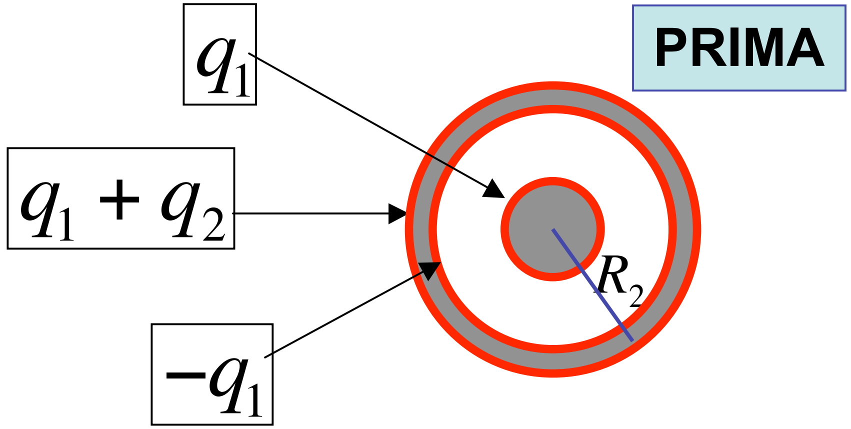
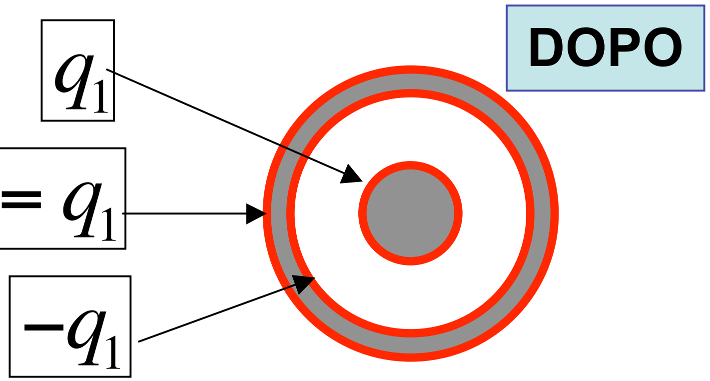
Esercizio

Sono date 2 sfere concentriche conduttrici di raggio R_1 ed R_2=9*cm*
Su quella esterna - che e' cava - viene depositata una carica $q_2=-2\times10^{-9}$C;
su quella interna una carica $q_1=10^{-9}$C.
Successivamente si aggiunge sulla sfera esterna una carica $q_3=-q_2$.
Calcolare di quanto varia il potenziale della sfera interna.

Per induzione elettrostatica compare $-q_1$ ($+q_1$) sulla superficie interna (esterna) della sfera cava.

La carica esterna q_2 non induce: effetto dello schermo elettrostatico !





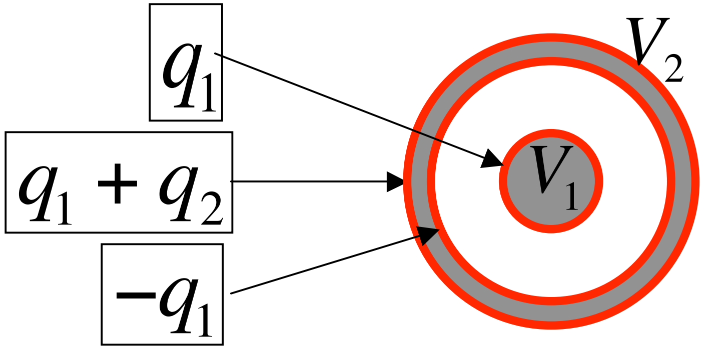
$$q_1 + q_2 + q_3 = q_1 + \cancel{q_2} - \cancel{q_2} = q_1$$

RISPOSTA :

Il guscio sferico esterno e' uno schermo elettrostatico per cui le variazioni di potenziale della sfera interna sono uguali a quelle della sfera esterna ! Quindi:

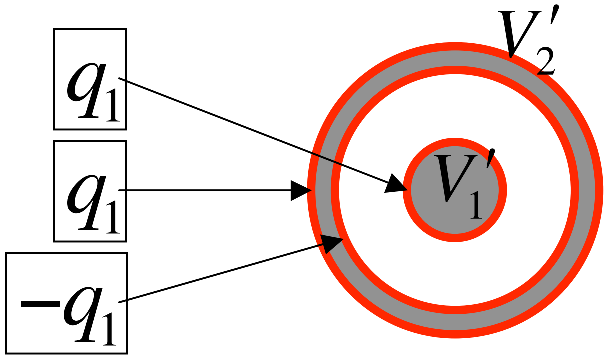
$$\Delta V_1 = \Delta V_2 \equiv V_2' - V_2$$

N.B.: non serve conoscere R_1 (non dato)



PRIMA

$$V_2 = \frac{(q_1 + q_2)}{4\pi\varepsilon_o R_2}$$



DOPO

$$V_2' = \frac{q_1}{4\pi\varepsilon_o R_2}$$

$$\Delta V_2 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{-q_2}{R_2}$$

$$\cong 9 \cdot 10^9 \frac{Nm^2}{C^2} \cdot \frac{-(-2 \cdot 10^{-9} C)}{9 \cdot 10^{-2} m}$$

$$\cong 2 \cdot 10^2 \frac{J}{C} = 200V$$

RICORDA che :

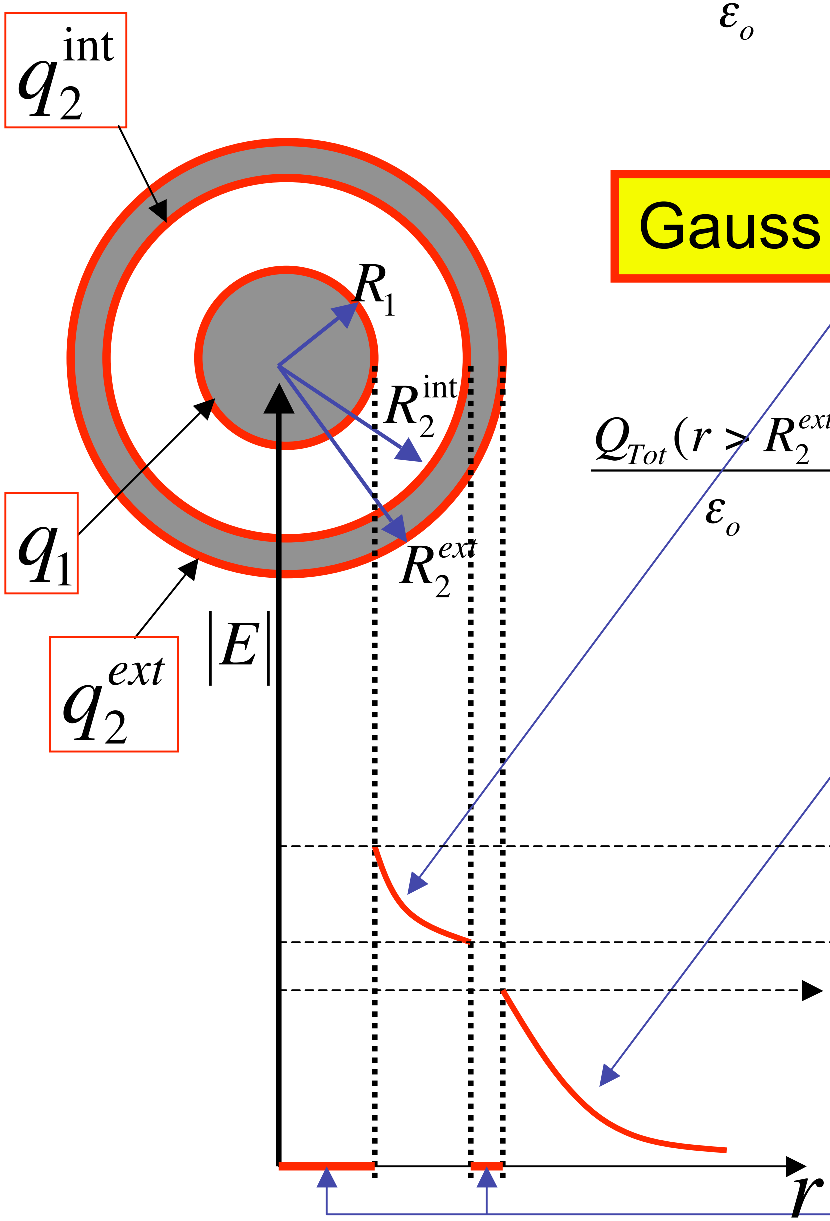
$$\frac{Q_{Tot}^{\text{int}}(R_1 < r < R_2^{\text{int}})}{\varepsilon_o} = \Phi(E(R_1 < r < R_2^{\text{int}})) = E(R_1 < r < R_2^{\text{int}}) \cdot S_r \Leftrightarrow$$

$$\boxed{\Leftrightarrow E(R_1 < r < R_2^{\text{int}}) = \frac{q_1}{4\pi\varepsilon_o r^2}}$$

Gauss

$$\frac{Q_{Tot}(r > R_2^{ext})}{\varepsilon_o} = \Phi(E(r > R_2^{ext})) = E(r > R_2^{ext}) \cdot S_r \Leftrightarrow$$

$$\boxed{\Leftrightarrow E(r > R_2^{ext}) = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r^2}}$$



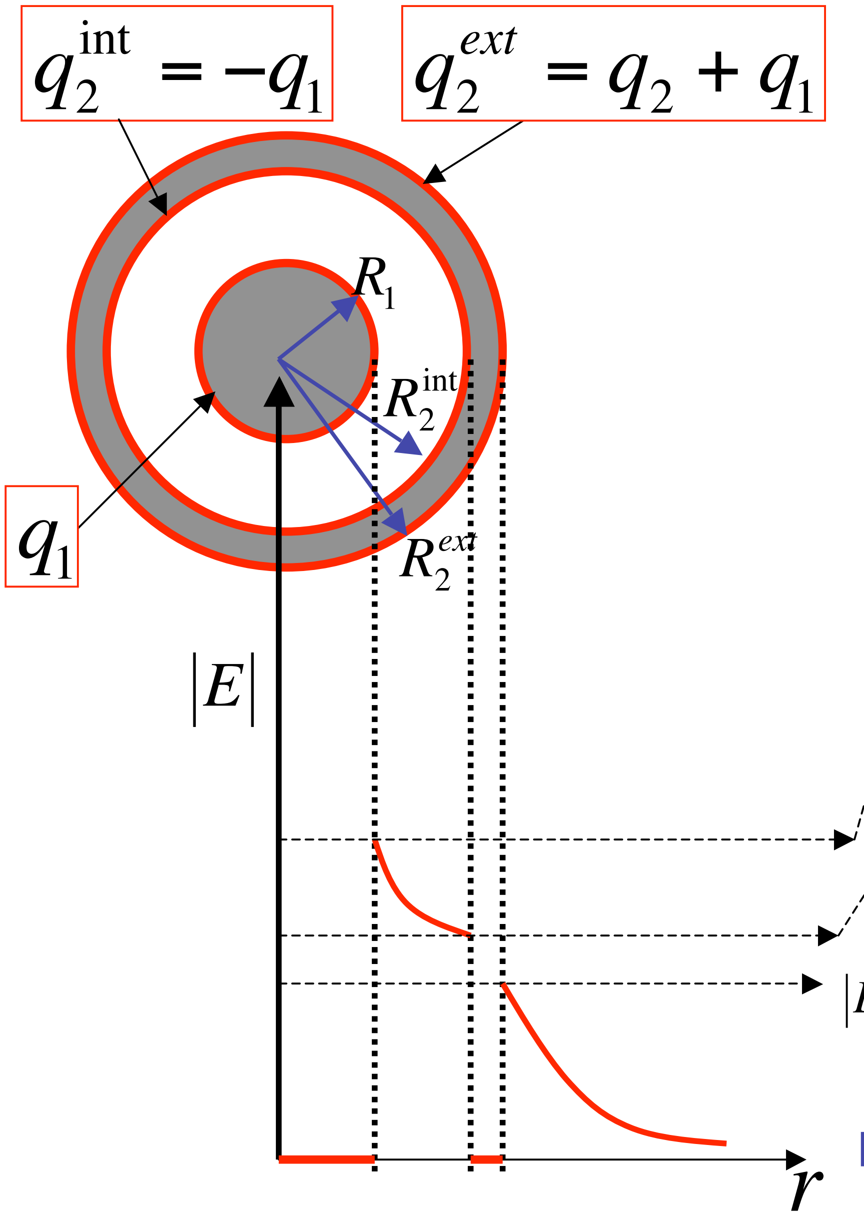
$$|E| = \lim_{r \to R_1} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o R_1^2}$$

$$|E| = \lim_{r \to R_2^{\text{int}}} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o R_2^{\text{int}\,2}}$$

$$|E| = \lim_{r \to R_2^{ext}} \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r^2} = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o R_2^{ext\,2}}$$

E=0 all’interno di un conduttore

Nell’esempio I.6.2 del Guerriero (e nella configurazione iniziale dell’esercizio):



$$|E| = \lim_{r\to R_1} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o R_1^2}$$

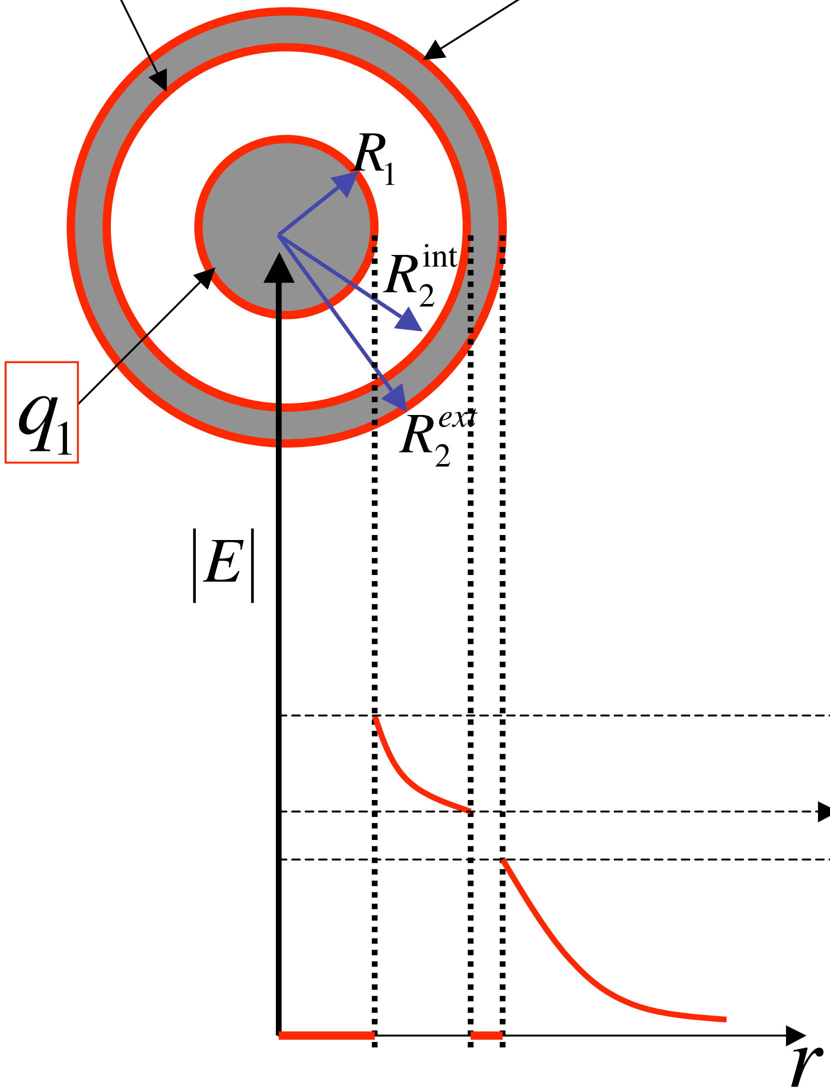
$$|E| = \lim_{r\to R_2^{\text{int}}} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o R_2^{\text{int}\,2}}$$

$$|E| = \lim_{r\to R_2^{ext}} \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r^2} = \frac{q_1 - q_1 + q_2 + q_1}{4\pi\varepsilon_o R_2^{ext\,2}} = \frac{q_1 + q_2}{4\pi\varepsilon_o R_2^{ext\,2}}$$

N.B.: nel nostro esercizio $q_2^{ext} = -2q_1 + q_1 = -q_1$

Nella configurazione finale dell'esercizio:

$$\boxed{q_2^{\text{int}} = -q_1} \qquad \boxed{q_2^{ext} = q_2 + q_1 - q_2 = q_1}$$



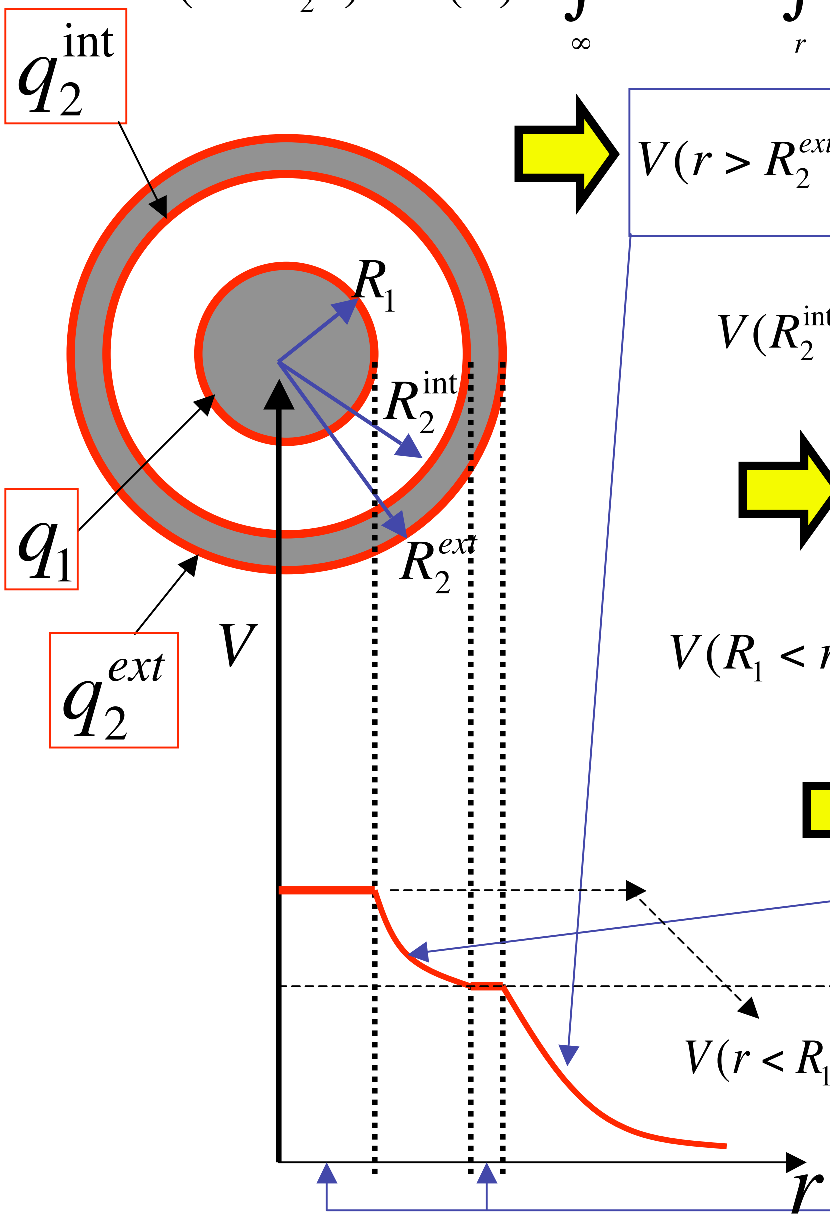
$$|E| = \lim_{r \to R_1} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o R_1^2}$$

$$|E| = \lim_{r \to R_2^{\text{int}}} \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{q_1}{4\pi\varepsilon_o {R_2^{\text{int}}}^2}$$

$$|E| = \lim_{r \to R_2^{ext}} \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r^2} = \frac{q_1 - q_1 + q_1}{4\pi\varepsilon_o {R_2^{ext}}^2} = \frac{q_1}{4\pi\varepsilon_o {R_2^{ext}}^2}$$

RICORDA ANCHE che :

$$V(r>R_2^{ext}) = V(\infty) - \int_\infty^r \vec{E}\cdot d\vec{\ell} = \int_r^\infty \vec{E}\cdot d\vec{\ell} = \int_r^\infty \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r^2} dr = -\frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o} \int_r^\infty \frac{-dr}{r^2}$$



$$\boxed{V(r>R_2^{ext}) = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r}}$$

$$V(R_2^{\text{int}}) = V(R_2^{\text{int}} < r < R_2^{ext}) = V(R_2^{ext}) = \lim_{r\to R_2^{ext}} V(r>R_2^{ext})$$

$$\boxed{V(R_2^{\text{int}}) = V(R_2^{ext}) = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o R_2^{ext}}}$$

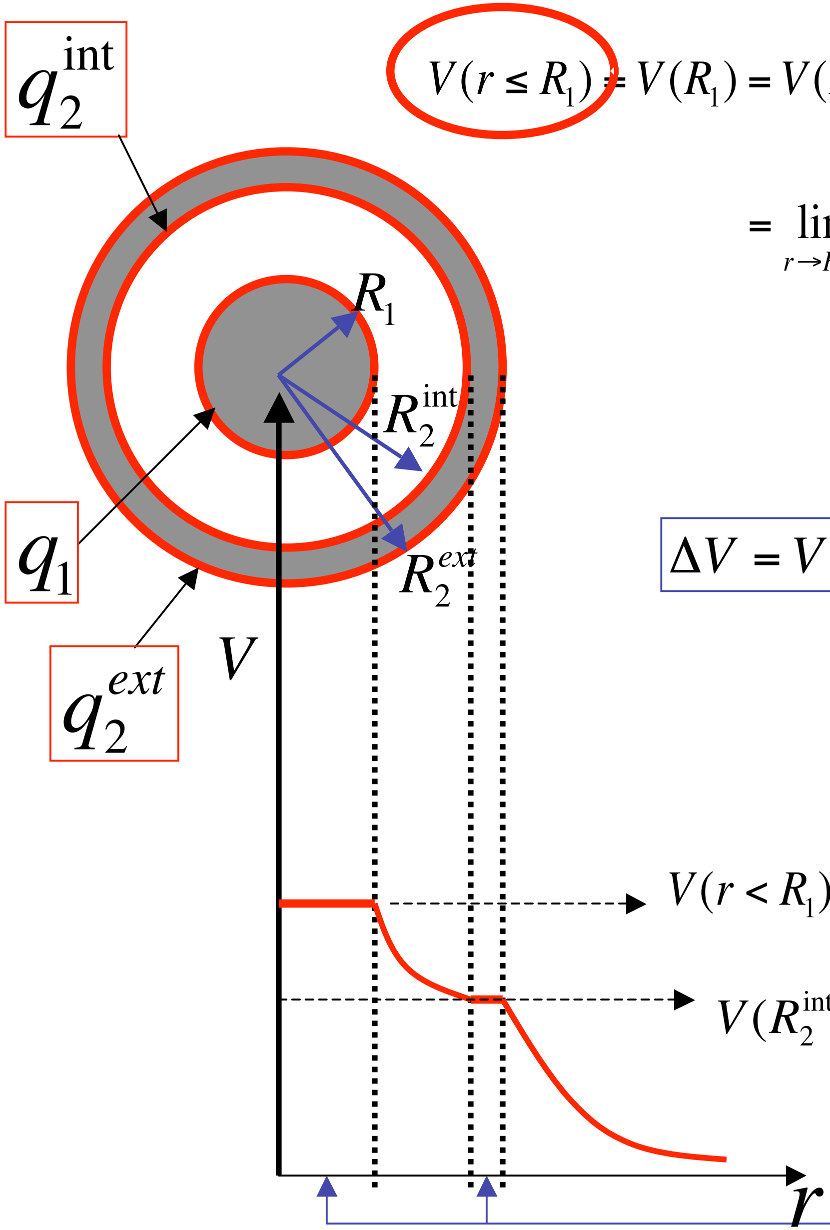
$$V(R_1 < r < R_2^{\text{int}}) = V(R_2^{\text{int}}) - \int_{R_2^{\text{int}}}^{r} \vec{E}\cdot d\vec{\ell} = V(R_2^{\text{int}}) + \int_r^{R_2^{\text{int}}} \vec{E}\cdot d\vec{\ell}$$

$$\boxed{V(R_1 < r < R_2^{\text{int}}) = V(R_2^{\text{int}}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R_2^{\text{int}}}\right)}$$

$$V(r<R_1) = V(R_1) = \lim_{r\to R_1} V(R_1 < r < R_2^{\text{int}}) = V(R_2^{\text{int}}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2^{\text{int}}}\right)$$

V=cost all’interno di un conduttore

Ecco dunque che il potenziale della sfera interna segue quello esterno al guscio:



$$V(r \le R_1) = V(R_1) = V(R_2^{\text{int}}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2^{\text{int}}}\right) = V(R_2^{ext}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2^{\text{int}}}\right) =$$

$$= \lim_{r \to R_2^{ext}} V(r > R_2^{ext}) + \underbrace{\frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2^{\text{int}}}\right)} \equiv \lim_{r \to R_2^{ext}} V(r > R_2^{ext}) + \cos t.$$

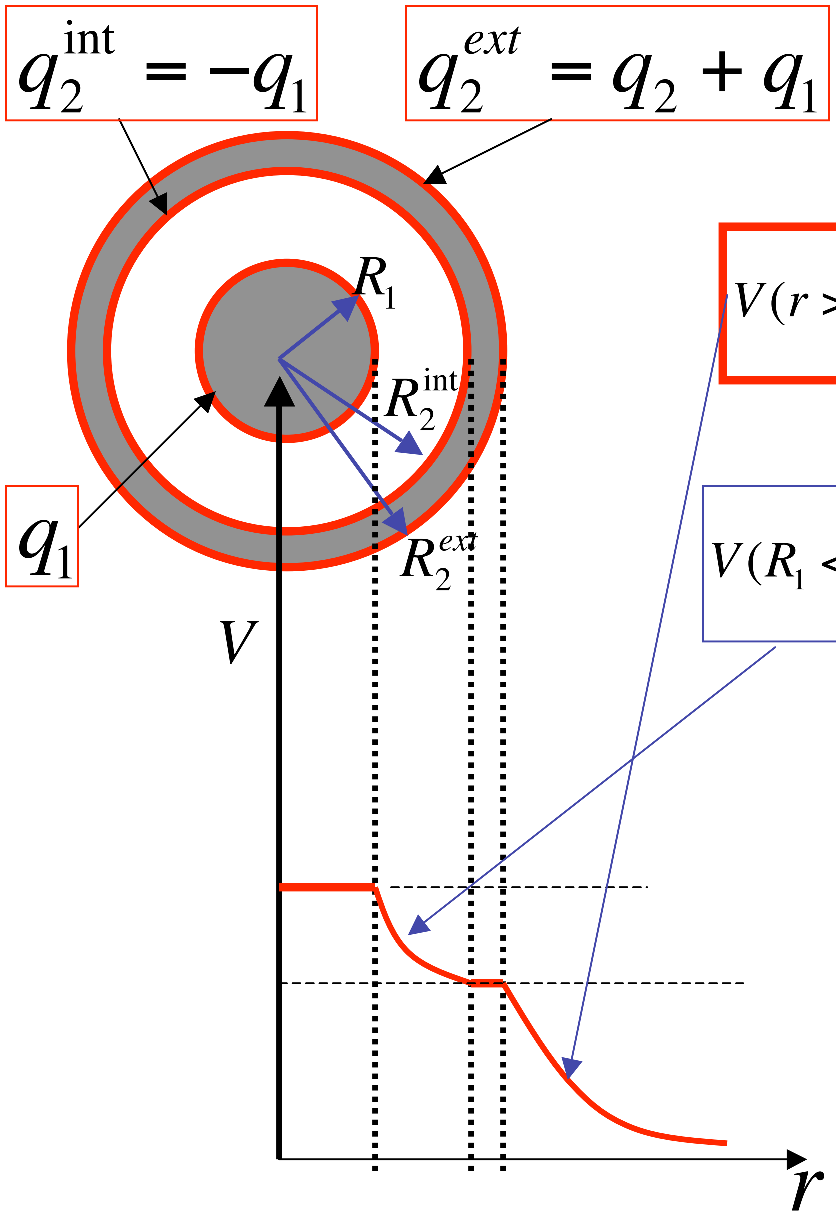
$$\boxed{\Delta V = V(R_1) - V(R_2^{\text{int}}) = V(R_1) - V(R_2^{ext}) \equiv V_{SFERA} - V_{GUSCIO}}$$

$$V(r < R_1) = V(R_1) = \lim_{r \to R_1} V(R_1 < r < R_2^{\text{int}})$$

$$V(R_2^{\text{int}}) = V(R_2^{\text{int}} < r < R_2^{ext}) = V(R_2^{ext}) = \lim_{r \to R_2^{ext}} V(r > R_2^{ext})$$

V=cost all’interno di un conduttore

Nell’esempio I.6.2 del Guerriero (e nella configurazione iniziale dell’esercizio):

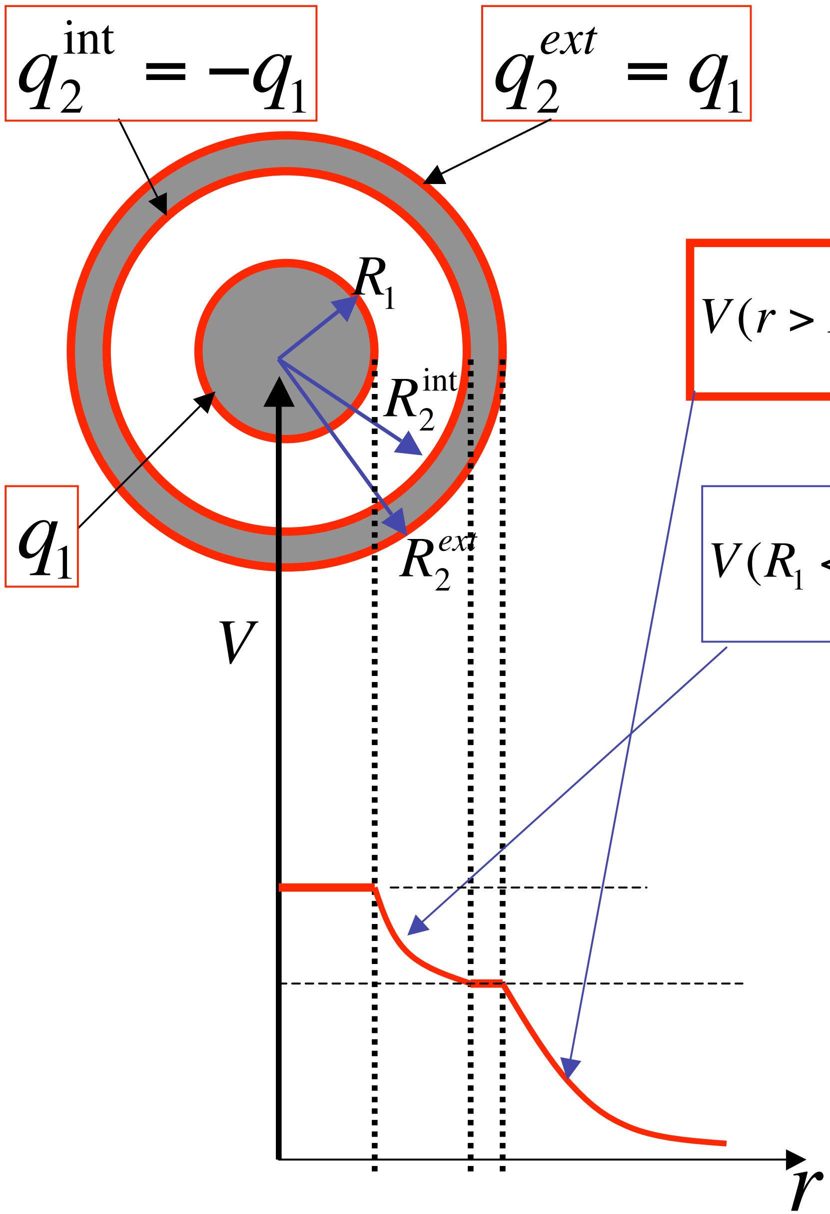


$$V(r > R_2^{ext}) = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r} = \frac{q_1 - q_1 + q_2 + q_1}{4\pi\varepsilon_o r} = \frac{q_1 + q_2}{4\pi\varepsilon_o r}$$

$$V(R_1 < r < R_2^{\text{int}}) = V(R_2^{\text{int}/ext}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R_2^{\text{int}}}\right) =$$

$$= \frac{q_1 + q_2}{4\pi\varepsilon_o R_2^{ext}} + \frac{q_1}{4\pi\varepsilon_o r} - \frac{q_1}{4\pi\varepsilon_o R_2^{\text{int}}}$$

Nella configurazione finale dell’esercizio:



$$V(r > R_2^{ext}) = \frac{q_1 + q_2^{\text{int}} + q_2^{ext}}{4\pi\varepsilon_o r} = \frac{q_1 - q_1 + q_1}{4\pi\varepsilon_o r} = \frac{q_1}{4\pi\varepsilon_o r}$$

$$V(R_1 < r < R_2^{\text{int}}) = V(R_2^{\text{int}/ext}) + \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} - \frac{1}{R_2^{\text{int}}}\right) =$$

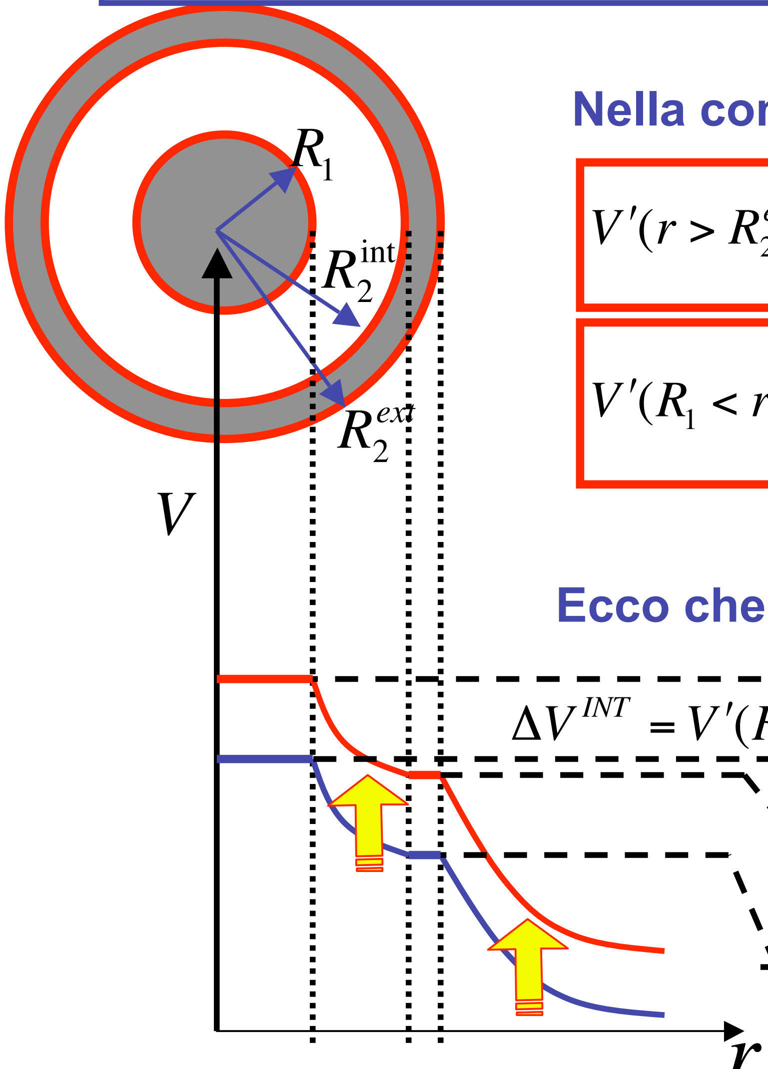
$$= \frac{q_1}{4\pi\varepsilon_o R_2^{ext}} + \frac{q_1}{4\pi\varepsilon_o r} - \frac{q_1}{4\pi\varepsilon_o R_2^{\text{int}}}$$

$$= \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} + \frac{1}{R_2^{ext}} - \frac{1}{R_2^{\text{int}}}\right)$$

Nella configurazione iniziale dell'esercizio:

$$V(r > R_2^{ext}) = \frac{q_1 + q_2}{4\pi\varepsilon_o r}$$

$$V(R_1 < r < R_2^{\text{int}}) = \frac{q_1 + q_2}{4\pi\varepsilon_o R_2^{ext}} + \frac{q_1}{4\pi\varepsilon_o r} - \frac{q_1}{4\pi\varepsilon_o R_2^{\text{int}}} = \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} + \frac{1}{R_2^{ext}} - \frac{1}{R_2^{\text{int}}}\right) + \frac{q_2}{4\pi\varepsilon_o R_2^{ext}}$$



Nella configurazione finale dell'esercizio:

$$V'(r > R_2^{ext}) = \frac{q_1}{4\pi\varepsilon_o r}$$

$$V'(R_1 < r < R_2^{\text{int}}) = \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r} + \frac{1}{R_2^{ext}} - \frac{1}{R_2^{\text{int}}}\right)$$

Ecco che la variazione (si ripercuote all'interno):

$$\Delta V^{INT} = V'(R_1 < r < R_2^{\text{int}}) - V(R_1 < r < R_2^{\text{int}}) = \frac{-q_2}{4\pi\varepsilon_o R_2^{\text{int}}} > 0$$

$$\Delta V^{EXT} = V'(r > R_2^{ext}) - V(r > R_2^{ext}) = \frac{-q_2}{4\pi\varepsilon_o R_2^{\text{int}}}$$

$$R_2^{\text{int}} \approx R_2^{ext} \equiv R_2 \Rightarrow \boxed{\Delta V^{INT} = \Delta V^{EXT} = -\frac{q_2}{4\pi\varepsilon_o R_2} \cong 200V}$$