapsed after the measurement and

$$\rho^{(\lambda)}(\tau) = \frac{1}{2}[\mathbf{1} + \mathbf{x}(\tau; (0,0,\lambda)) \cdot \boldsymbol{\sigma}] \qquad (\lambda = \pm 1) \tag{27}$$

is the density matrix of the system after the measurement, $\mathbf{x}(\tau; (0, 0, \lambda))$ being the solution of the stochastic differential equation (12) with initial condition $(0, 0, \lambda)$. By (18), $\overline{\mathbf{x}}(\tau; (0, 0, \lambda)) = -\overline{\mathbf{x}}(\tau; (0, 0, -\lambda))$ and (26) yields, on the average,

$$\overline{\rho}(t) = \frac{1}{2} [\mathbf{1} + \overline{z}(\delta t_1)\overline{\mathbf{x}}(t - \delta t_1; (0, 0, 1)) \cdot \boldsymbol{\sigma}] \qquad (t > \delta t_1) \tag{28}$$

If another σ_3 measurement is performed at time $t = \delta t_1 + \delta t_2$,

$$\rho \rightarrow \overline{\rho}^{(2)} = \frac{1}{2} [\mathbf{1} + \overline{z}(\delta t_1)\overline{z}(\delta t_2)\sigma_3] \tag{29}$$

and the evolution starts again with the new initial conditions. If N measurements are performed in time $t = \Sigma_{j=1}^{N} \delta t_j$, the final state is

$$\bar{\rho}^{(N)} = \frac{1}{2}\left[\mathbf{1} + \prod_{j=1}^{N} \bar{z}(\delta t_j)\sigma_3\right] \tag{30}$$

Take, for simplicity, $\delta t_j = \delta t$ ($\forall j$), so that the time interval between successive measurements is constant. Then, from (24),

$$\bar{z}(t) = \mathrm{Tr}(\bar{\rho}^{(N)}\sigma_3) = [\bar{z}(\delta t = t/N)]^N \xrightarrow{N\to\infty} e^{-(2\alpha^2\delta t)t} \xrightarrow{\delta t\to 0} 1, \tag{31}$$

for $t < \infty$. Once again, the oscillations are hindered. One can say that the two situations analyzed in this section, large coupling with the environment (23) and frequent measurements (31), are equivalent in that they yield the same physical effect. There is an analogy with what Schulman calls "continuous versus pulsed observations" [14] and a link with the phenomenon of "dominated evolution," analyzed in [15]. The two regimes can be quantitatively compared: if

$$\beta^{-2} = \delta t, \tag{32}$$

(23) and (31) are identical. In other words, a (σ_3) white noise of large strength β and a series of frequent (σ_3) observations at short time intervals δt slow down (and eventually halt) the evolution of an *eigenstate* of σ_3 [initial condition $\bar{z}(0) = 1$].

It is worth estimating the decoherence parameter ε introduced in the previous section, which measures the degree of quantum coherence lost during the interaction with the environment. Recall that the decoherence parameter ε is essentially given by the Bloch vector $\bar{\mathbf{x}}(t)$. See (21). Since $1 = \|\mathbf{x}(t)\|^2 \geq \bar{\mathbf{x}}^2(t) \geq \bar{z}^2(t)$ and $\bar{z}(t) \to 1$ in both limits considered above,

$$\varepsilon \to 0 \tag{33}$$

in both cases. This implies, in particular, in the latter case of frequent *measurements*, that the system remains in a pure state in the $N = \infty$ limit, as if no measurements were performed on it. This observation is in line with the claim [16] that von Neumann's projection postulate is not necessary to realize the quantum Zeno phenomenon.

5. ZENO DYNAMICS AND (DE)COHERENCE

The previous discussion is valid when the system is prepared in the particular initial state $\bar{x}(0) = \bar{y}(0) = 0, \bar{z}(0) = 1$, that is, in an eigenstate of σ_3. Since our Hamiltonian is given by (9), the system undergoes coherent (Rabi) oscillations around the first axis, modulated by the stochastic vibrating force along the third axis. This is why the system tends to remain in the initial state when the stochastic force becomes infinitely strong ($\beta \to \infty$), as in (23). In the previous section, this phenomenon of hindered transition was compared with the Zeno dynamics, i.e., the short-time characteristics of quantum systems.

The question we wish to investigate here is whether such a similarity between the effect of the strong environmental force, here modeled by the stochastic term in the Hamiltonian, and the Zeno dynamics can be given a more general meaning. For this purpose, we consider a system characterized by the same Hamiltonian H (9), but prepared in different initial states.

If the system is prepared in an eigenstate of σ_1, i.e., $\bar{x}(0) = 1$, its dynamics is trivial, as one easily understands either from the structure of H or from the solutions (18):

$$\bar{x}(t) = e^{-2\beta^2 t} \qquad \bar{y}(t) = \bar{z}(t) = 0 \tag{34}$$

The system decays into a completely mixed state in the large-t limit, which is a consequence of the fact that the decoherence parameter ε approaches 1:

$$\varepsilon = 1 - \bar{\mathbf{x}}^2(t) \xrightarrow{t\to\infty} 1. \tag{35}$$

If, on the other hand, the same system is frequently monitored to check whether it remains in the initial state, an argument similar to that in the previous section leads to the final density matrix,

$$\bar{\rho}^{(N)} = \frac{1}{2}[\mathbf{1} + [\bar{x}(t/N)]^N \sigma_1] \tag{36}$$

when N σ_1 measurements are performed at equal time intervals within the finite time t. Notice that $\bar{x}(t)$ is just the exponential function (34) and therefore the above quantity is independent of N:

$$\bar{\rho}^{(N)} = \frac{1}{2}[\mathbf{1} + e^{-2\beta^2 t}\sigma_1] \tag{37}$$

This result was to be expected, since the Hamiltonian is nothing but a fluctuating force, for a system prepared in one of the eigenstates of σ_1, so that the system decays exponentially at all times, regardless of the frequency of monitoring. In conclusion, the system exponentially loses quantum coherence:

$$\varepsilon = 1 - e^{-4\beta^2 t} \xrightarrow{t\to\infty} 1 \tag{38}$$

As a final simple example, prepare the system in the state $\overline{y}(0) = 1$. This case represents an interesting situation, since the system undergoes both coherent and stochastic forces from the beginning. If $\beta^4 < 2\alpha^2$, the solutions is

$$\begin{aligned} \overline{x}(t) &= 0 \\ \overline{y}(t) &= e^{-\beta^2 t}[\cos\omega t - (\beta^2/\omega)\sin\omega t] \\ \overline{z}(t) &= (2\alpha/\omega)e^{-\beta^2 t}\sin\omega t \end{aligned} \tag{39}$$

If, on the other hand, $\beta^4 > 2\alpha^2$, the solution is

$$\overline{\mathbf{x}}(t) = e^{-\beta^2 t}\begin{pmatrix} 0 \\ \cosh\omega t - (\beta^2/\omega)\sinh\omega t \\ (2\alpha/\omega)\sinh\omega t \end{pmatrix} \tag{40}$$

which reduces for large $t \gg \omega^{-1} \simeq \beta^{-2}$, to

$$\overline{\mathbf{x}}(t) \xrightarrow{\text{large}\, t} e^{-(2\alpha^2/\beta^2)t}\begin{pmatrix} 0 \\ -\alpha^2/\beta^4 \\ \alpha/\beta^2 \end{pmatrix} \tag{41}$$

Notice that unlike the case considered in the previous section (22)–(23), in this example even a very strong noise ($\beta \to \infty$) does not yield a pure state. Instead, the system finally loses quantum coherence and the decoherence parameter tends to 1 as a power function of β:

$$\varepsilon \overset{\text{large}\,\beta}{\simeq} 1 - (\alpha^2/\beta^4)e^{-4(\alpha^2/\beta^2)t} \tag{42}$$

What happens if the system prepared in such initial state is frequently monitored, by measuring σ_2? The answer is easily inferred from the final density matrix,

$$\overline{\rho}^{(N)} = \frac{1}{2}[\mathbf{1} + [\overline{y}(t/N)]^N\sigma_2] \tag{43}$$

which is easily derived like (36). Since (40) yields, for small t,

$$\overline{y}(t) \simeq 1 - 2\beta^2 t \tag{44}$$

the above density matrix becomes again independent of N,

$$\overline{\rho}^{(N)} \sim \frac{1}{2}[\mathbf{1} + e^{-2\beta^2 t}\sigma_2] \tag{45}$$

and the decoherence parameter is given again by (38).

In both cases considered in this section, neither a strong coupling with the environment, nor a frequent series of measurements can halt the evolution. There is no quantum Zeno effect.

6. SUMMARY AND COMMENTS

We have studied the model proposed by Blanchard, Bolz, Cini, De Angelis, and Serva [6], showing its explicit solution and discussing the loss of quantum mechanical coherence. The coherence properties of the system are well described by a single parameter ε, which is defined in terms of the (averaged) Bloch vector $\overline{\mathbf{x}}$ (t). Coherence is lost exponentially as $t \to \infty$. It is remarkable that ε, which can be evaluated from $\overline{\mathbf{x}}(t)$, yields a quantitative estimate of the quantity $\overline{\rho} - \overline{\rho}^2$, even though the term $\overline{\rho}^2$ is sometimes claimed not to be directly measurable.

We have concentrated our attention, in particular, on the short-time dynamics of the system. With the Hamiltonian (9), if the system is prepared in a particular state, i.e., one of the σ_3 eigenstates, the quantum coherence can be preserved by increasing either the strength of the noise term ($\beta \to \infty$) or the frequency of σ_3 observations. The latter case is a realization of the quantum Zeno phenomenon. For different initial conditions and different types of measurements, the system generally loses its quantum coherence due to the action of the stochastic force. The degree of the loss of coherence is manifest in the value of the decoherence parameter ε; since the latter behaves exponentially in such cases, *no* Zeno effect can be obtained even under very frequent observations.

In conclusion, the occurrence of the Zeno phenomenon in the present context requires the concomitance of several factors: in particular, the noise term (or a series of frequent observations) prevents the system from decohering if and only if it is applied in an appropriate direction with respect to the initial state of the system.

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