Non-Markovian Counting Processes

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Collaborations with:

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A.-P. Jauho (Copenhagen), M. Sassetti (Genoa)

Outline

- Theory
  - Counting problems & FCS in a nutshell
  - FCS for non-Markovian Master Equation
  - Self Consistent Equation
  - Non-Markovian expansion & theorem
  - Cumulants via PT & recursion

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- Theory
  - Counting problems & FCS in a nutshell
  - FCS for non-Markovian Master Equation
  - Self Consistent Equation
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  - Cumulants via PT & recursion
- Applications
  - Counting statistics in DQD with decoherence

Why Counting Processes ?

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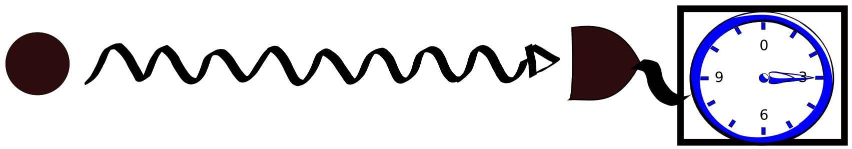
- Photon-counting experiments



Glauber,Mandel,Arecchi,...

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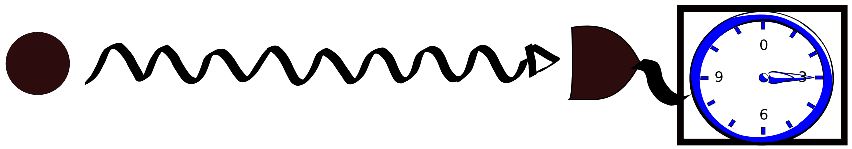

Glauber,Mandel,Arecchi,...

- Quantum dot counters

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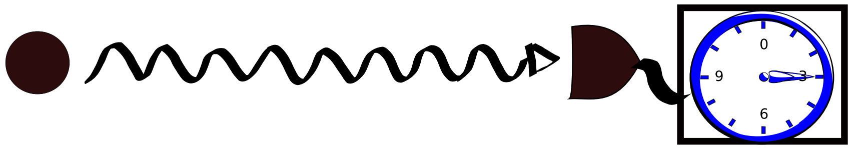

Glauber,Mandel,Arecchi,...

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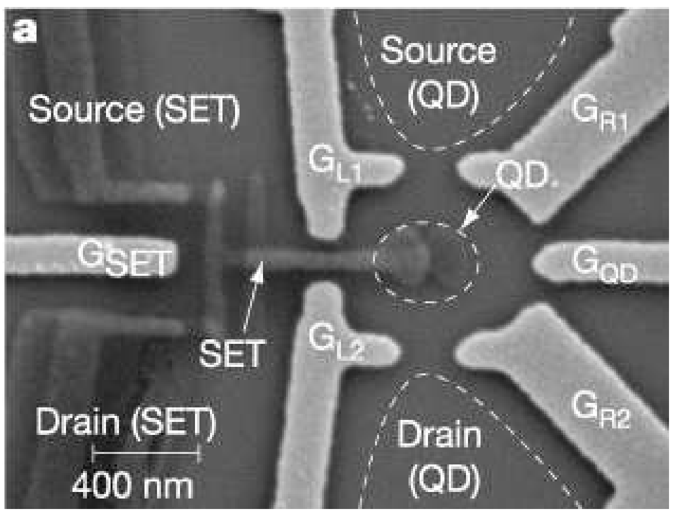

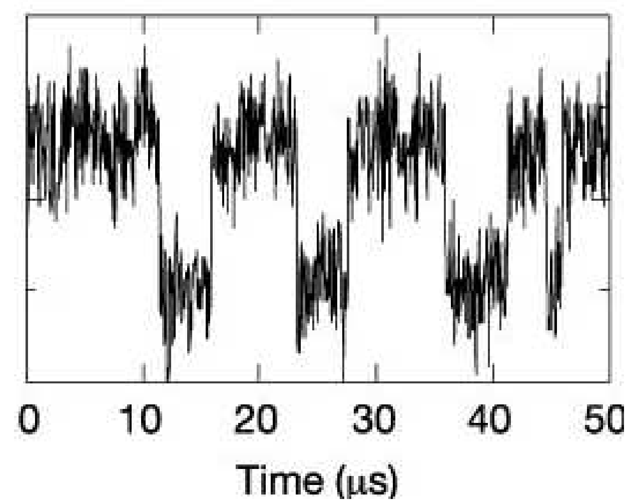

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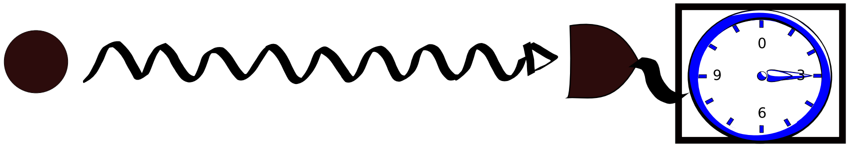

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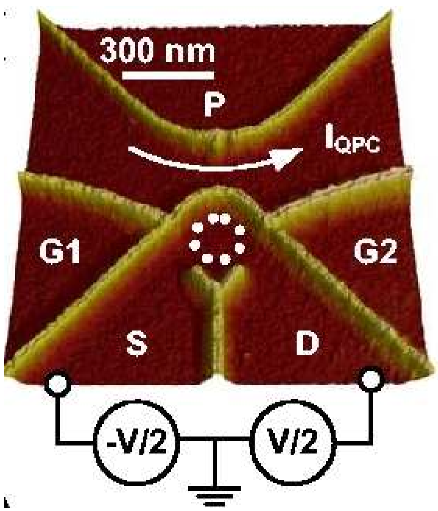

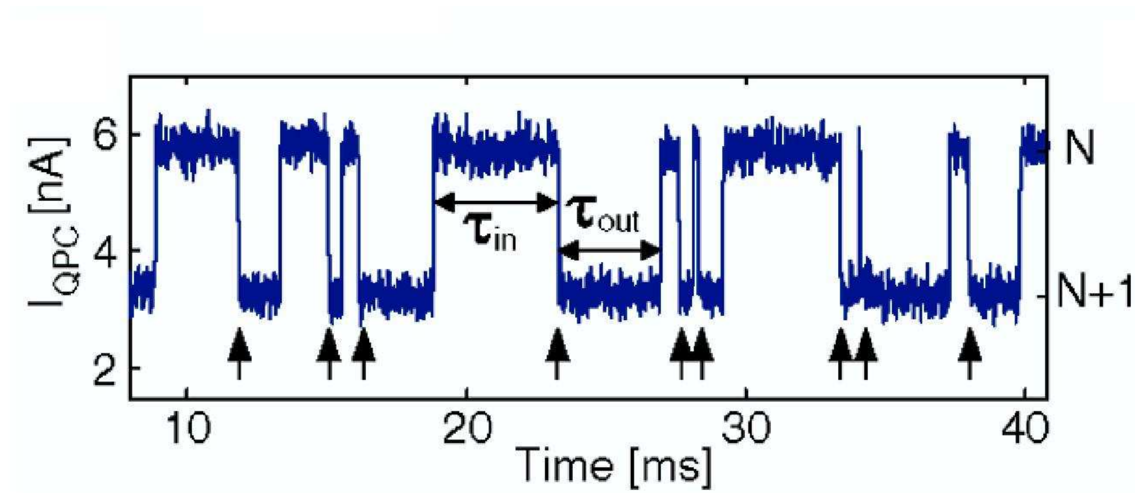

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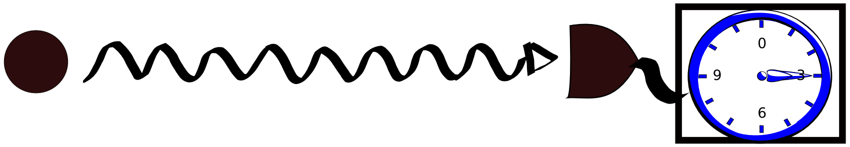

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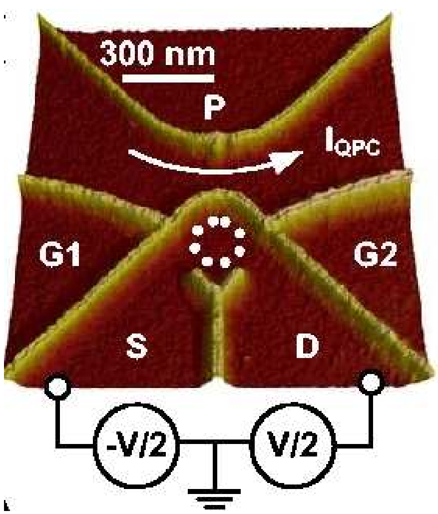

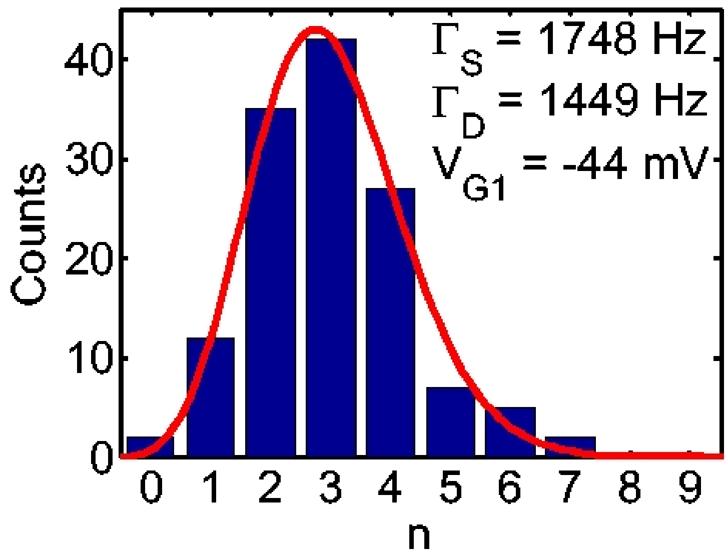

Counting Probability distribution $P(n,t)$

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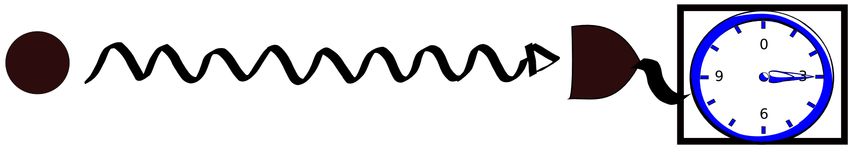

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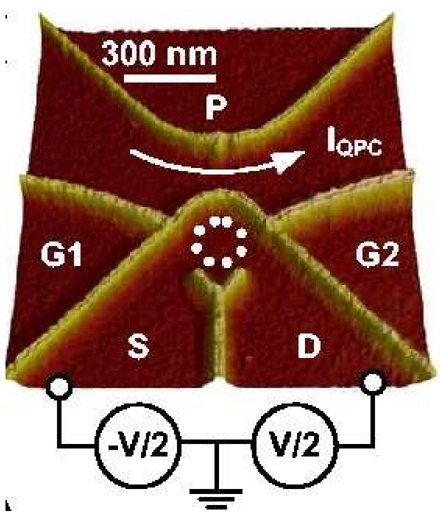

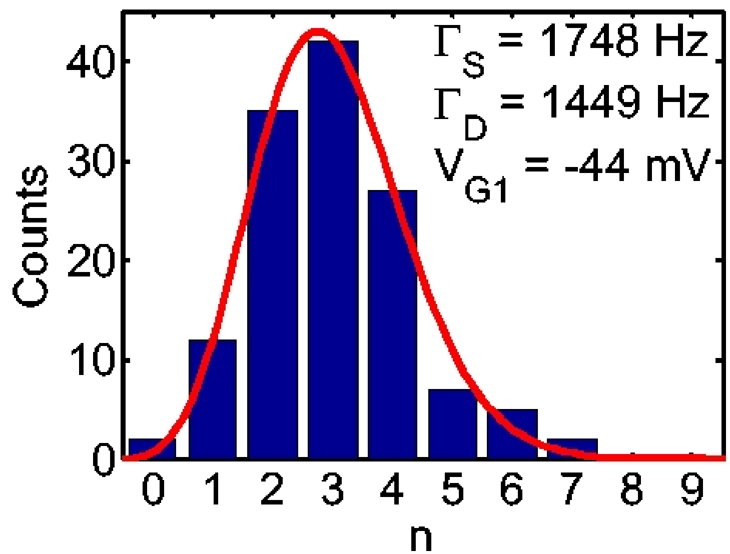

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- Phonon counting (Yoctocalorimetry) M. L. Roukes, Physica B '99

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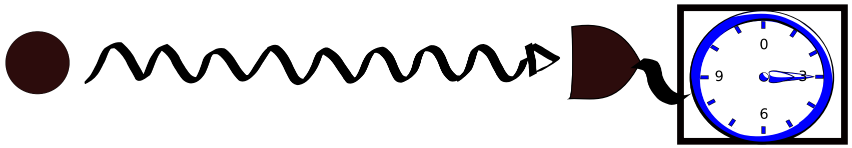

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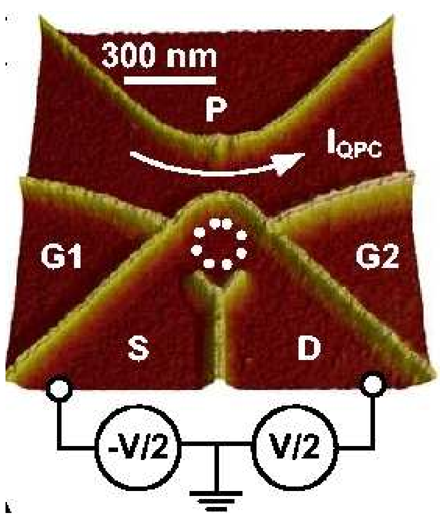

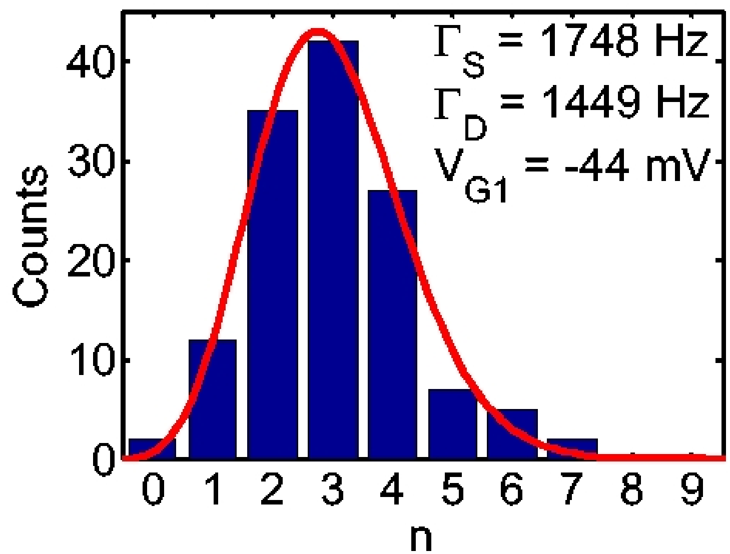

Counting Probability distribution $P(n,t)$

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- Phonon counting (Yoctocalorimetry) M. L. Roukes, Physica B '99
- Other countable quantities (Flow Cytometry,...)

Full Counting Statistics

$$P(\chi, t) = \mathrm{e}^{S(\chi,t)} = \sum_n P(n,t)\, \mathrm{e}^{in\chi}$$

$S(\chi, t)$ Cumulant Generating Function (CGF)

Full Counting Statistics

$$P(\chi, t) = \mathrm{e}^{S(\chi,t)} = \sum_n P(n,t)\, \mathrm{e}^{in\chi}$$

$S(\chi, t)$ Cumulant Generating Function (CGF)

Irreducible moments: cumulants

$$\langle\langle n(t) \rangle\rangle_k = \frac{\partial^k S(\chi,t)}{\partial^k (i\chi)}\Big|_{\chi \to 0}$$

Full Counting Statistics

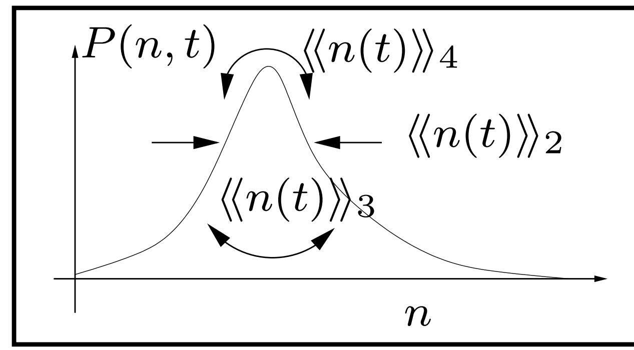
$$P(\chi,t) = \mathrm{e}^{S(\chi,t)} = \sum_n P(n,t)\,\mathrm{e}^{in\chi}$$

$S(\chi,t)$ Cumulant Generating Function (CGF)

- Mean Current

  $$\langle\langle n(t)\rangle\rangle_1 = \frac{1}{e}\int_0^t dt' \langle I(t')\rangle \underset{t\ggg\tau_c}{=} \frac{t}{e}\langle I\rangle$$

  $t \ggg \tau_c$ correlation time



L.S. Levitov and G. B. Lesovik, JETP Lett. '93

"Quantum Noise" Ed. Yu. Nazarov '03

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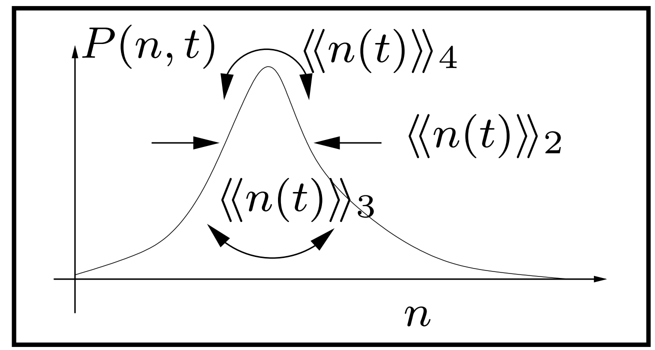

- Mean Current
  $\langle I \rangle = \frac{e}{t} \langle\langle n(t) \rangle\rangle_1$

Full Counting Statistics

$$P(\chi, t) = \mathrm{e}^{S(\chi,t)} = \sum_n P(n,t)\, \mathrm{e}^{in\chi}$$

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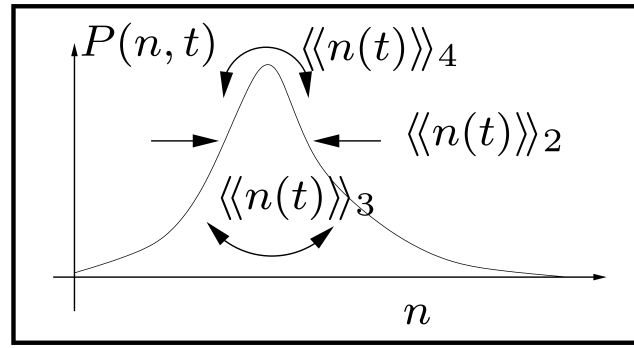


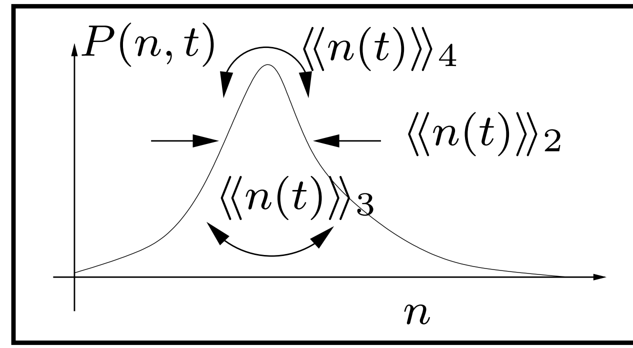
- Mean Current
  $\langle I \rangle = \frac{e}{t}\langle\langle n(t)\rangle\rangle_1$

- Current Noise $S(0) = \langle\langle I \rangle\rangle_2 \underset{t \ggg \tau_c}{=} \frac{e^2}{t}\langle\langle n(t)\rangle\rangle_2$

Full Counting Statistics

$$P(\chi,t) = \mathrm{e}^{S(\chi,t)} = \sum_n P(n,t)\,\mathrm{e}^{in\chi}$$

$S(\chi,t)$ Cumulant Generating Function (CGF)



- Mean Current
  $\langle I\rangle = \frac{e}{t}\langle\langle n(t)\rangle\rangle_1$

- Zero frequency irreducible moments (cumulants)

$$\langle\langle I\rangle\rangle_k = \frac{(e)^k}{t}\frac{\partial^k S(\chi,t)}{\partial^k(i\chi)}\Big|_{\chi\to 0}$$

FCS as large deviation function

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- Poisson distribution $P(n,t) = \frac{(\gamma t)^n}{n!} e^{-\gamma t}$ $\bar{I} = \gamma t$

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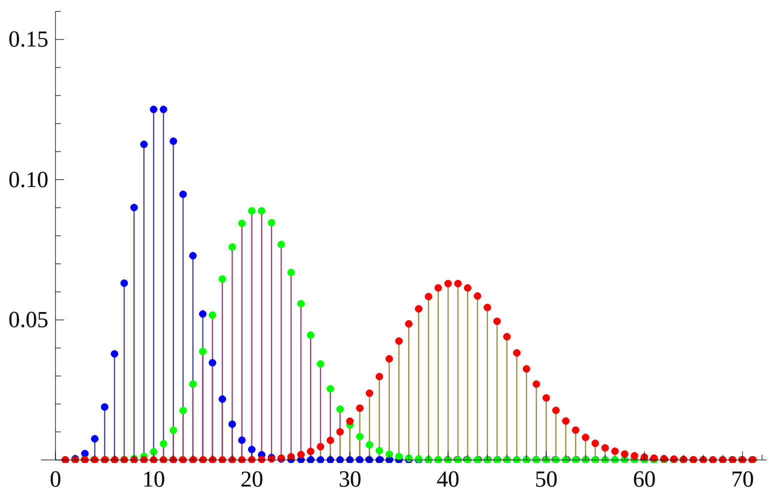
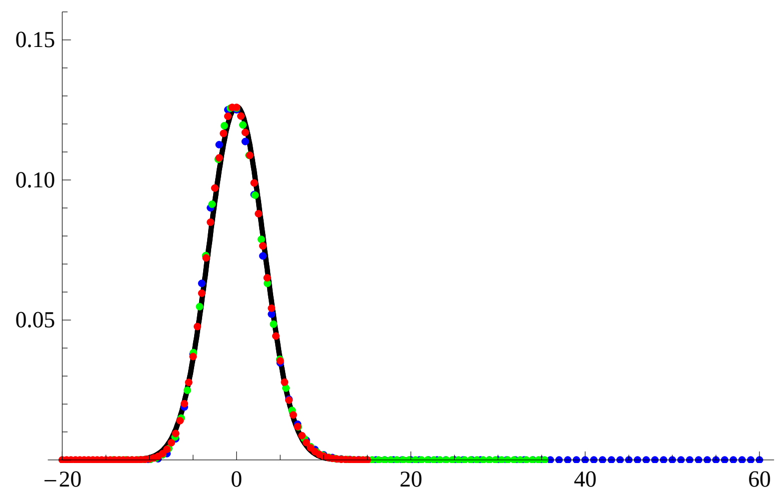
- Particle view & Central limit

$P(n)$ $\qquad\qquad$ $P(n)\sqrt{\gamma t}$





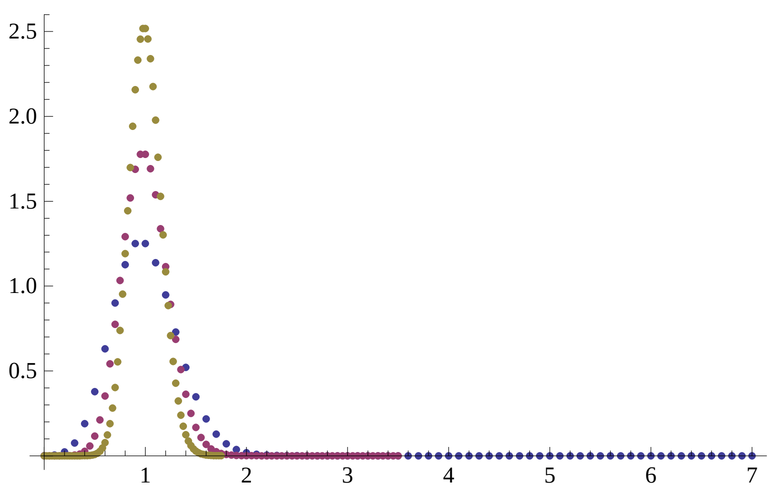
n $\qquad\qquad$ $(n-\gamma t)/\sqrt{\gamma t}$

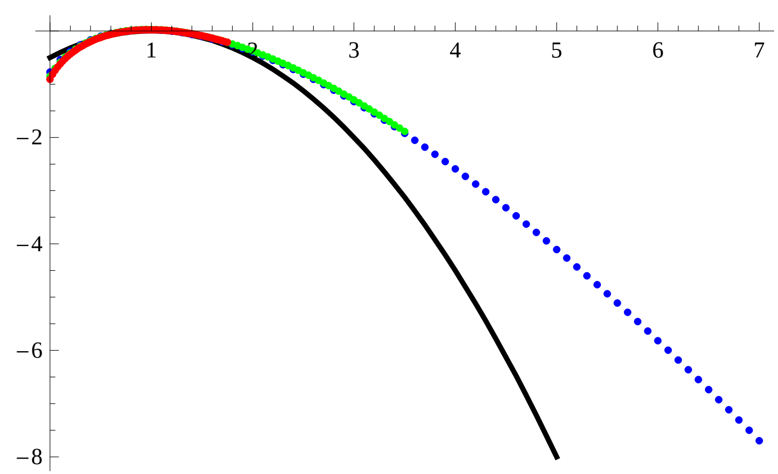
$\gamma t = 10, 20, 40$ $\qquad$ – **Gaussian fit**

FCS as large deviation function

- Poisson distribution $P(n,t) = \dfrac{(\gamma t)^n}{n!} e^{-\gamma t}$ $\qquad \bar{I} = \gamma t$
- Particle view
- Current view & Large deviation

$P(I)$ $\qquad\qquad$ $\log P(I)/\bar{I}$





$I/\bar{I}$ $\qquad\qquad$ $I/\bar{I}$

$\bar{I} = \gamma t = 10, 20, 40$ $\qquad$ – **Gaussian fit**

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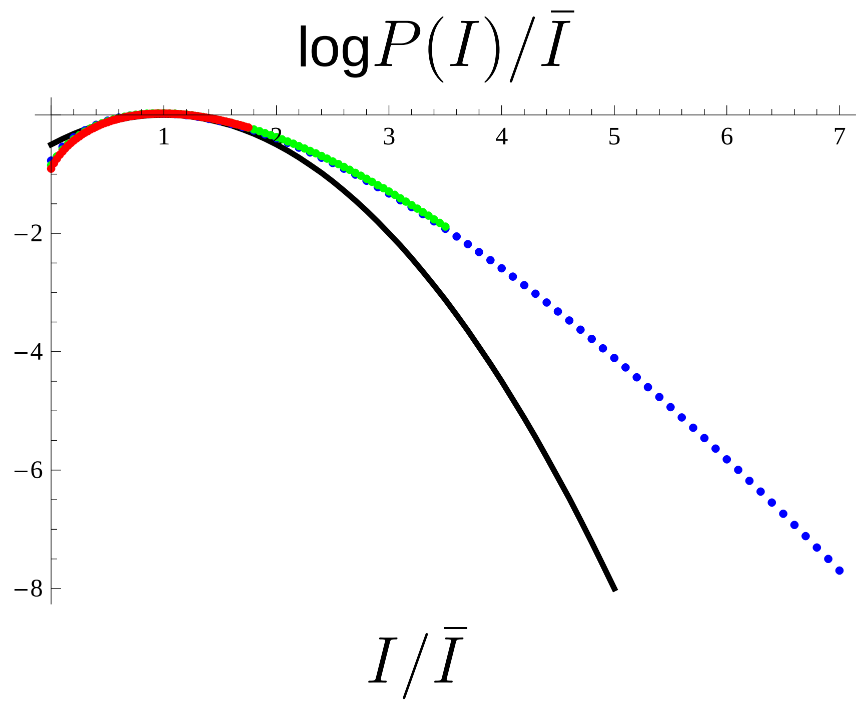
- Poisson distribution $P(n,t) = \dfrac{(\gamma t)^n}{n!} e^{-\gamma t}$ $\qquad \bar{I} = \gamma t$
- Particle view
- Current view
- FCS is the large deviation function



FCS as large deviation function

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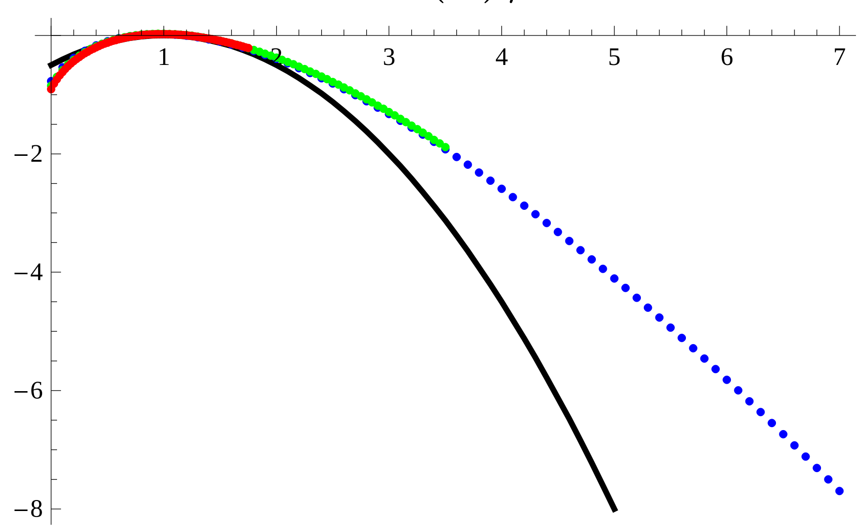
$\log P(I)/\bar{I}$



$I/\bar{I}$

- Rare fluctuations

Markovian vs Non-Markovian

Quantum mechanics is **Markovian**

Markovian vs Non-Markovian

Quantum mechanics is **Markovian**

$$H,\ \Psi(0) \Longrightarrow \Psi(t)$$

Markovian vs Non-Markovian

Quantum mechanics is **Markovian**

but

Markovian vs Non-Markovian

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but

real life is **non-Markovian**!

Markovian vs Non-Markovian

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One has to trace out some external degrees of freedom

Markovian vs Non-Markovian

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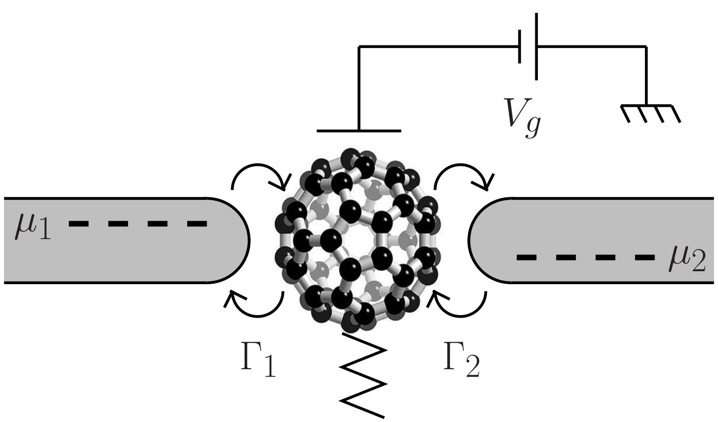
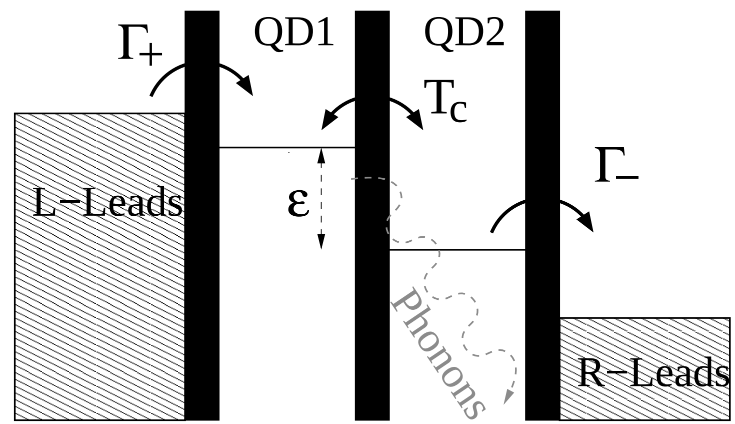
NEMS



Qubit



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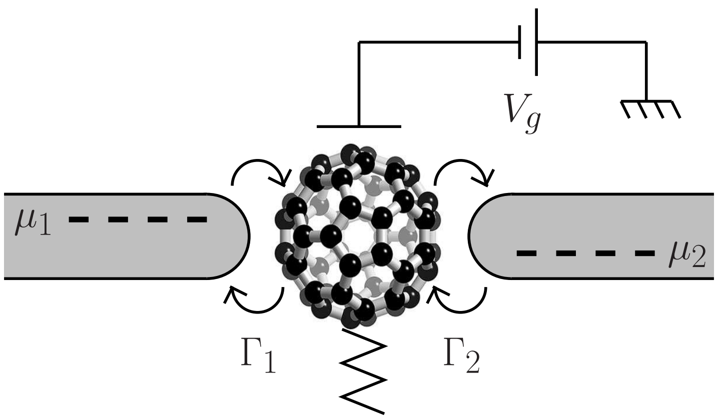

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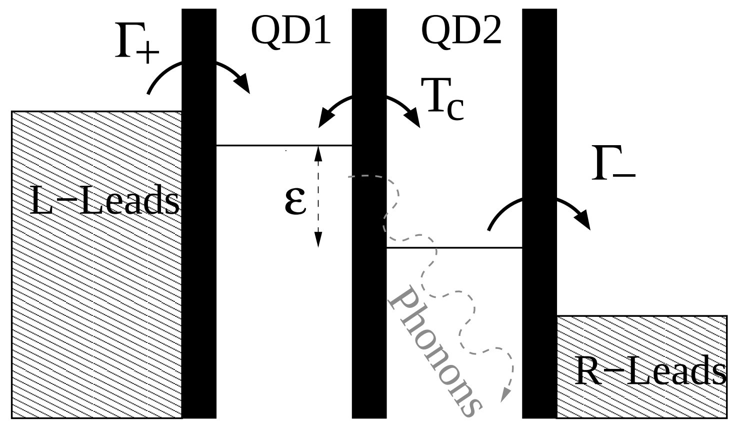

Open systems

$$H = H_0 + H_B + H_I$$

$$\rho(t) = \mathrm{Tr}_B\left[\rho_{tot}(t)\right]$$

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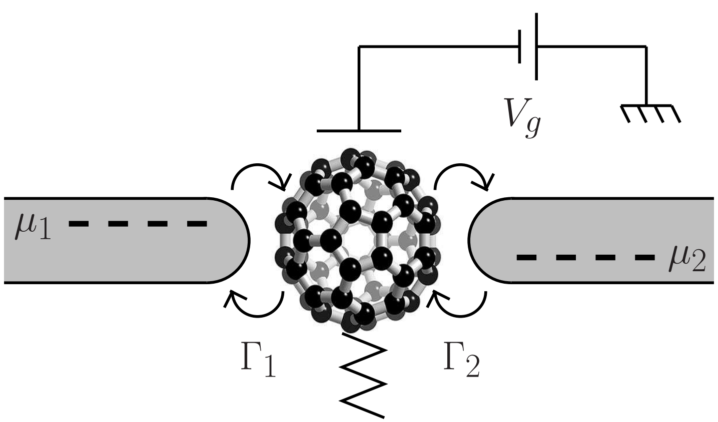

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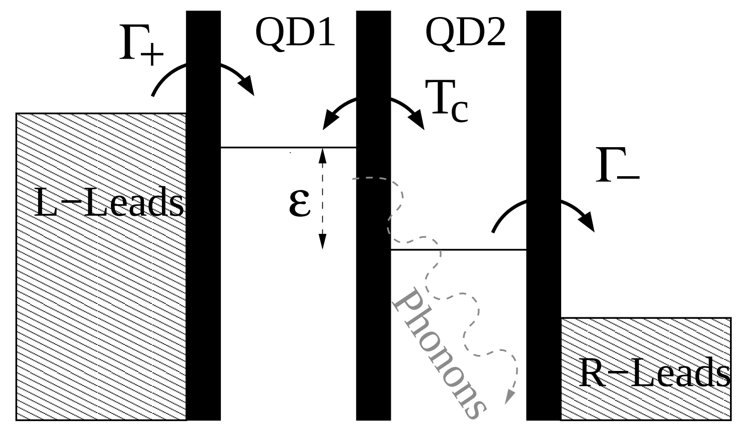

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$$\dot{\rho}(t) = \int_0^t dt' \mathrm{Tr}_B\left[\left[H_I(t), \left[H_I(t'), \rho(t')\right]\right]\right]$$

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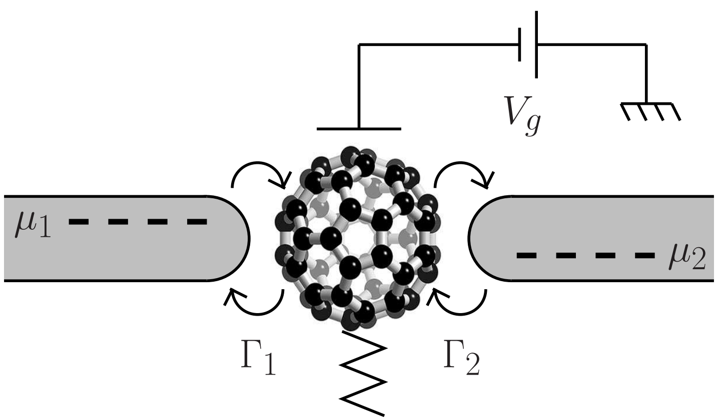

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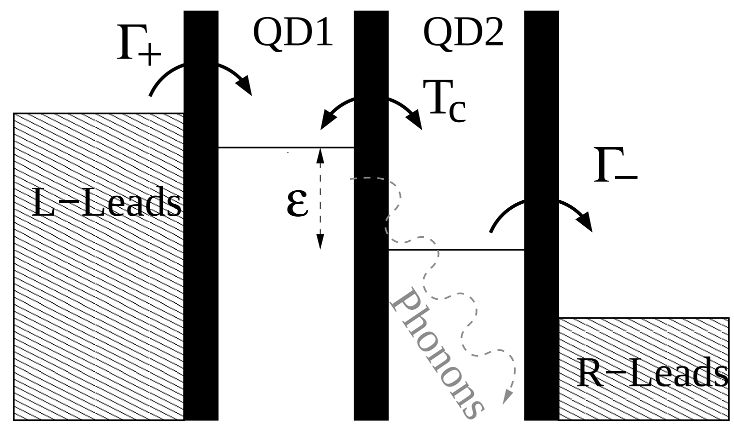

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n-Resolved Generalized Master Equation

Quantum optics

$|\rho(n,t)\rangle = \mathrm{Tr}_B[\mathbb{P}_n \chi(t)]$

M.B. Plenio and P. L. Knight, **70**, 101 (1998).

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$$\frac{d}{dt}|\rho(n,t)\rangle = \sum_{n'} \int_0^t dt' \, \mathbf{W}(n-n', t-t')|\rho(n',t')\rangle + |\gamma(n,t)\rangle$$

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  Non-Markovian Exp.

- $$S(\chi) = \sum_{j=0} S_j(\chi)$$

A.B., J.König, R. Fazio, PRL (2006)

n-Resolved Generalized Master Equation

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- $$S(\chi) = \sum_{j=0} S_j(\chi) \qquad \Longleftrightarrow$$

A.B., J.König, R. Fazio, PRL (2006)

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Non-Markovian Exp.

Self-consistent Eq.

- $$S(\chi) = \sum_{j=0} S_j(\chi) \qquad \Longleftrightarrow \qquad z^\star - \lambda(\chi, z^\star) = 0$$

A.B., J.König, R. Fazio, PRL (2006)

C. Flindt, A.B. *et al.*, PRL (2008)

n-Resolved Generalized Master Equation

Quantum optics

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- Fourier-Laplace Transformations in χ and z

n-Resolved Generalized Master Equation

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- Fourier-Laplace Transformations in χ and z
- Dyson equation

$$\boxed{z|\rho(\chi,z)\rangle - |\rho^{in}\rangle = \mathbf{W}(\chi,z) \cdot |\rho(\chi,z)\rangle + |\gamma(\chi,z)\rangle}$$

n-Resolved Generalized Master Equation

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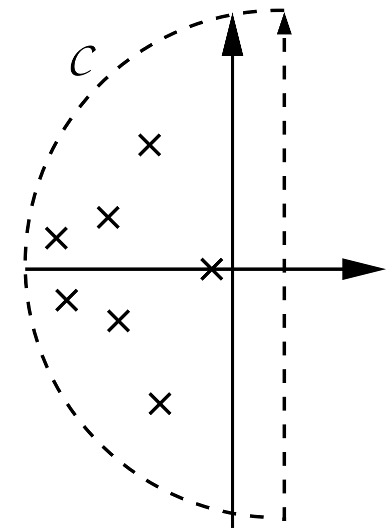

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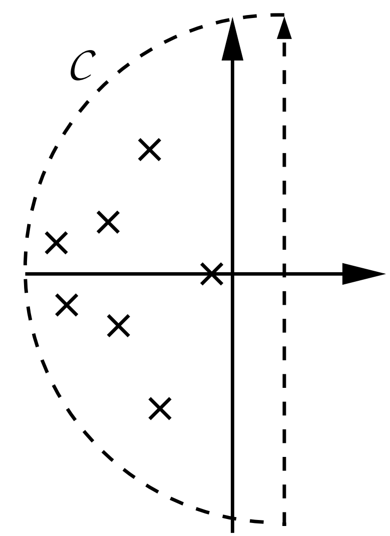

Long time limit

- $S(\chi) \equiv z^*(\chi)$ Self consistent Eq.

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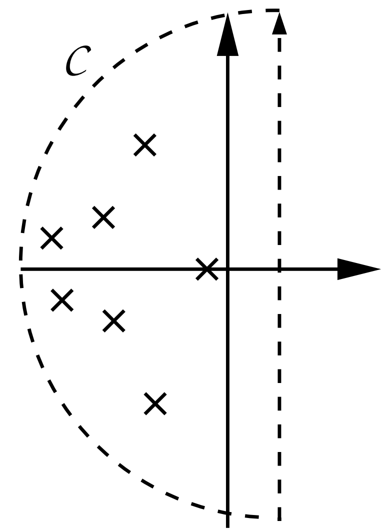

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C. Flindt, A.B. *et al.*, PRL '08 $\longrightarrow$ Bagrets& Nazarov, PRB '03

Finite Frequency Noise

- Mac Donald formula

$$S_{II}(\omega) = \omega \int_0^{\infty} dt \sin(\omega t) \frac{d}{dt} \langle\langle n \rangle\rangle_2(t)$$

Finite Frequency Noise

- Laplace domain

$$S_{II}(\omega) = -\frac{\omega^2}{2}\left[\langle\langle n\rangle\rangle_2(z = i\omega) + (\omega \to -\omega)\right]$$

Finite Frequency Noise

- Second cumulant

$$\langle\langle n \rangle\rangle_2(z) = \frac{\partial^2}{\partial(i\chi)^2} \langle \tilde{0} | \mathbf{G}(\chi, z) | \tilde{\rho}^{in}(\chi, z) \rangle \Big|_{\chi \to 0}$$

- Knowledge of full pole structure

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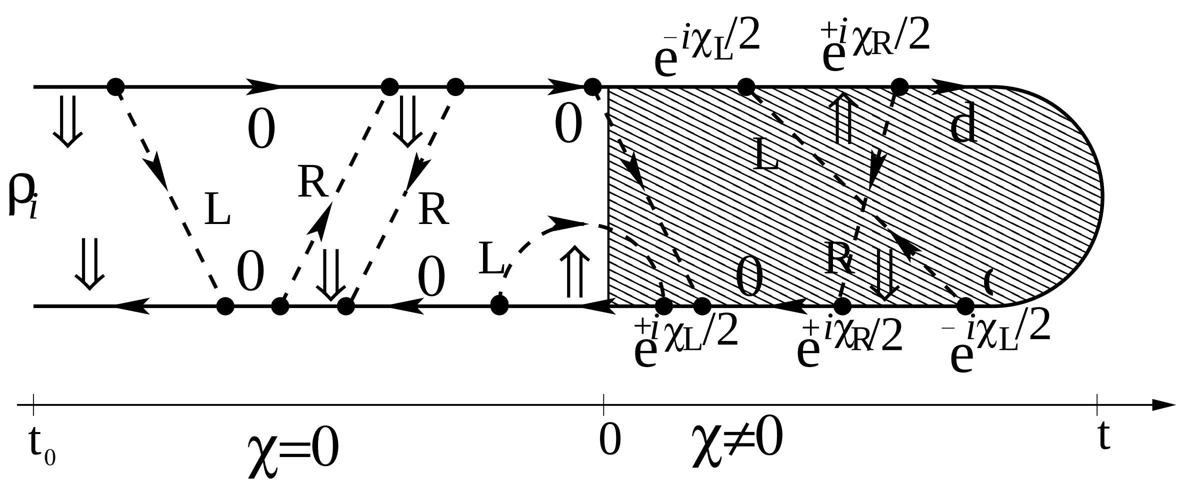
- Knowledge of initial correlations $|\tilde{\rho}^{in}(\chi,z)\rangle$



Keldysh countour

J. K onig et al PRL '96

A.B. et al. PRL'06

Non-Markovian Expansion

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A.B., J.König, R. Fazio, PRL (2006)

Non-Markovian Theorem

- Perturbation theory vs Higher Cumulants

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Evaluation n-th cumulants in m-th order perturbation requires k-th order non-Markovian term with $k = \min\{n,m\} - 1$

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- Evaluation n-th cumulants in m-th order perturbation requires k-th order non-Markovian term with $k = \min\{n, m\} - 1$
- $n = 1$ and m $\rightsquigarrow k = 0$
  Current doesn't require any Non-Markovian corrections!
  20 yrs of quantum transport

Non-Markovian Theorem

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- Evaluation n-th cumulants in m-th order perturbation requires k-th order non-Markovian term with $k = \min\{n, m\} - 1$
- $n = 2$ and $m = 1 \rightsquigarrow k = 0$
  First order noise doesn't require any Non-Markovian terms!

  S. Hershfield *et al.* PRB 1993, A. N. Korotkov PRB 1994

Non-Markovian Theorem

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- Evaluation n-th cumulants in m-th order perturbation requires k-th order non-Markovian term with $k = \min\{n, m\} - 1$
- $n = 2$ and m $\rightsquigarrow k = 1$
  Second order noise requires Non-Markovian corrections! Thielmann *et al.* PRL 2005

Non-Markovian Theorem

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- Evaluation n-th cumulants in m-th order perturbation requires k-th order non-Markovian term with $k = \min\{n, m\} - 1$
- First order non-Markovian GME can be written as a Markovian GME

Cumulants: Recursive Scheme

Two step problem:

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- 1. Calculate the eigenvalue expansion
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- Algebraical scheme convenient for *analitycal* calculations and also in *numerical* evaluations
- We do not need numerical derivatives
- Tested for many different cases

Cumulants

If $\lambda_0(\chi, z) = \sum_{k,l=0}^{\infty} \frac{(i\chi)^k}{k!} \frac{z^l}{l!} c^{(k,l)}$

- Current
  $\langle\langle I \rangle\rangle_1 = c^{(1,0)}$

Cumulants

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- Skewness
  $\langle\langle I^3 \rangle\rangle =$
  $c^{(3,0)} + 3c^{(2,0)}c^{(1,1)} + 3c^{(1,0)} \left[c^{(1,0)}c^{(1,2)} + 2(c^{(1,1)})^2 + c^{(2,1)} \right]$

Cumulants

If $\lambda_0(\chi, z) = \sum_{k,l=0}^{\infty} \frac{(i\chi)^k}{k!} \frac{z^l}{l!} c^{(k,l)}$

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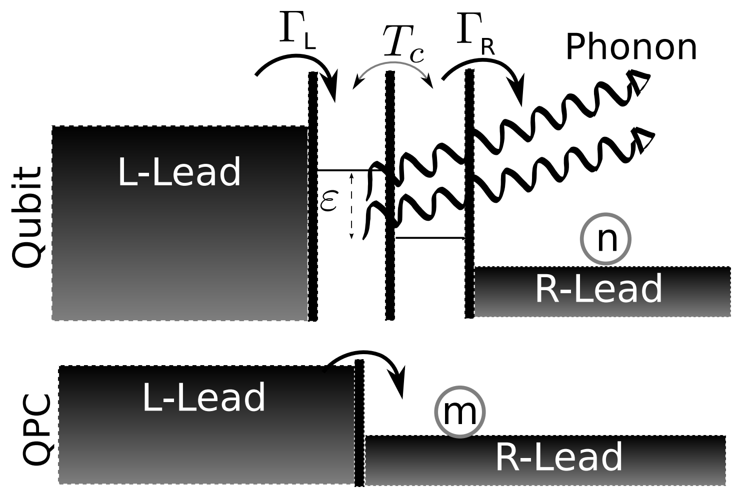
- Doing perturbation theory in $\mathbf{W}'$
  $\mathbf{W}'(\chi, z) \equiv \mathbf{W}(\chi, z) - \mathbf{W}(0,0)$

$$[\mathbf{W}(0,0) + \mathbf{W}'(\chi, z)] |0(\chi, z)\rangle = \lambda_0(\chi, z) |0(\chi, z)\rangle$$

we obtain $c^{(k,l)}$ as function of $\mathbf{R} = \mathbf{Q}\mathbf{W}^{-1}\mathbf{Q}$

Dissipative DQD coupled with a QPC

$$\hat{H} = \hat{H}_{DD} + \hat{V}_B + \hat{H}_B + \hat{H}_{\text{res}} + \hat{H}_T + + \hat{H}_{DD-QPC} + \hat{H}_{QPC}$$



$$|0\rangle, \ |+\rangle, \ |-\rangle, \ \not{|2\rangle}$$

Stoof&Nazarov PRB ’96

Aguado&Brandes PRL ’04

Kießlich PRL ’07

Gurvitz PRB ’97

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⋮ ⋮ ⋮

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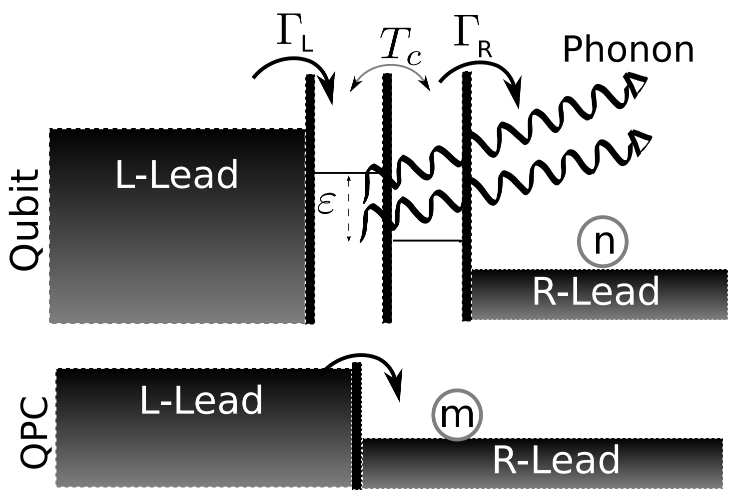

Spin Boson

$$\hat{H}_{DD} + \hat{V}_B + \hat{H}_B$$

$$|0\rangle, \ |+\rangle, \ |-\rangle, \ |2\rangle$$

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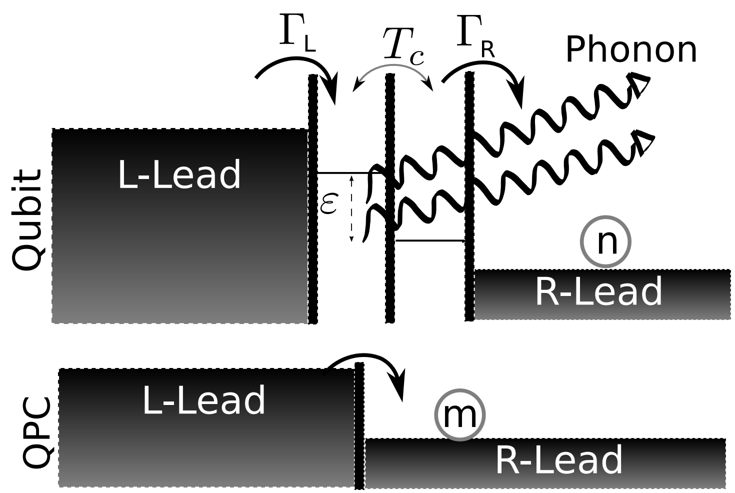

Spin

$$\hat{H}_{DD} = \frac{\varepsilon}{2}\hat{\sigma}_z + T_c\hat{\sigma}_x$$

$$|0\rangle,\ |+\rangle,\ |-\rangle,\ |2\rangle$$

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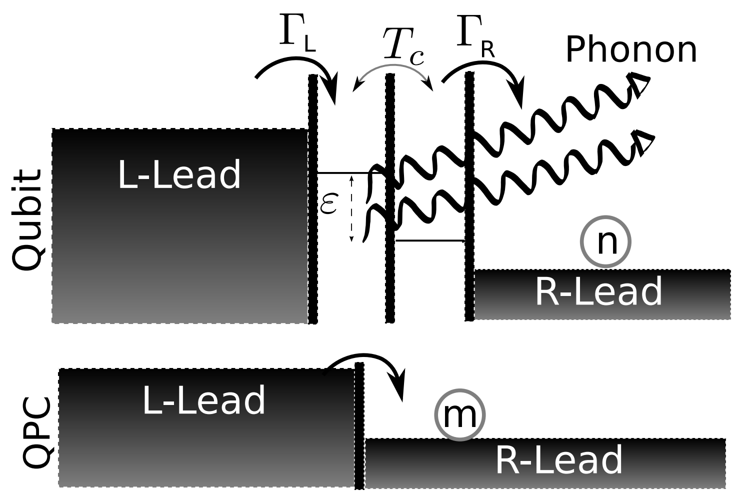

Coupling σ_z

$$\hat{V}_B = \hat{\sigma}_z \sum_j \frac{c_j}{2}(\hat{a}_j^\dagger + \hat{a}_j)$$

$$|0\rangle, \; |+\rangle, \; |-\rangle, \; |2\rangle$$

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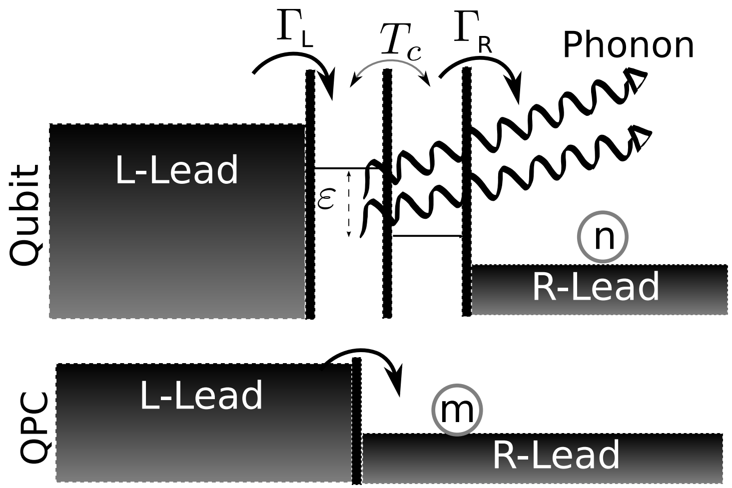

Boson bath

$$H_B = \sum_j \hbar\omega_j \hat{a}_j^\dagger \hat{a}_j$$

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Dissipative DQD coupled with a QPC

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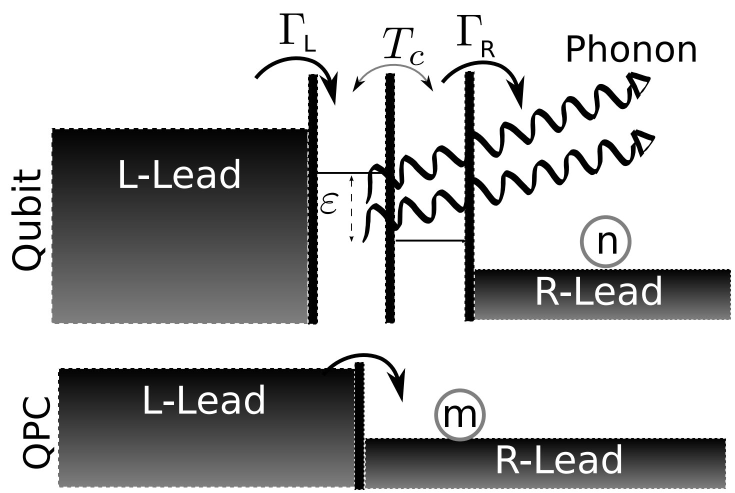

R/L Leads

$$\hat{H}_{\rm res} = \sum_{k,\alpha=\pm} \epsilon_{k,\alpha} \hat{c}^\dagger_{k,\alpha} \hat{c}_{k,\alpha}$$

$$|0\rangle, \ |+\rangle, \ |-\rangle, \ \not{|2\rangle}$$

Stoof&Nazarov PRB ’96

Aguado&Brandes PRL ’04

Kießlich PRL ’07

Gurvitz PRB ’97

Korotkov PRB ’01

⋮ ⋮ ⋮

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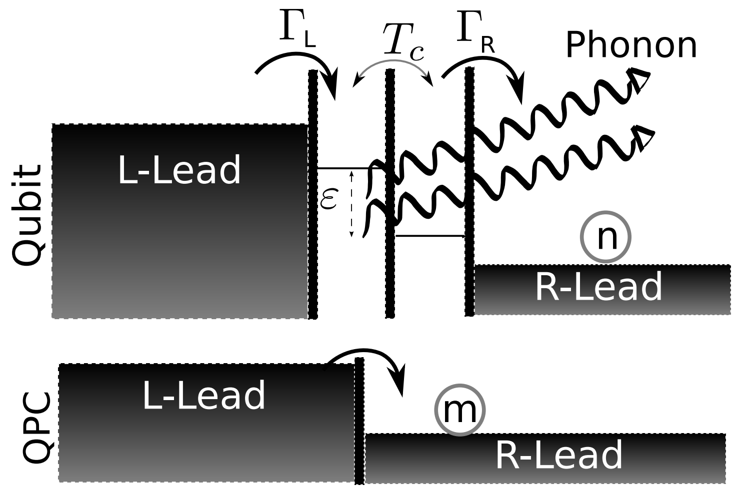

$$|0\rangle, \; |+\rangle, \; |-\rangle, \; |2\rangle$$

Tunneling DQD-Leads

$$\hat{H}_T = \sum_{k,\alpha=\pm} (t_\alpha \hat{c}^\dagger_{k_\alpha} |0\rangle\langle\alpha| + \text{h.c.})$$

Stoof&Nazarov PRB '96

Aguado&Brandes PRL '04

Kießlich PRL '07

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Korotkov PRB '01

⋮ ⋮ ⋮

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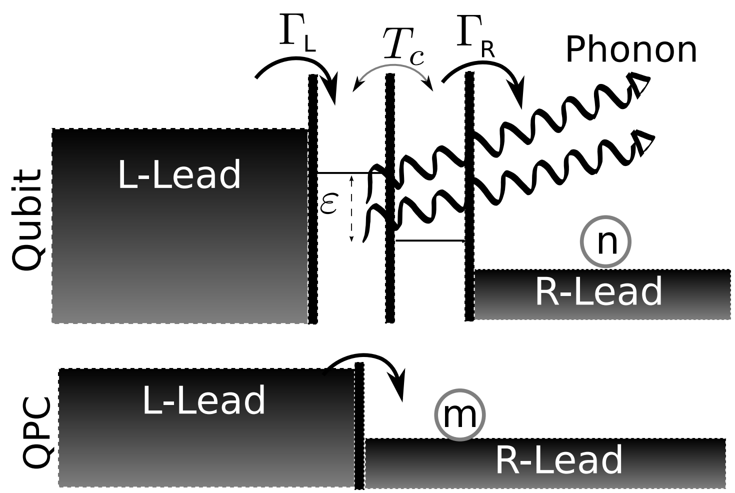

QPC

$$\hat{H}_{QPC} = \sum_{k,\alpha=L,R} \varepsilon_{k\alpha} \hat{d}^\dagger_{k\alpha} \hat{d}_{k\alpha} + \sum_{k,k'} T_0\, \hat{d}^\dagger_{kL} \hat{d}_{k'R} + h.c.$$

$$|0\rangle,\ |+\rangle,\ |-\rangle,\ \cancel{|2\rangle}$$

QPC current

$$I^{QPC} = 2\pi T_0^2 \mathcal{D}_L \mathcal{D}_R e^2 V/\hbar$$

Stoof&Nazarov PRB '96

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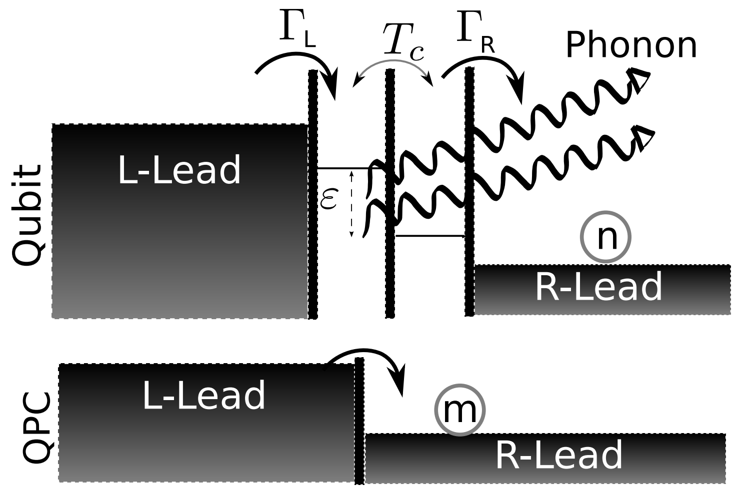

DD-QPC

$$\hat{H}_{QPC-DD} = \sum_{k,k',j=R,L} \delta T_j \, \hat{d}^\dagger_{kL} \hat{d}_{k'R} |j\rangle\langle j| + h.c.$$

$$|0\rangle, \ |+\rangle, \ |-\rangle, \ |2\rangle$$

QPC current for $|j\rangle$ DQD state

$$I_j^{QPC} = 2\pi (T_0 + \delta T_j)^2 \mathcal{D}_L \mathcal{D}_R e^2 V/\hbar$$

Stoof&Nazarov PRB '96

Aguado&Brandes PRL '04

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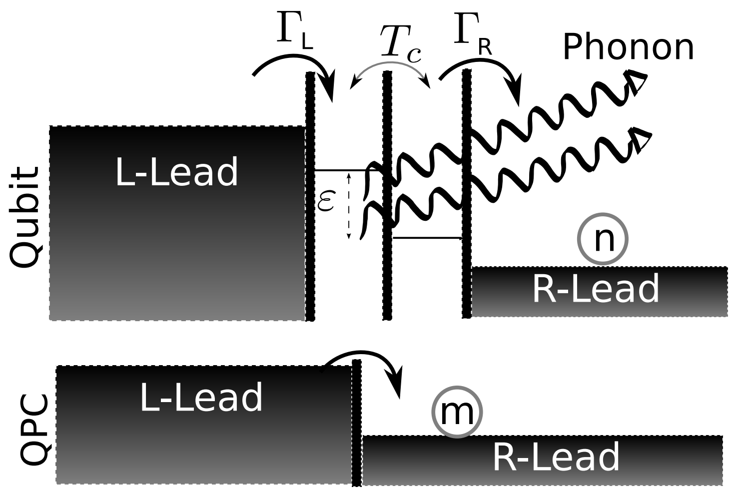

$$|0\rangle,\ |+\rangle,\ |-\rangle,\ \not{|2\rangle}$$

Stoof&Nazarov PRB '96

Aguado&Brandes PRL '04

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⋮ ⋮ ⋮

Weak responding limit

$$|I_R^{QPC} - I_L^{QPC}| \ll I^{QPC}$$

Ruskov& Korotkov PRB '03

Dissipative DQD coupled with a QPC

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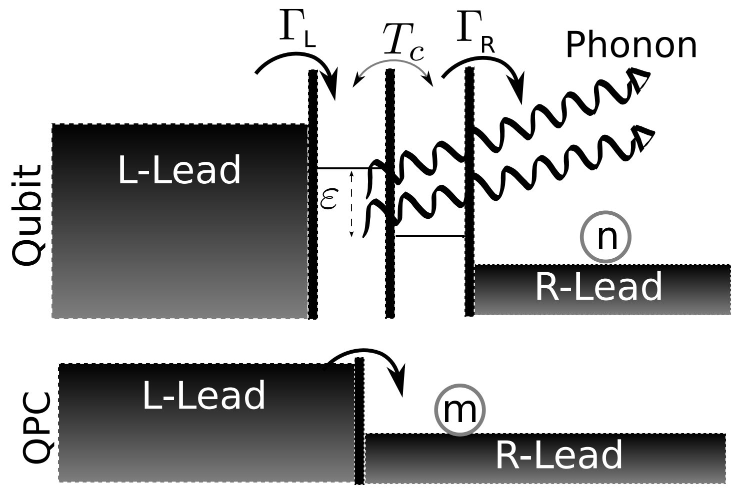

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Ruskov& Korotkov PRB ’03

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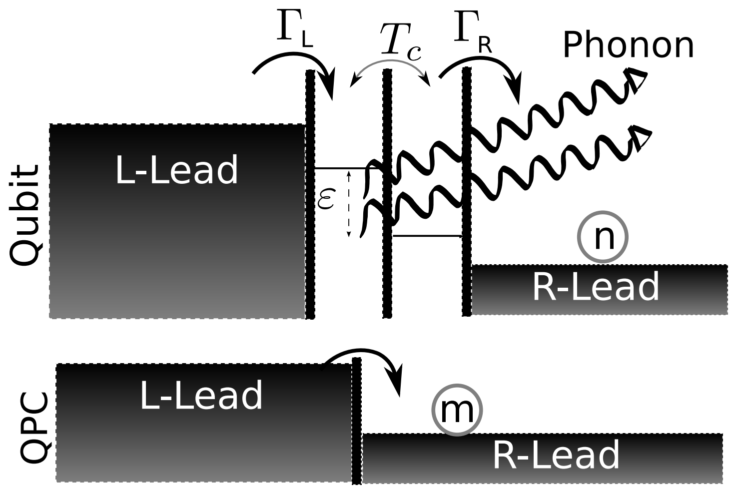

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n, m right-lead electrons

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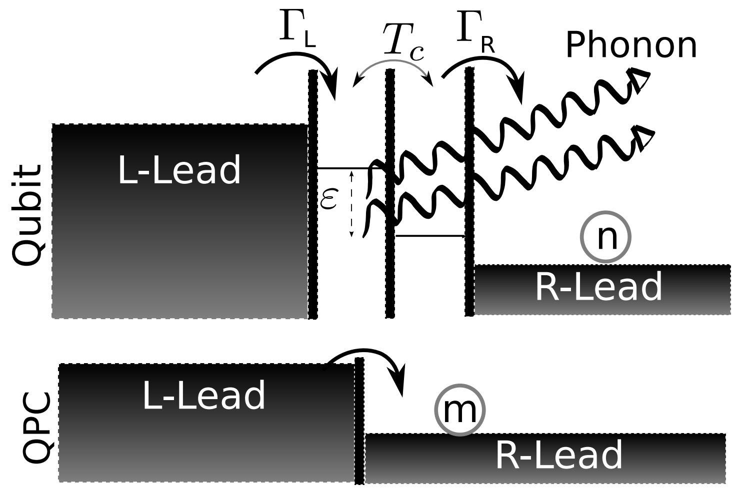

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$$V_{sd} \to \infty$$

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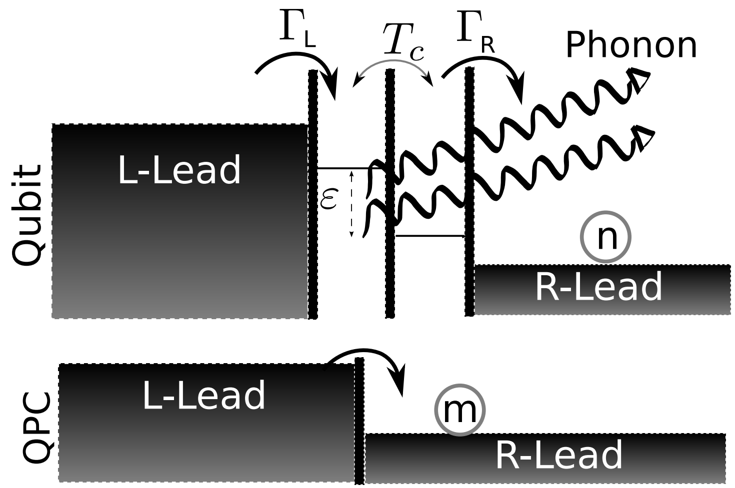

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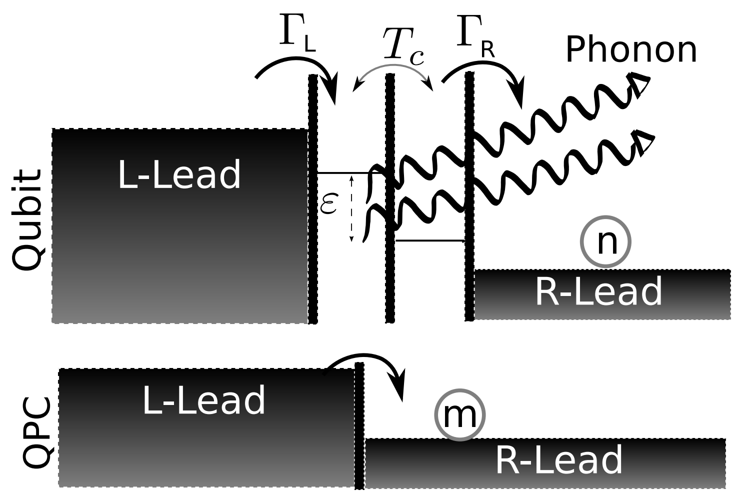

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$$\sigma^{(n,m)} = \{\sigma_{00}^{(n,m)}, \sigma_{++}^{(n,m)}, \sigma_{--}^{(n,m)}, \sigma_{+-}^{(n,m)}, \sigma_{-+}^{(n,m)}\}$$

EOM technique Gurvitz&Prager PRB (1996)

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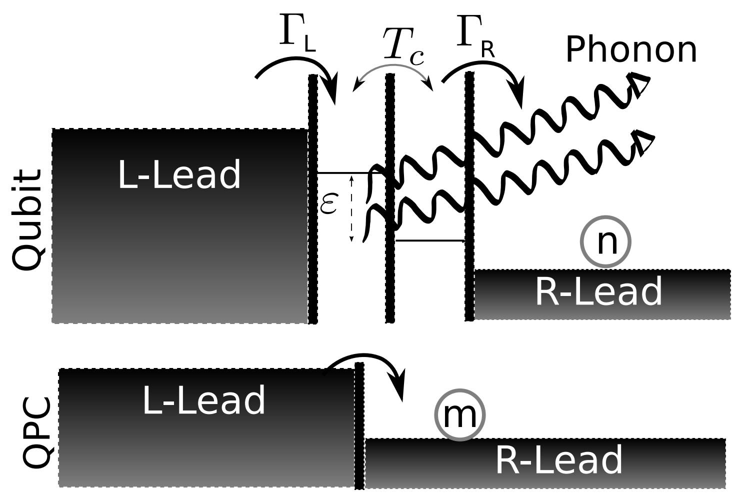

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$$\sigma^{(n,m)} = \{\sigma_{00}^{(n,m)}, \sigma_{++}^{(n,m)}, \sigma_{--}^{(n,m)}, \sigma_{+-}^{(n,m)}, \sigma_{-+}^{(n,m)}\} \rightarrow \sigma(\chi,\phi) \xrightarrow{\chi,\phi \rightarrow 0} \sigma$$

EOM technique Gurvitz&Prager PRB (1996)

Dissipative DQD coupled with a QPC

- $$\dot{\sigma}(t) = \mathcal{L}(\chi, \phi)\sigma(t) - i[\hat{H}_B + \hat{V}_B \hat{s}_z, \sigma(t)]$$

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- $$\dot{\sigma}(t) = \mathcal{L}(\chi, \phi)\sigma(t) - i[\hat{H}_B + \hat{V}_B \hat{s}_z, \sigma(t)]$$
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- $$\frac{d}{dt}\mathbf{P}(\chi,\phi,t) = \int_0^t \mathbf{W}(\chi,\phi,t-\tau)\mathbf{P}(\chi,\phi,\tau)$$

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- $$\mathbf{W}(\chi,z) \begin{pmatrix} -\Gamma_L & 0 & \Gamma_R e^{i\chi} \\ \Gamma_L & -\Gamma_B^{(+)}(z) & \Gamma_B^{(-)}(z) \\ 0 & \Gamma_B^{(+)}(z) & -\Gamma_B^{(-)}(z) - \Gamma_R \end{pmatrix}$$

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- Bath-assisted hopping rates are
  $\Gamma_B^{(\pm)}(z) = T_c^2\{\check{g}^{(+)}[z_\pm] + \check{g}^{(-)}[z_\mp]\}$ with $z_\pm = z \mp i\varepsilon + \Gamma_d$
  $g^{(\pm)}(t) = \mathrm{Tr}_B\{e^{-i\hat{H}_B^{(+)}t}\sigma_\beta^{(\pm)}e^{i\hat{H}_B^{(-)}t}\}$ A.B. et al. JSTAT '08

Some interesting limits

- $\alpha = 0$ No-phonons

Sequential $$\Gamma_z^{(\pm)} \underset{z \to 0}{\longrightarrow} \frac{2T_c^2 \Gamma_d}{\Gamma_d^2 + \varepsilon^2}$$

Coherent $$\Gamma_z^{(\pm)} = \frac{2T_c^2 (z + \Gamma_d)}{(z + \Gamma_d)^2 + \varepsilon^2}$$

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  Stoof & Nazarov PRB '96, Gurvitz & Prager PRB '96

  Elattari& Gurvitz PLA '02, Brandes Phys. Rep. '05

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  G. Kießlich et al. PRL ' 07, A.B et al Physica E '08, A.B. et al JSTAT '08

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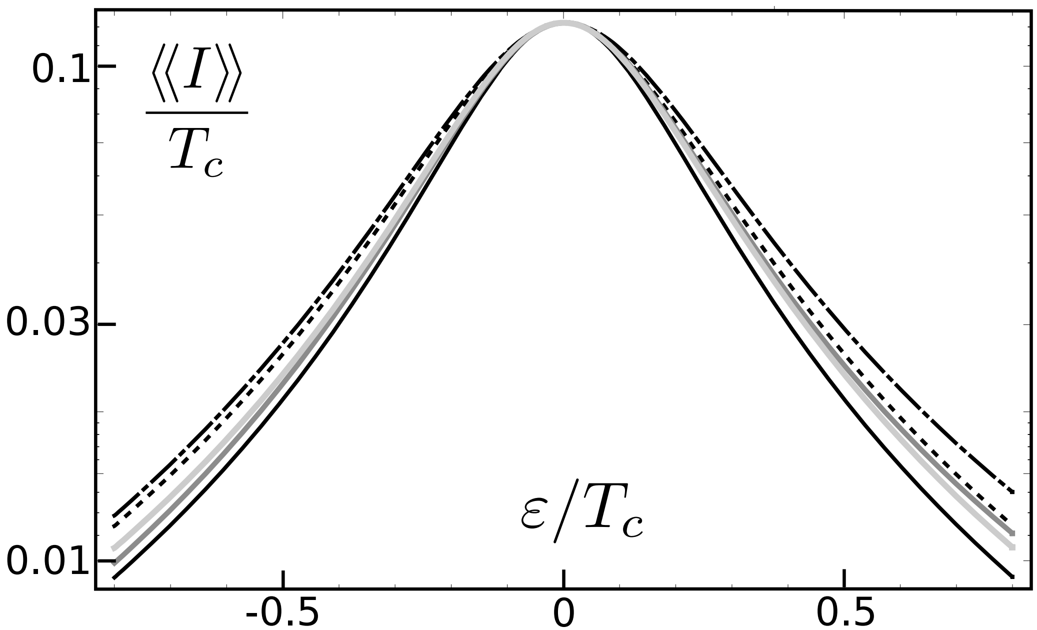
Cumulants and decoherence

Weak coupling

A.B et al. JSTAT '08

Current



$T = 0$

Coherent (Solid)

Incoherent (Dashed)

QPC (Dotted)

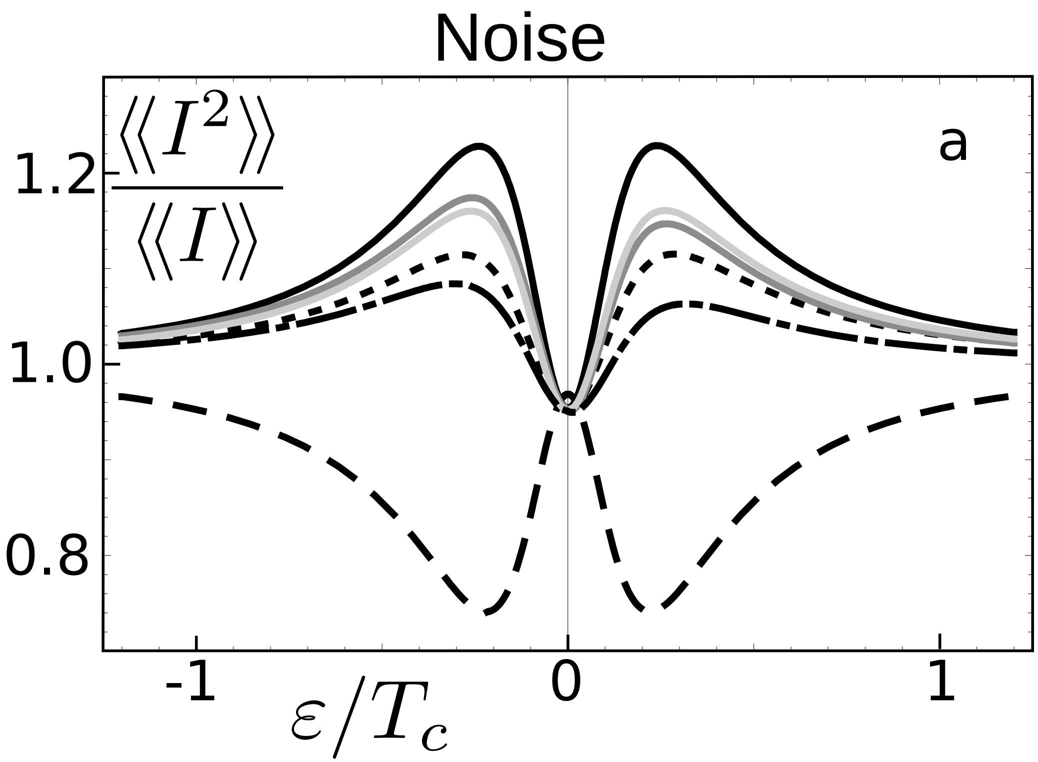
$T \neq 0$ Boson + State Dependent decoupling (dark grey)

Boson + State Independent (light grey)

QPC + Bosons + State Dependent decoupling (dot-dashed)

Cumulants and decoherence

Weak coupling A.B et al. JSTAT '08



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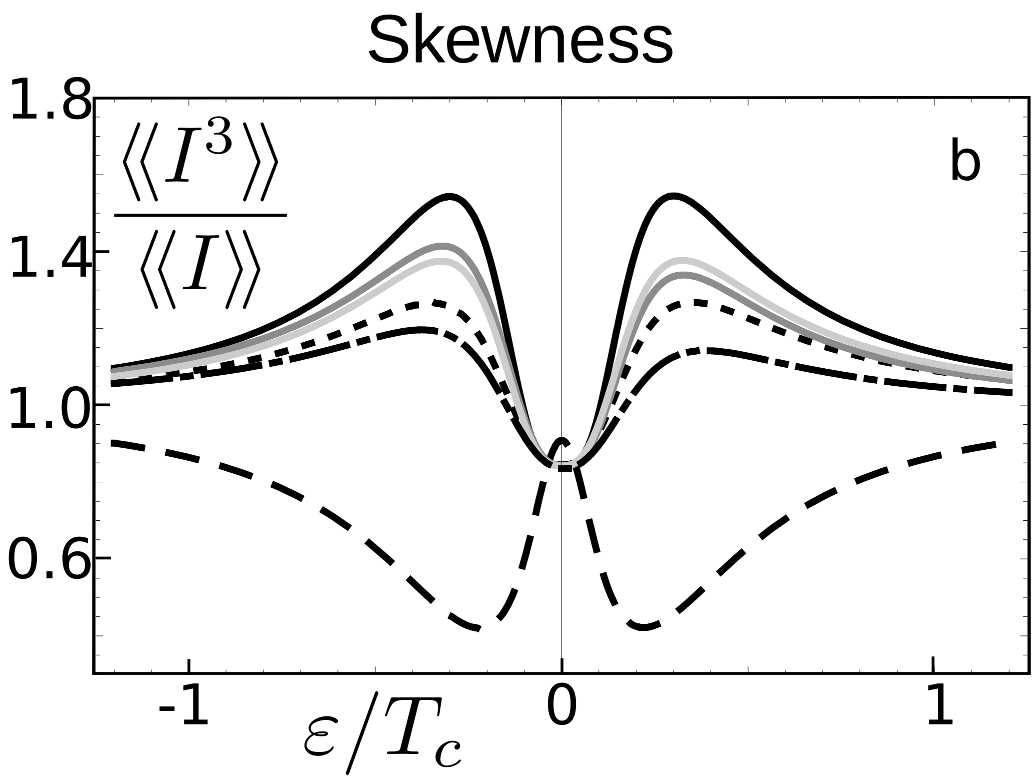

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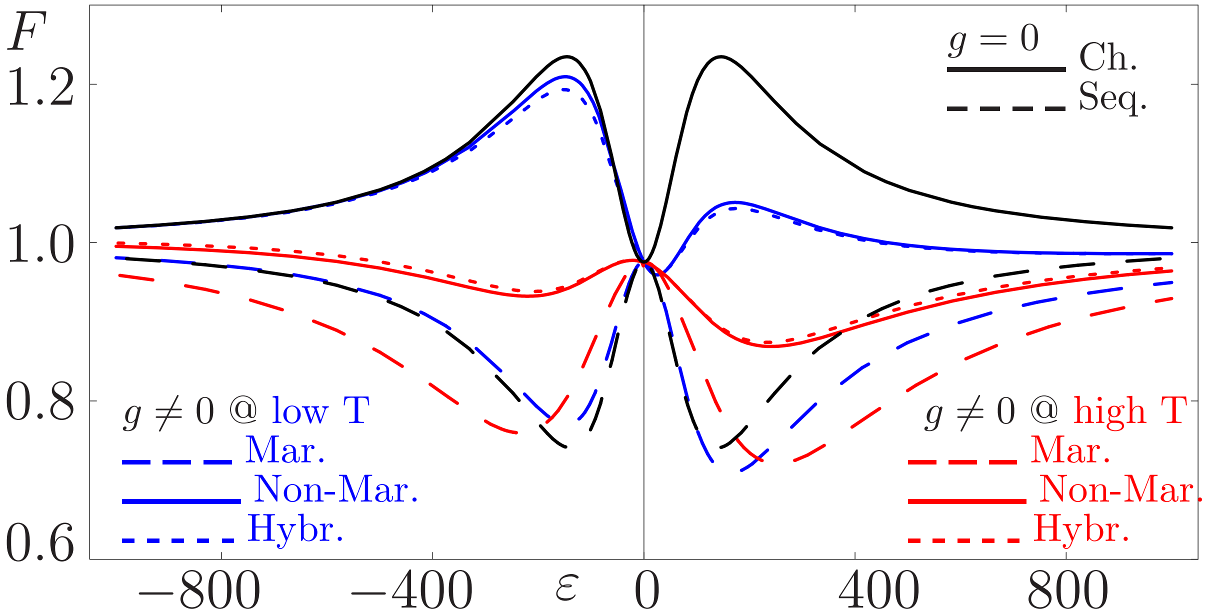
Superpoissonian and Coherent regime

Weak coupling



G. Kießlich et al. PRL '07

High $T = 12^\circ K$

Low $T = 1.4^\circ K$

$\Gamma_+ = 100\mu$eV,

$\Gamma_- = 2.5\mu$eV

Superpoissonian vs coherence?

Superpoissonian and Coherent regime

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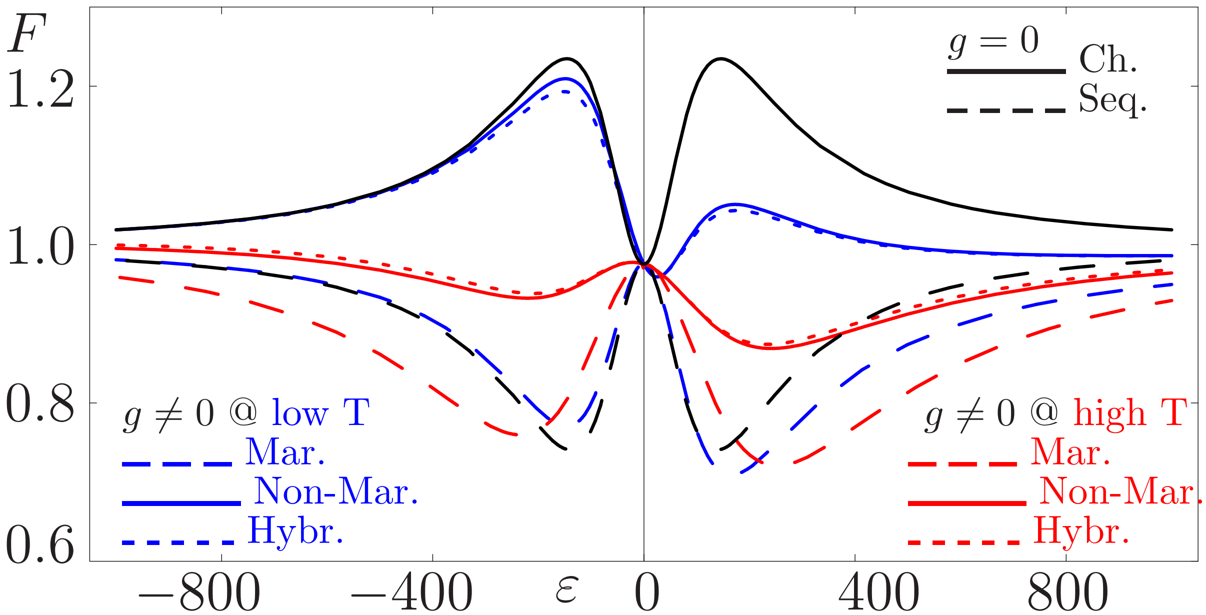

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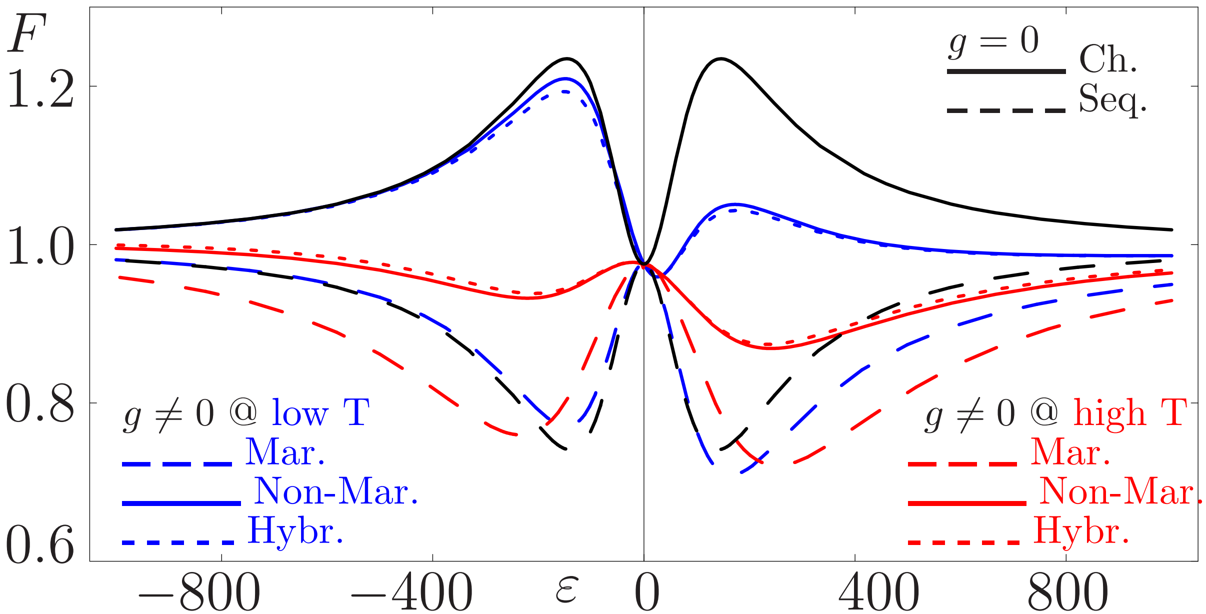
Superpoissonian vs coherence?

$$F = F_m + \underbrace{\frac{2\Gamma_-\Gamma_+^2[(\Gamma_- + \Gamma_0^{(-)})\partial_z\Gamma_0^{(+)} - \partial_z\Gamma_0^{(-)}\Gamma_0^{(+)}]}{\Gamma_t^4}}_{\text{Non-Markovian term}}$$

And changing the bath spectral density?

Superpoissonian and Coherent regime

Weak coupling



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Superpoissonian and Coherent regime

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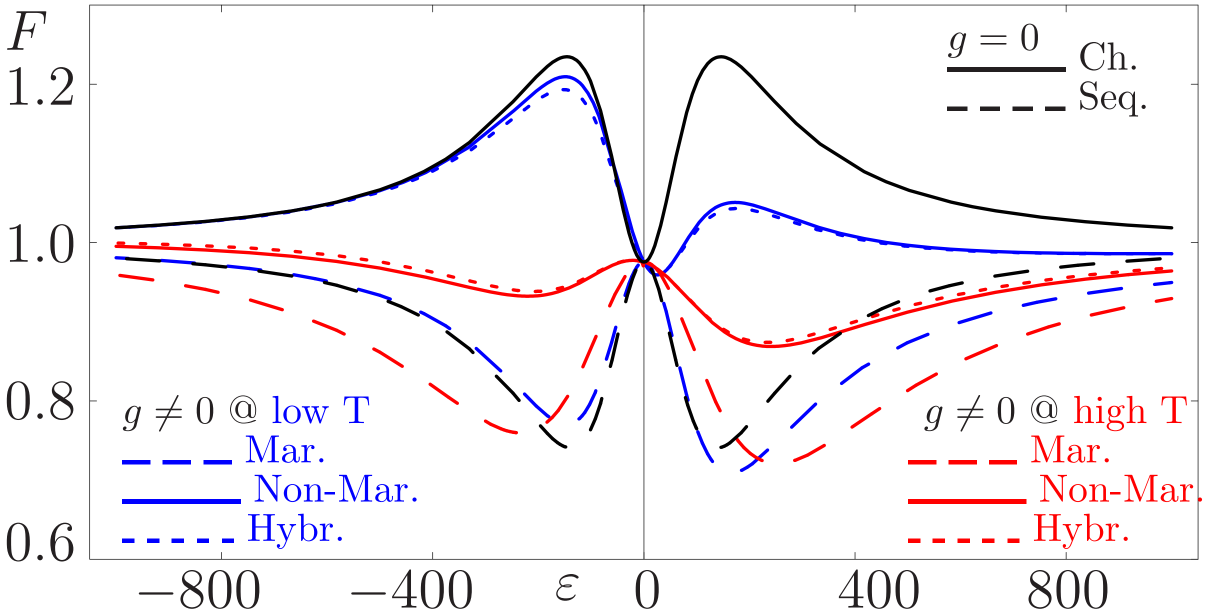

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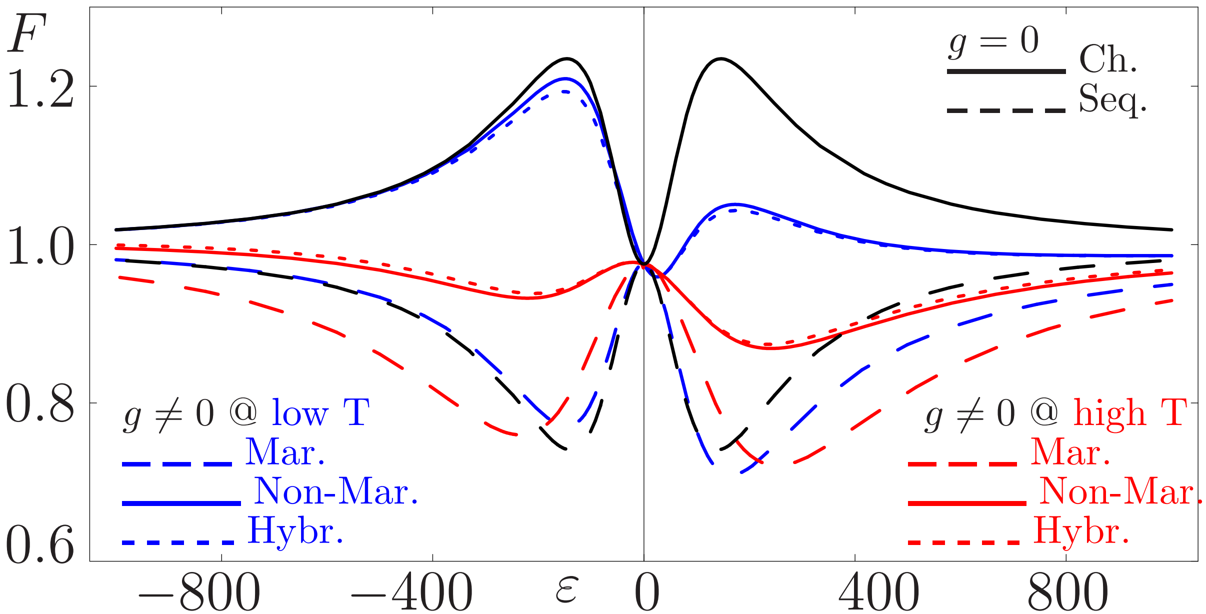
$$\partial_z \Gamma_0^{(+)} \propto -\int_0^\infty dt\, t\, \Gamma^{(+)}(t) = -\tau_m$$

Superpoissonian and Coherent regime

Weak coupling



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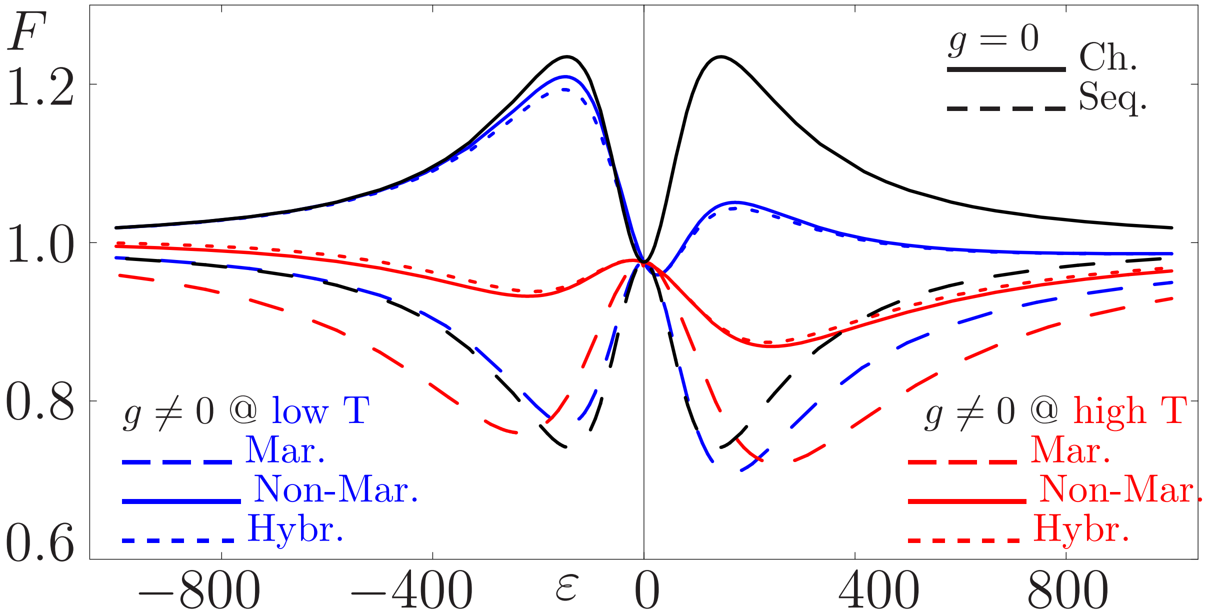
τ_m Mean memory time!

Superpoissonian and Coherent regime

Weak coupling



G. Kießlich et al. PRL '07

High $T = 12^\circ K$

Low $T = 1.4^\circ K$

$\Gamma_+ = 100\mu$eV,

$\Gamma_- = 2.5\mu$eV

Superpoissonian only if $\boxed{\partial_z \Gamma_0^{(+)} \gtrsim 0}$

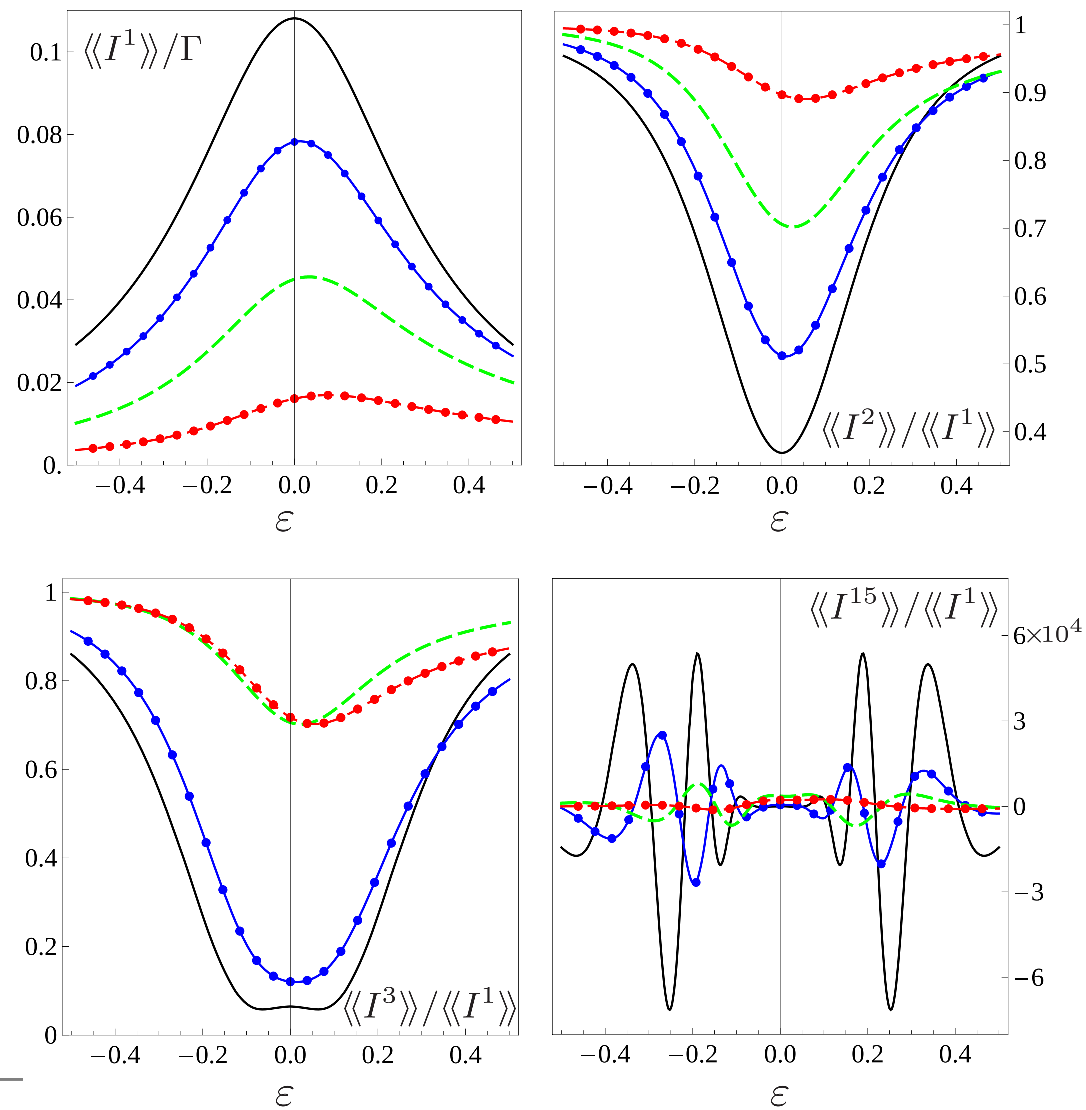
$$\partial_z \Gamma_0^{(+)} \propto - \int_0^\infty dt\ t\ \Gamma^{(+)}(t) = -\tau_m$$

Classical intuition fails ! A.B., et al. Physica E '08

Cumulants from Recursive scheme

Strong coupling

C. Flindt et al. PRL ’08



Parameters:

$T = 0 \quad \Gamma = \Gamma_L = \Gamma_R = 0.5$

$T_c = 0.1 \quad \omega_c = 500$

$2\pi\alpha = 0, 0.2, 0.5, 1$

Dissipation destroy coherent features

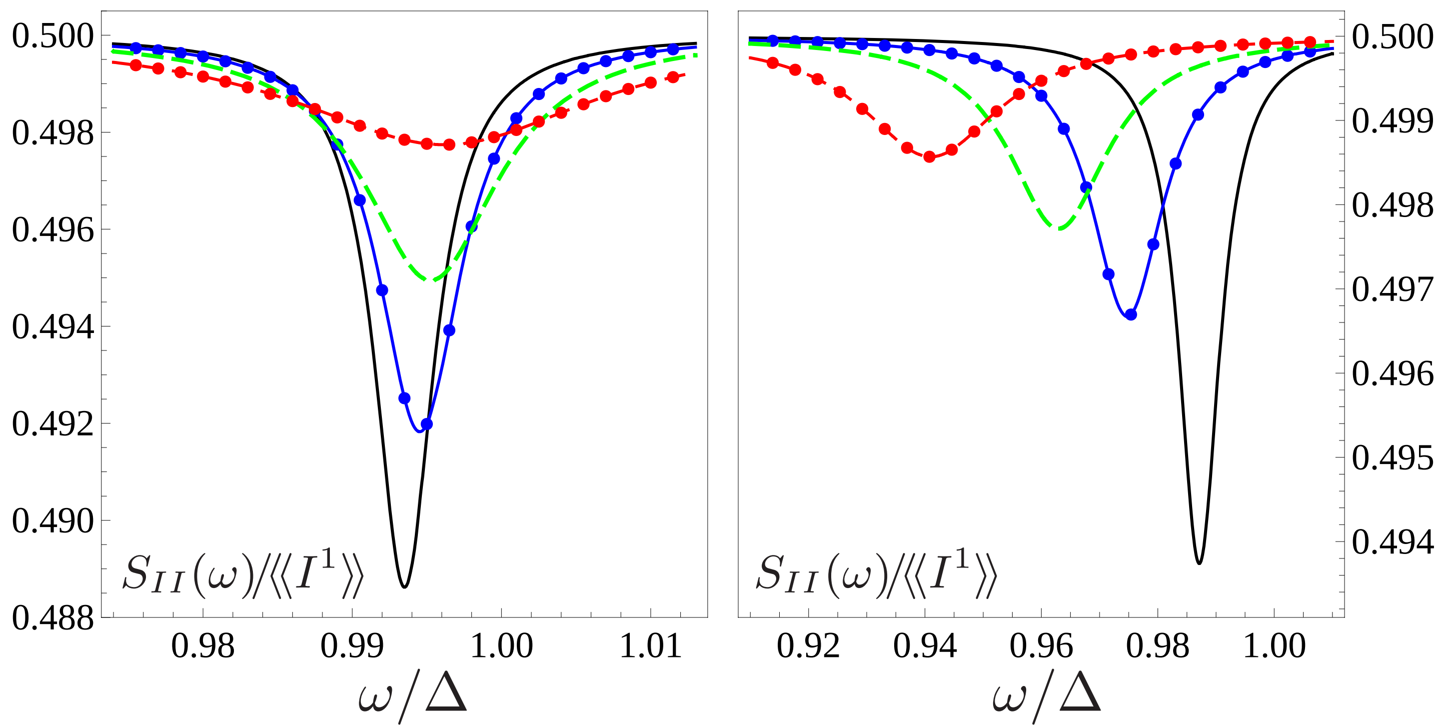
Factorial growth $\langle\langle I\rangle\rangle_n \approx cq^m m!$

Universal oscillations of high order cumulants C. Flindt et al.

Finite frequency noise

Strong coupling

C. Flindt et al. PRL ’08



Different Temperatures

$k_BT = 0, 1, 2, 5$

$\alpha = 0.005$

Different Couplings

$\alpha = 0.01, 0.02, 0.03, 0.05$

$k_BT = 0$

Other parameters: $\Gamma = \Gamma_L = \Gamma_R = 0.5, T_c = 0.1, \omega_c = 500$

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