On the behaviour of the flux tube thickness near the deconfinement transition.

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Bari, September 2008

Plan of the Talk

- Effective string models in Lattice Gauge Theories
  - zero temperature results: Wilson loops
  - finite temperature extension: Polyakov loop correlators

- Effective string predictions for the flux tube thickness
  - high temperature behaviour of the flux tube thickness
  - dimensional reduction and the Svetistky-Yaffe conjecture

Main goal: show that the effective string approach predicts a log to linear transition in the R dependence of the flux tube width and test this result with MC simulations and dimensional reduction .

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References

M.C, F.Gliozzi, U.Magnea, S.Vinti
, "Width of long colour flux tubes in lattice gauge systems"
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M.C., M. Hasenbusch and M.Panero
"Comparing the Nambu–Goto string with LGT results."
JHEP 0503 (2005) 026

M. Billo' and M.C.
Polyakov loop correlators from D0-brane interactions in bosonic string theory
JHEP 0507 (2005) 038

M.C., P. Grinza and N. Magnoli
" Study of the flux tube thickness in 3d LGT's by means of 2d spin models."
J. Stat. Mech. (2006) P11003

A. Allais and M.C
"Linear increase of the flux tube width at high temperature."
In preparation

String theory and Lattice Gauge Theories

Conjecture: Two color sources in a confining gauge theory are bound together by a thin flux tube, which can fluctuate like a massless string.

Main consequence:

- linearly rising potential
- quantum corrections to the interquark potential (Lüscher term)
- log increase of the flux tube thickness

- Potential $V(R)$ between two external, massive quark and anti-quark sources from Wilson loops

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-LV(R)} \text{ (large } L\text{)}$$

$$V(R) = -\lim_{L\to\infty}\frac{1}{L}\log\langle W(L,R)\rangle$$

  In the limit of infinite mass quarks (pure gauge theory) we find the famous "area law" for the Wilson loop

- Area law $\leftrightarrow$ linear potential

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-\sigma RL}; \qquad V(R) = \sigma R + \ldots$$

  σ is the string tension

Quantum corrections and effective models

The area law is exact in the strong coupling limit ($\beta \to 0$). As the continuum limit ($\beta \to \infty$) is approached quantum corrections become important.

- Leading correction for large R: The "Lüscher term"

$$V(R) = \sigma\, R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) \; .$$

It can be obtained summing up the quantum fluctuations of the transverse degrees of freedom of the string treated as $d-2$ massless modes [Lüscher, Symanzik and Weisz, (1980)]. This id the "gaussian approximation" of the effective string:

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-\sigma RL}\int DX^i \mathrm{e}^{-\frac{\sigma}{2}\int dx^0 dx^1\{(\partial_0\vec{X})^2+(\partial_1\vec{X})^2+\text{int.s}\}}$$

- Can also be derived assuming a "SOS picture" for the fluctuations of the surface bordered by the Wilson loop $\rightarrow$ two-dimensional conformal field theory of $d-2$ free bosons
- Consequence: Using known results of CFT's we can predict the behaviour of the Wilson loop for finite values of L.

$$< W(R,L) >= e^{-(\sigma RL)} Z_q(R,L)$$

Where $Z_q(R,L)$ is the partition function of $d-2$ free massless scalar fields living on the rectangle defined by the Wilson loop: $R \times L$

$$Z_q(R,L) \propto \left[\frac{\eta(\tau)}{\sqrt{R}}\right]^{-\frac{d-2}{2}}$$

where $\eta(\tau)$ is the Dedekind η function and $\tau = iL/R$.

- The $L \leftrightarrow R$ simmetry is ensured by this identity

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\ \eta(\tau)$$

known as the "modular" transformation of the η.

- Defining: $F(R,L) \equiv -\log < W(R,L) >$ and expanding the Dedekind function

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n) \quad ; \quad q = e^{2\pi i \tau} \qquad ,$$

one finds

$$F(R,L) = \sigma RL - (d-2)\left[\frac{\pi L}{24R} + \frac{1}{4}\log R\right] + \ldots$$

From which we find as anticipated:

$$V(R) = \sigma\, R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) \ .$$

Montecarlo test of the Lüscher term.

The following ratio is particularly useful to single out the effective string contribution from a collection of Wilson loops (it requires a very precise knowledge of σ):

$$R(L,n) \equiv \frac{\langle W(L+n, L-n)\rangle}{\langle W(L,L)\rangle} \exp(-n^2\sigma)$$

It is easy to see that $R(L,n)$ depends only on $t = n/L$:

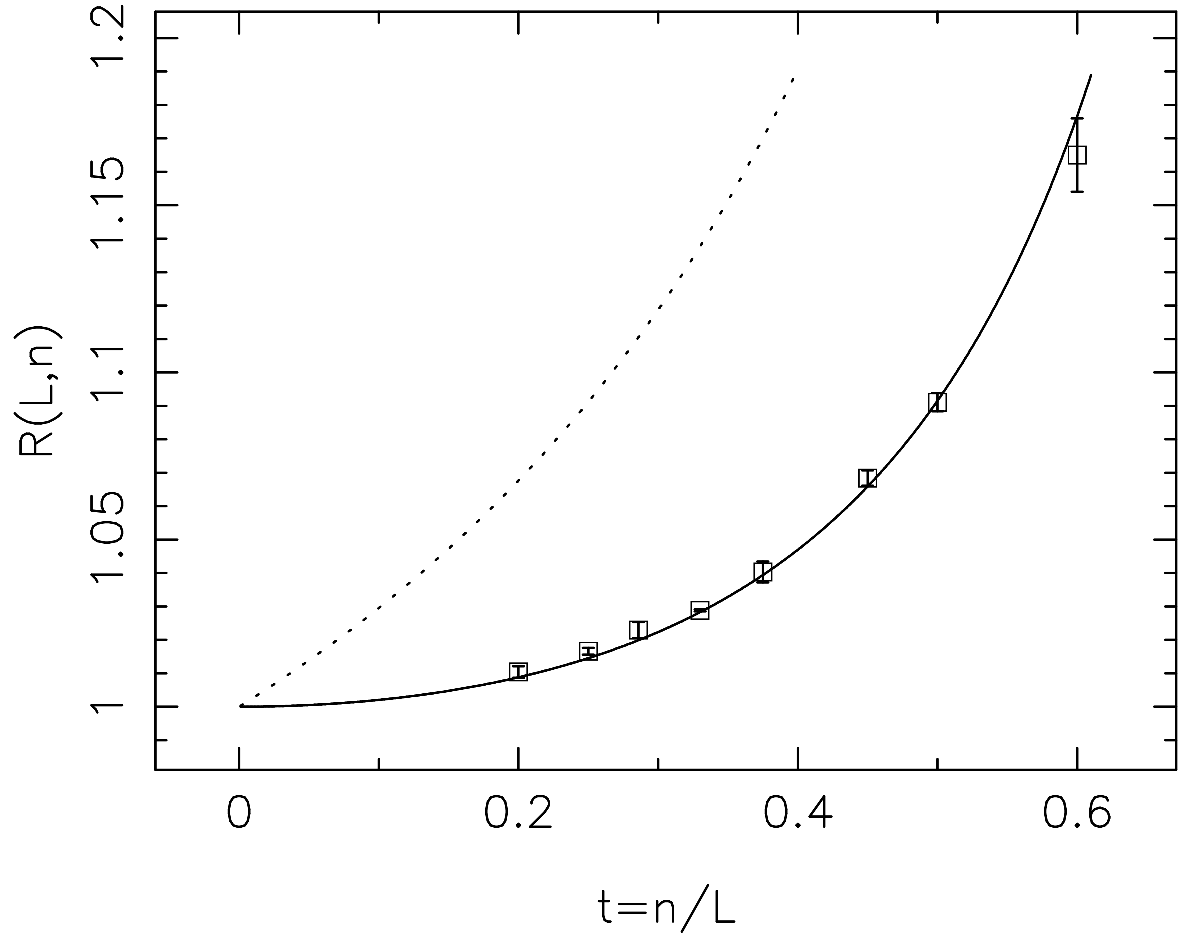
$$R(L,n) = F(t) = \left[\frac{\eta(i)\sqrt{1-t}}{\eta\left(i\frac{1+t}{1-t}\right)}\right]^{1/2}$$

The effective string model assumes that all confining gauge theories share the same behaviour for the interquark potential (for large enough R) with no dependence on the gauge group and a trivial (linear) dependence on the space-time dimensions.

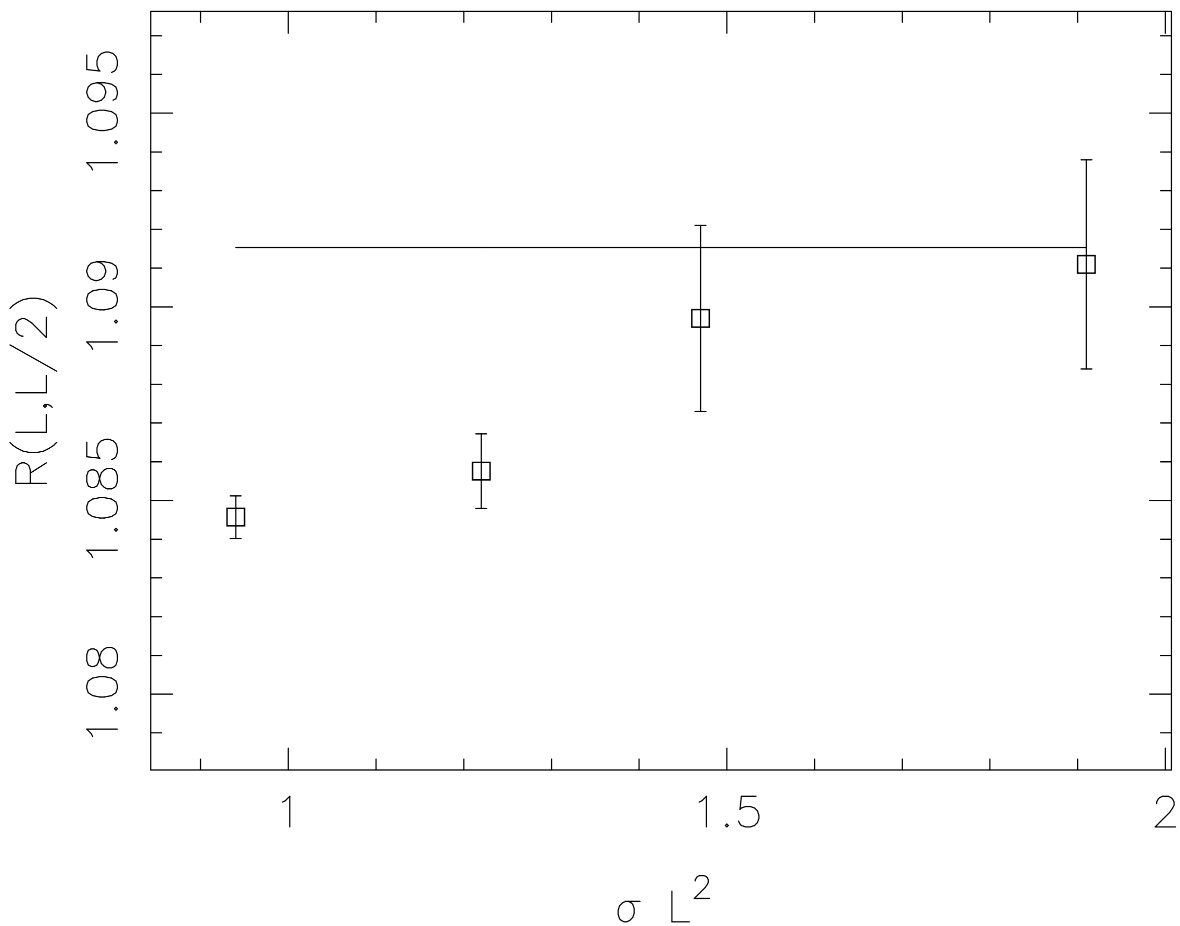
The 3d gauge Ising model (dual of the standard 3d spin Ising model) is a perfect choice for testing the string predictions.

Duality allows high precision simulations for very large values of R and L

Very good agreement in the large R regime is found but corrections appear as R decreases and suggest that a more sophisticated effective description is needed.



R(L,n)
1
1.05
1.1
1.15
1.2
0
0.2
0.4
0.6
t=n/L



R(L,L/2)
1.08
1.085
1.09
1.095
1
1.5
2
σ L^2

The Nambu-Goto string

Action $\sim$ area of the surface spanned by the string in its motion:

$$S = -\sigma \int d\xi^0 d\xi^1 \sqrt{\det g_{\alpha\beta}} \tag{1}$$

where $g_{\alpha\beta}$ is the metric “induced” on the w.s. by the embedding:

$$g_{\alpha\beta} = \frac{\partial X^M}{\partial \xi^\alpha}\frac{\partial X^N}{\partial \xi^\beta} G_{MN} \tag{2}$$

ξ^α = world-sheet coords. (ξ^0 = proper time, ξ^1 spans the extension of the string)

Connection with the gaussian model

- One can use the world-sheet re-parametrization invariance of the NG action to choose the so called "physical gauge":
  - The w.s. coordinates ξ^0, ξ^1 are identified with two target space coordinates x^0, x^1
- One can study the 2d QFT for the $d-2$ transverse bosonic fields with the gauge-fixed NG action

$$Z = \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \sqrt{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+(\partial_0 \vec{X} \wedge \partial_1 \vec{X})^2}}$$

$$= \mathrm{e}^{-\sigma RL} \int DX^i \mathrm{e}^{-\frac{\sigma}{2} \int dx^0 dx^1 \left\{(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+\text{int.s}\right\}}$$

The anomaly problem

- Problem: This gauge fixing is anomalous (unless we are in $d = 26$)

- "Effective" solution: It can be shown (Olesen, 1985) that the corrections due to the anomaly decay as $1/R^3$ thus maybe one can trust the perturbative expansion up to the order $1/R^2$ i.e. the first order beyond the gaussian approximation.

- "Stringy" solution: The Nambu-Goto action can be rewritten (order by order in $1/L$) so as to be anomaly-free in any dimension (Polchinski and Strominger, 1991):

Polchinski and Strominger action:

$$S_{\text{eff}} = \frac{1}{4\pi}\int d\tau^+ d\tau^- \left[\frac{1}{a^2}(\partial_+ X \cdot \partial_- X) + \left(\frac{D-26}{12}\right)\frac{(\partial_+^2 X \cdot \partial_- X)(\partial_+ X \cdot \partial_-^2 X)}{(\partial_+ X \cdot \partial_- X)^2} + O(L^{-3})\right], \quad (3)$$

where $\tau^{\pm}$ are light-cone world-sheet coordinates, and a is a length scale related to the string tension.

As a consistency check, it can be shown (Lüscher, Weisz, Drummond, Hari Dass, Mattlock, 2004) that the first perturbative correction beyond the gaussian contribution in both frameworks is the same.

Finite Temperature LGTs

- Finite temperature can be realized by imposing periodic boundary conditions in the (euclidean) “time” direction

- The (finite) temperature is given by the inverse of the lattice length in the compactified time direction $T = 1/L$

- A new set of observables can be constructed: the Polyakov loop which is the trace of the ordered product of timelike variable along a timelike axis of the lattice and behaves as an order parameter for the deconfinement phase transition

- The interquark potential is given by the correlator of two Polyakov loops.

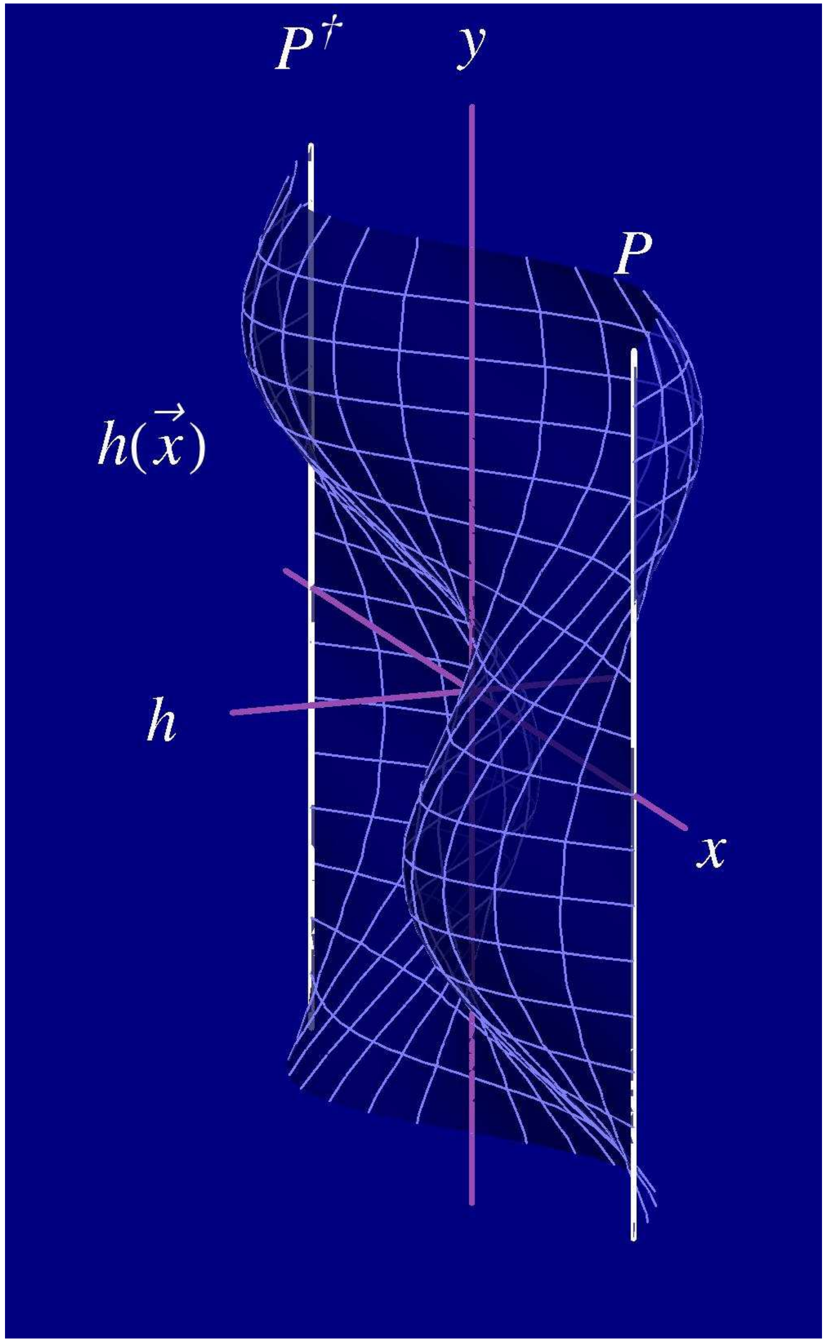
$$V(R,T) = -\log(< P(x)P^{+}(x+R) >) \qquad T = 1/L$$

Effective string description for Finite Temperature LGTs

The main change with respect to the zero temperature case is that now the string world sheet is bordered by the two Polyakov loops in one direction but is periodic in the "time" direction, thus it has a cylindric geometry.

There is no more $L \leftrightarrow R$ simmetry: the modular transformation of the Dedekind function $\tau \to -\frac{1}{\tau}$ allows to relate the T and R dependences of the interquark potential, i.e. to predict the behaviour of σ as a function of the temperature T

In string theory this modular transformation is known as open $\leftrightarrow$ closed string duality


$P^\dagger$
y
P
$h(\vec{x})$
h
x

Using again the gaussian approximation and 2d CFT we find in this case

$$< P(x)P^{+}(x+R) >\propto e^{-(\sigma RL)} \left[\eta(i\frac{L}{2R})\right]^{-(d-2)}$$

to be compared with

$$< W(R,L) >\propto e^{-(\sigma RL)} \left[\frac{\eta(i\frac{L}{R})}{\sqrt{R}}\right]^{-\frac{d-2}{2}}$$

the two expression give the same Lüscher term

Nambu-Goto contribution to the Polyakov loop correlators

In the Polyakov loop case the Nambu Goto partition function can be evaluated exactly if one neglects the anomaly

$$\mathcal{F} = \frac{1}{2}\sum_k c_k \mathrm{e}^{-LE_k(R)} \ ,$$

where the coefficient c_k are the partitions of integers (they come form the expansion of the Dedekind function).

The spectrum is

$$E_k(R) = \sigma R \sqrt{1 + \frac{2\pi}{\sigma R^2}\left(k - \frac{d-2}{24}\right)}$$

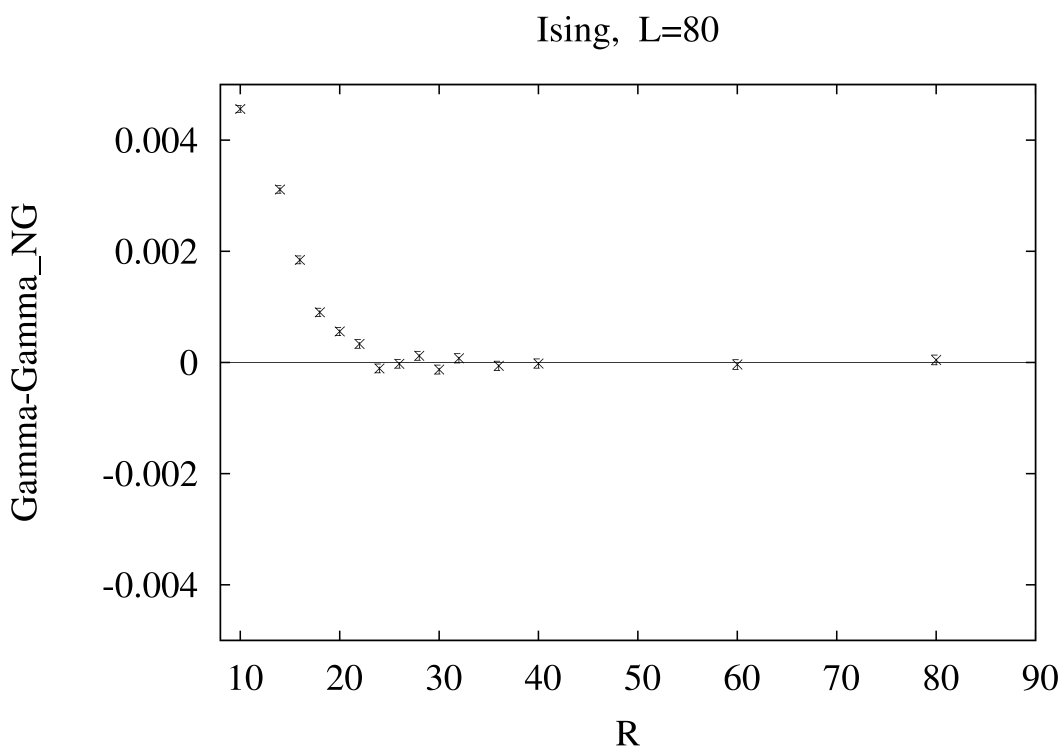
This spectrum coincides with that conjectured by Arvis long ago (1982) by (formal) quantization of the NG action in the physical gauge.

Comparison with MC simulations

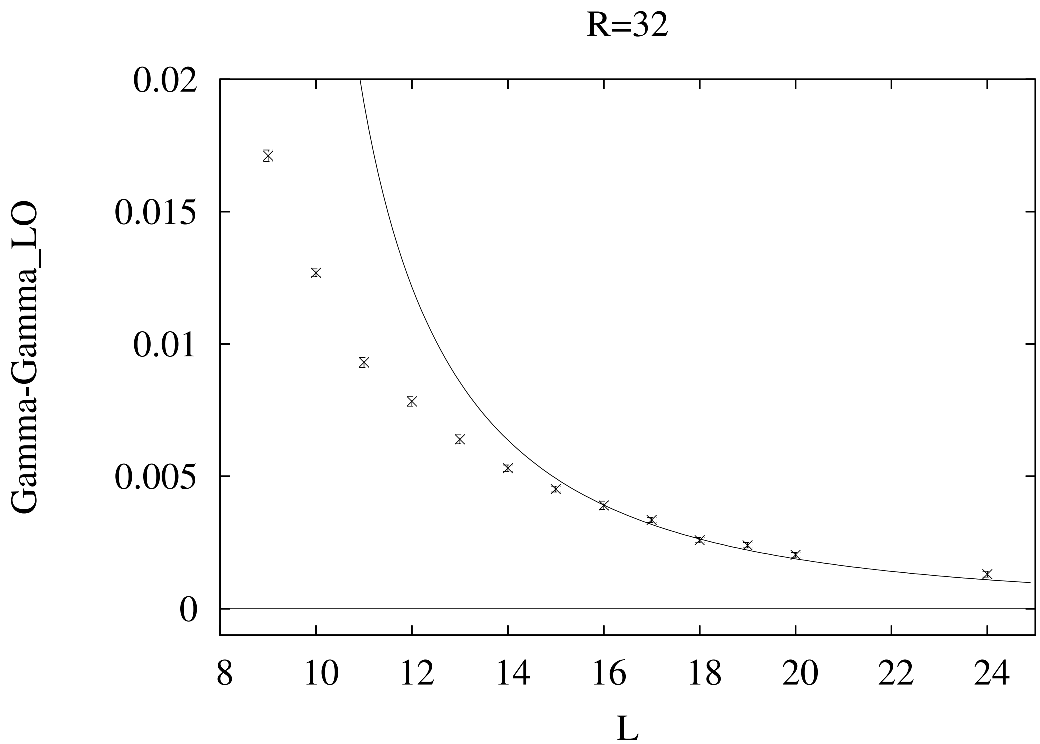
Duality allows to test the above predictions can be tested with a precision $\frac{\delta G}{G}$ which in some cases reaches 10^{-4}.

In the comparison there is no free parameter. The figures are not the result of a fitting procedure.

The agreement at large distance with NG is impressive. At shorter distances deviations appear. Liouville mode?



Polyakov loop correlators in the (2+1) dimensional gauge Ising model at $T = T_c/10$ (corresponding for this β at $L = 80$). $10 < R < 80$ is the interquark distance. In the figure is plotted the deviation of Γ (the ratio $G(R+1)/G(R)$ of two correlators shifted by one lattice spacing) with respect to the Nambu-Goto string expectation Γ_{NG} (which with this definition of observables corresponds to the straight line at zero).



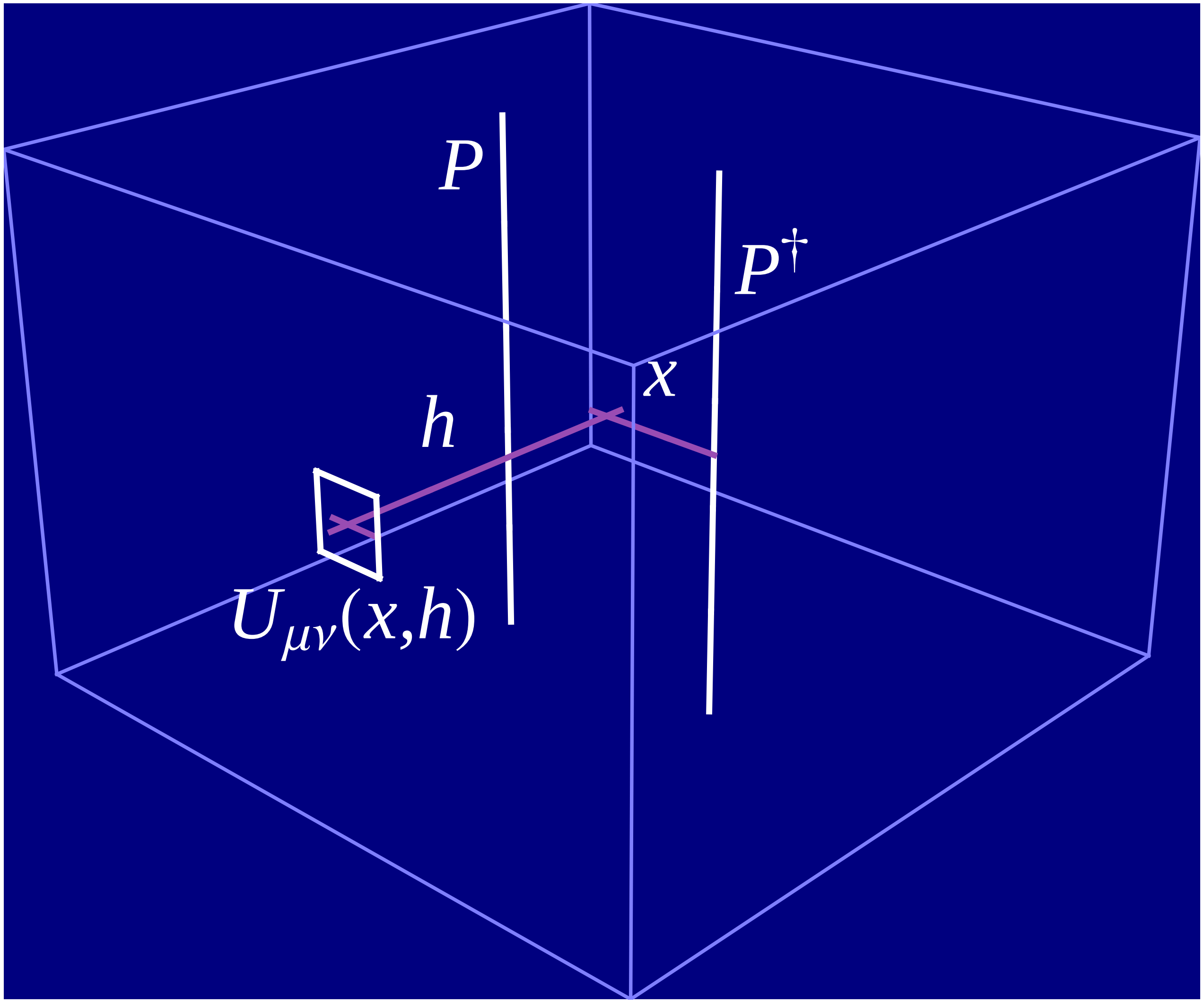
Polyakov loop correlators in the (2+1) dimensional gauge Ising model at a fixed interquark distance $R = 32$ varying inverse-temperature size $(8 < L < 24)$. In the figure is plotted the deviation of Γ with respect to the asymptotic free string expectation Γ_{LO} (which with this definition of observables corresponds to the straight line at zero). The curve is the Nambu-Goto prediction for this observable.

The flux tube thickness.

The flux density in presence of a pair of Polyakov loops is:

$$< \mathcal{F}_{\mu,\nu}(x_0, x_1, h, R) >= \frac{< P(0,0)P^{+}(0,R)U_{\mu,\nu}(x_0, x_1, h) >}{< P(0,0)P^{+}(0,R) >} - < U_{\mu,\nu} >$$

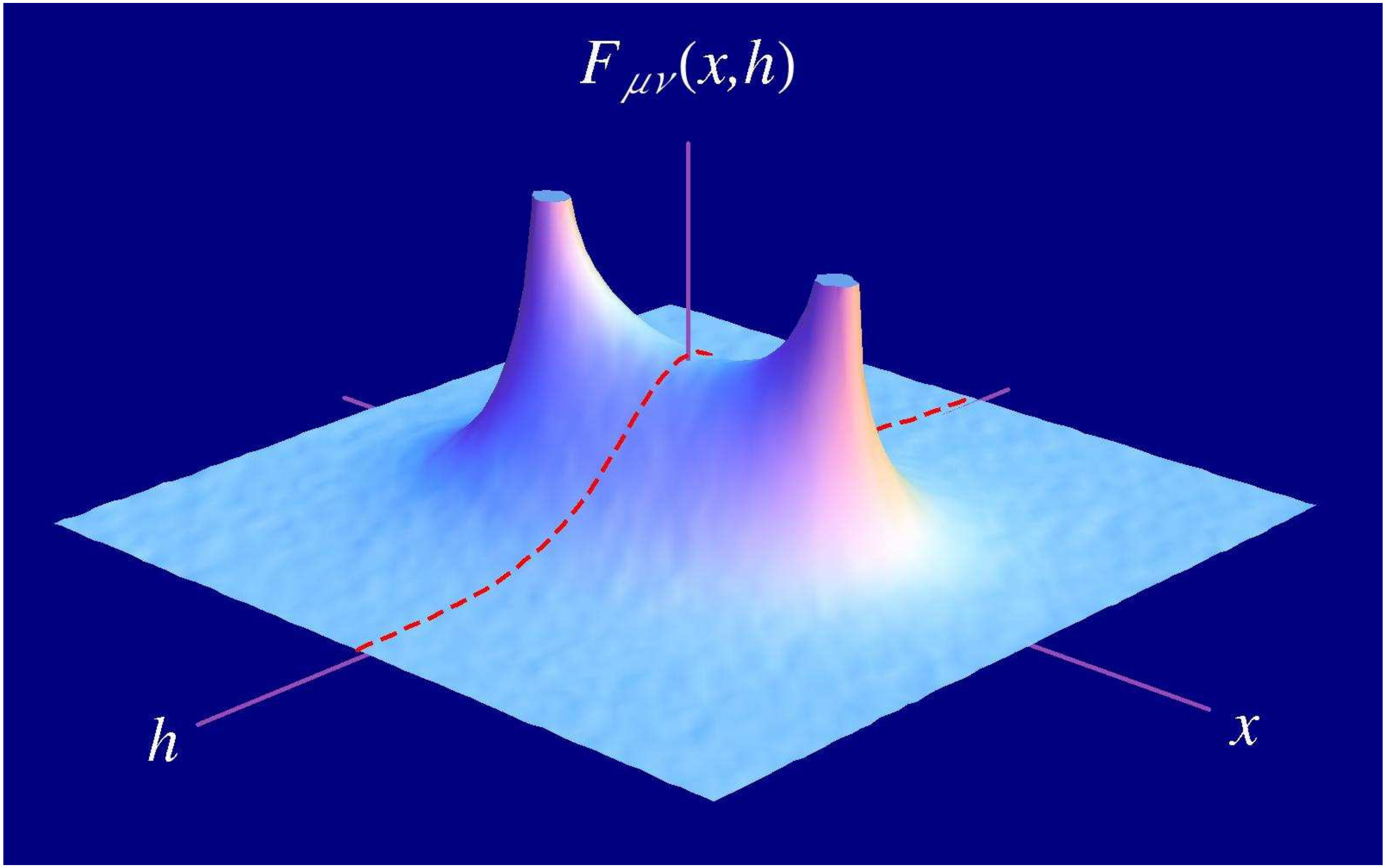
where x_0 denotes the timelike direction, x_1 is the direction of the axis joining the two Polyakov loops and h denotes the transverse direction.

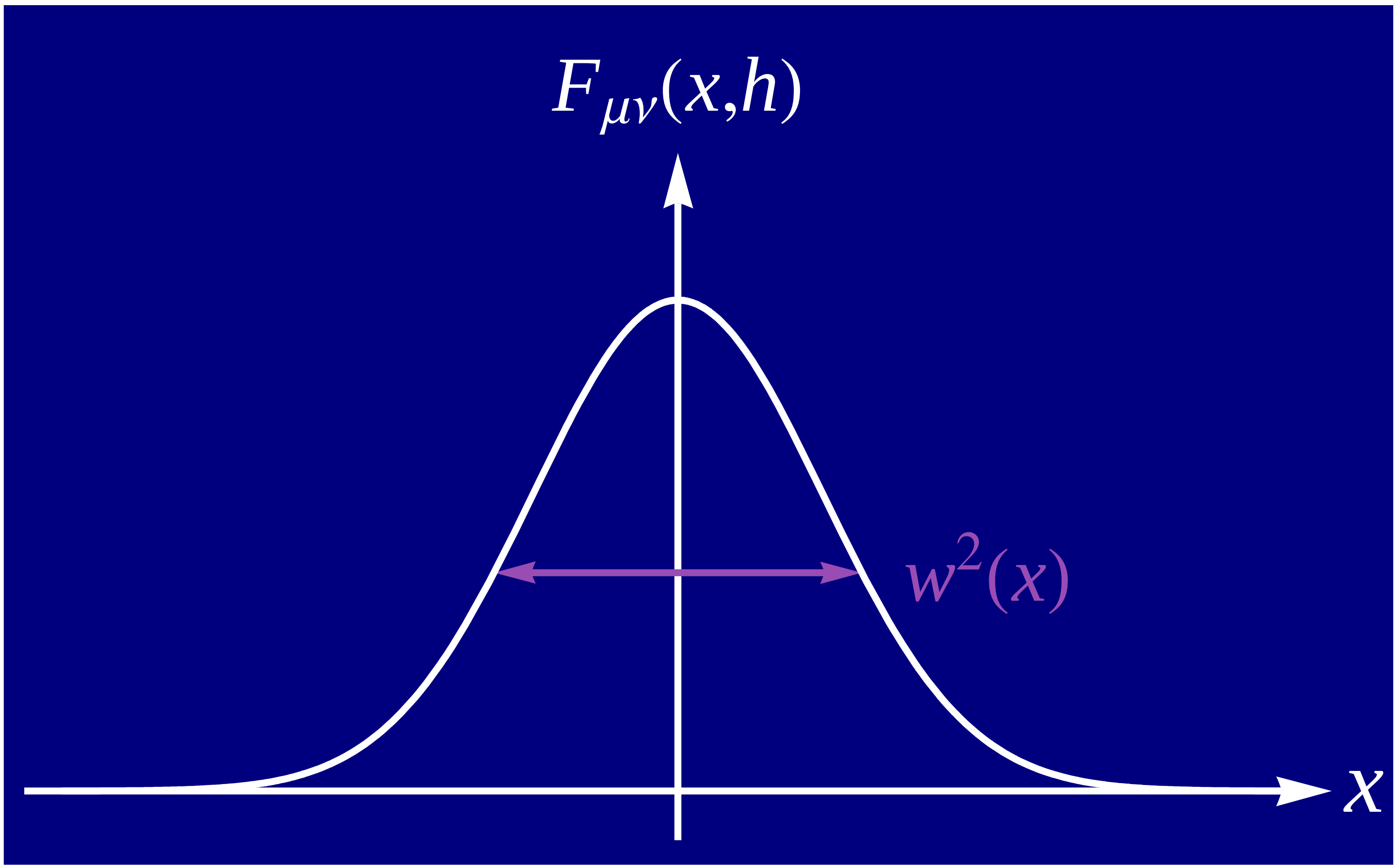

P
$P^\dagger$
x
h
$U_{\mu\nu}(x,h)$

To evaluate the flux tube thickness we fix $x_2 = R/2$ to minimize boundary effects. (thanks to the periodic b.c. in the "temperature" direction we can instead choose any value of x_0)

In the x_1 direction the flux density shows a gaussian like shape, the width of this gaussian is the "flux tube thickness": $w(R, L)$. $w(R, L)$ only depends on the interquark distance R and on the lattice size in the compactified timelike direction L, i.e. on the inverse temperature of the model

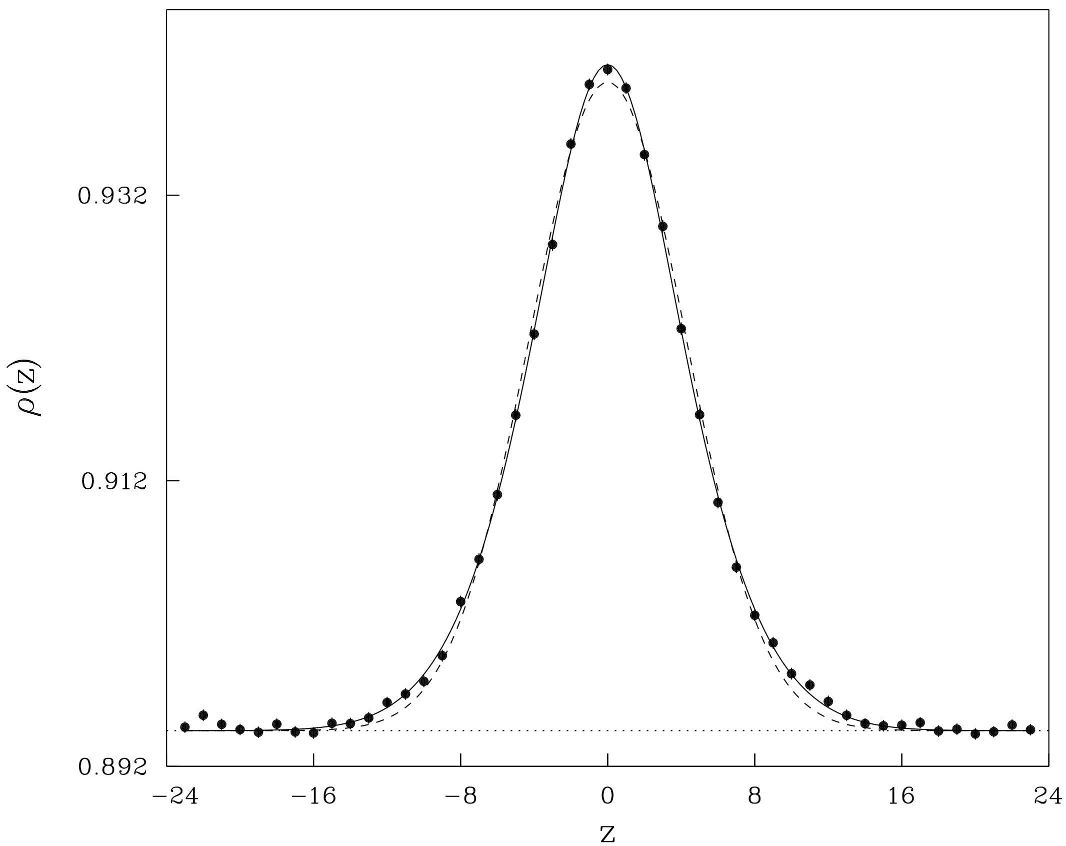
By tuning L we can thus study the flux tube thickness in the vicinity of the deconfinement transition


$F_{\mu\nu}(x,h)$
h
x


$F_{\mu\nu}(x,h)$
$w^2(x)$
x

Shape of the flux density generated by a 30×30 Wilson loop in the Ising gauge model (at $\beta = 0.7460$). The dashed line is the gaussian fit.

fig. 2

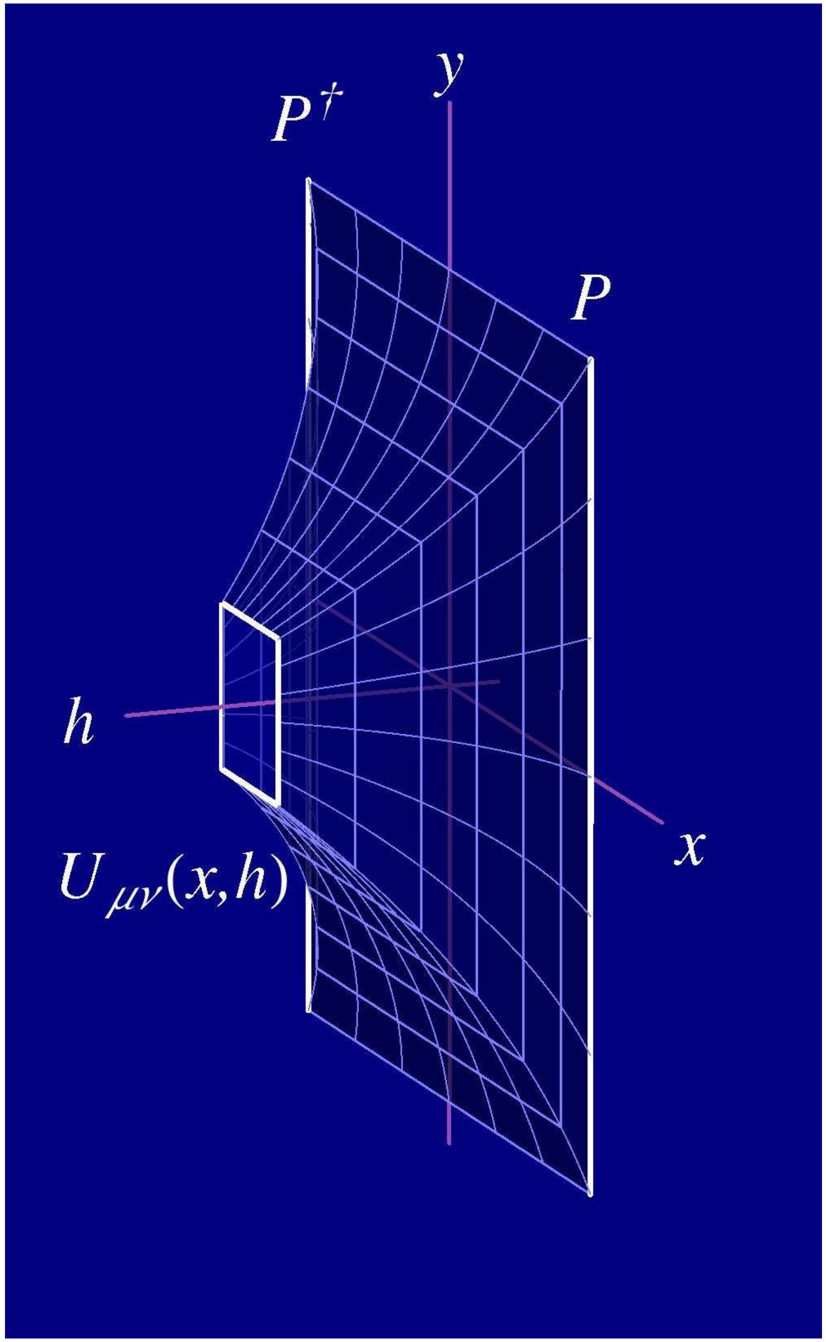


Effective string predictions for the flux tube thickness.

In the Nambu-Goto framework one should sum over all the surfaces bordered by the Polyakov loops and the plaquette with a weight proportional to the surface area.

$$\mathcal{F}_{\mu\nu}(x,h) = \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}$$

$$w^2(x) = \frac{\int \mathrm{d}h\; h^2\; \mathcal{F}_{\mu\nu}(x,h)}{\int \mathrm{d}h\; \mathcal{F}_{\mu\nu}(x,h)} = \frac{\int \mathrm{d}h\; h^2\; \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}}{\int \mathrm{d}h\; \sum_{\text{surf}} e^{-\sigma \text{Area[surf]}}}$$



y
$P^{\dagger}$
P
h
x
$U_{\mu\nu}(x,h)$

If we assume the size of the plaquette to be negligible with respect to the other scales, perform a "physical" gauge fixing and denote as h_0 the transverse coordinate of the plaquette then:

$$w^2(\vec{x}_0) = \frac{\int \mathrm{d}h_0 \; {h_0}^2 \int_{h(\vec{x}_0)=h_0} [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}}{\int \mathrm{d}h_0 \int_{h(\vec{x}_0)=h_0} [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}}$$

which can be rewritten as

$$w^2(\vec{x}_0) = \frac{\int [\mathcal{D}h(\vec{x})] \;\; h(\vec{x}_0)^2 e^{-\sigma S[h]}}{\int [\mathcal{D}h(\vec{x})] \;\; e^{-\sigma S[h]}} \equiv \langle h^2(\vec{x}) \rangle$$

with $S[h] = \sigma \int \mathrm{d}^2 x \; \sqrt{1 + (\nabla h)^2}$

This expectation value is singular and must be regularized using for instance a point splitting prescription.

$$\sigma w^2(\vec{x}) = \langle h(\vec{x}+\vec{\epsilon})h(\vec{x})\rangle \equiv \mathcal{G}(\vec{x}+\vec{\epsilon};\vec{x})$$

The U.V. cutoff ϵ has a natural physical meaning: $\epsilon \;\sim\;$ plaquette size.

Dealing with the whole NG action turns out to be too difficult. We resort again to the free boson approximation ("SOS picture")

$$S[h] \simeq \sigma \int \mathrm{d}^2x \; [1 + 1/2(\nabla h)^2]$$

Then $\mathcal{G}(\vec{x}+\vec{\epsilon};\vec{x})$ is the Green function of the Laplacian on the cylinder.

which can be written (choosing the reference frame so as to have the two loops located in $\pm R/2$) as:

$$G_2(z;z_0) = -\frac{1}{2\pi}\log\left|\frac{\theta_1\left[\pi(z-z_0)/2R\right]}{\theta_2\left[\pi(z+\bar{z}_0)/2R\right]}\right|$$

with $q = e^{-\pi L/2R}$

Setting $z_0 = z+\epsilon$ and performing an expansion in ϵ one finds:

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{R} + \frac{1}{2\pi}\log\left|2\theta_2\left(\pi \text{ Re } z/R\right)/\theta_1'\right|$$

A similar calculation in the Wilson loop case gives

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{R} + \frac{1}{2\pi}\log\left|\frac{2\theta_2(\pi \text{ Re } z/R)\theta_4(i\pi \text{ Im } z/R)}{\theta_1' \ \theta_3(\pi z/R)}\right|$$

with $q = e^{-\pi L/R}$.

In both cases the dominant term diverges as $\frac{1}{2\pi}\log R$

Both results assume $L >> R$

Performing a dual tranformation we can obtain the behaviour for $R >> L$ i.e. in the high T regime. In this case the Green function can be written as:

$$G(z;z_0) = -\frac{1}{2\pi}\log\left|\frac{\theta_1\left[\pi(z-z_0)/L\right]}{\theta_4\left[\pi(z-\bar{z}_0)/L\right]}\right| - \frac{\mathrm{Im}z\ \mathrm{Im}z_0}{LR} \qquad\qquad q = e^{-2\pi R/L}$$

and we have:

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{\pi|\epsilon|}{L} + \frac{1}{2\pi}\log|\theta_4(2\pi i\ \mathrm{Im}\ z/L)/\theta_1'| - \frac{(\ \mathrm{Im}\ z)^2}{LR}$$

Expanding in powers of R/L we find

$$\sigma w^2(z) = -\frac{1}{2\pi}\log\frac{2\pi|\epsilon|}{L} + \frac{R}{4L}$$

This time the R dependence is linear !!

Summary of the result for the Polyakov loop correlator

- Low temperature

$$w^2 \sim \frac{1}{2\pi\sigma} \log(\frac{R}{R_c}) + \dots \qquad\qquad (L >> R >> 0)$$

- High temperature (but below the deconfinement transition)

$$w^2 \sim \frac{1}{2\pi\sigma}(\frac{\pi R}{2L} + \log(\frac{L}{2\pi|\epsilon|}) + \dots) \qquad\qquad (R >> L)$$

Log increase of the flux tube width at zero temperature but Linear increase near the deconfinement transition!

Comparison with MC simulations.

The 3d gauge Ising model is perfectly suited for studying the flux tube width.

Thanks to duality one can create a "vacuum" containing the Wilson loop or the Polyakov loop correlators by simply frustrating the links in the dual lattice orthogonal to a surface bordered by the loops.

Any choice of the surface is equivalent.

Measuring the energy operator (which is the dual of the plaquette operator) in this vacuum one can thus evaluate the ratio:

$$\frac{< P(0,0)P^{+}(0,R)U_{\mu,\nu}(x_0,x_1,h) >}{< P(0,0)P^{+}(0,R) >}$$

for any value of R and L with the same statistical uncertainty of the expectation value of the plaquette in the usual vacuum $< U_{\mu,\nu} >$.

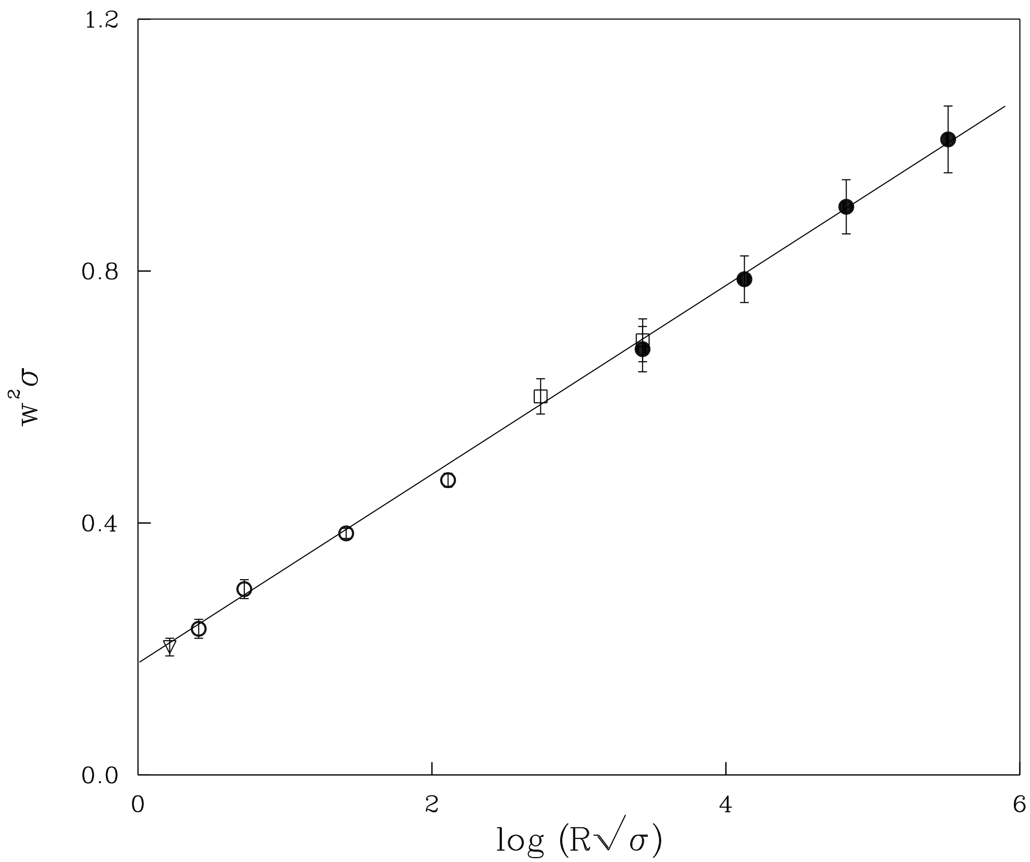
We used Wilson loops to test the low T predictions and Polyakov loop correlators for the high T ones.

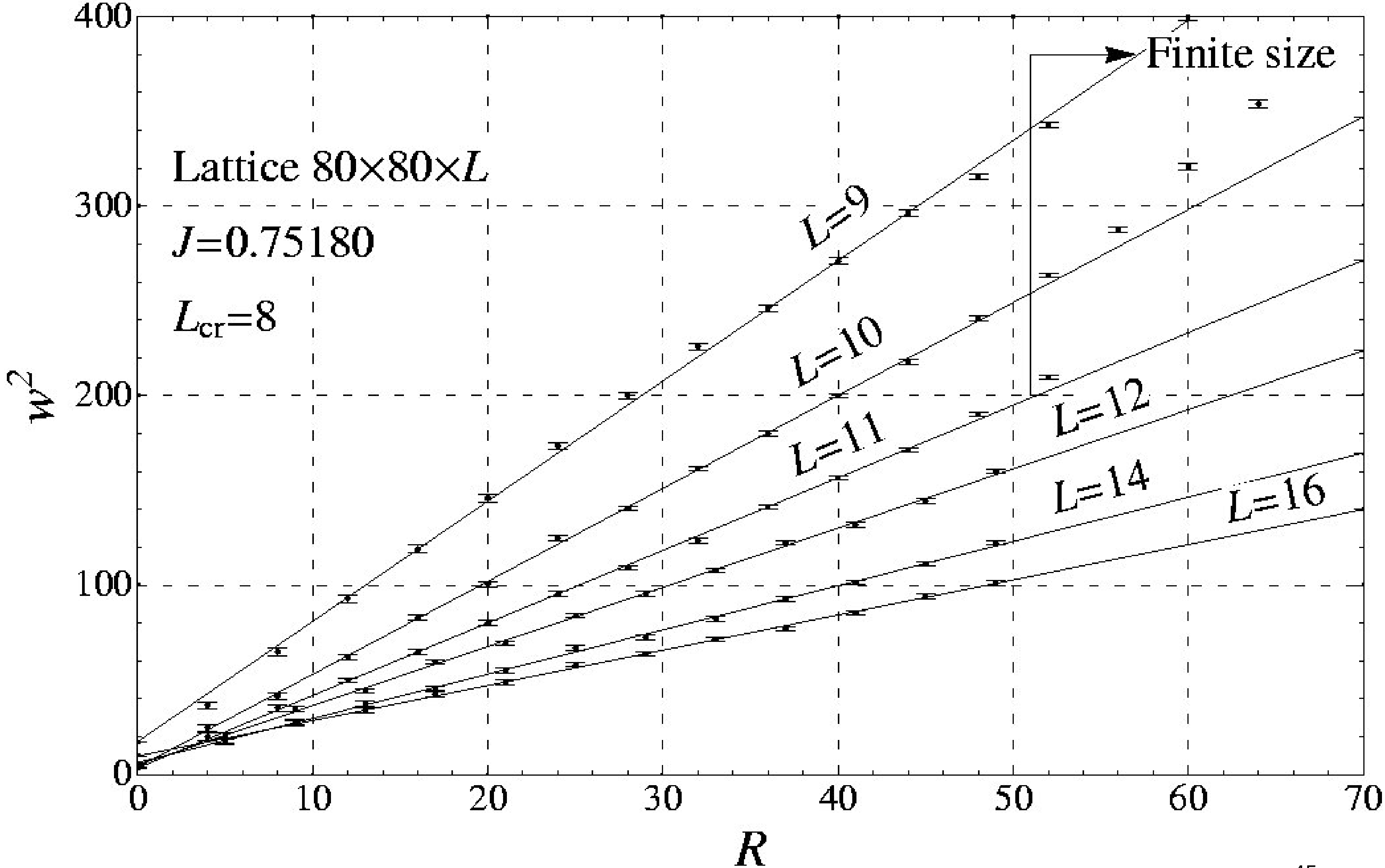
- **Low** T we tested several values of β in the range $0.6543 \leq \beta \leq 0.7516$ and several sizes of the Wilson loops (ranging from 12^2 to 64^2) so as to test a wide range of $R\sqrt{\sigma}$ values. The log fit of σw^2 as a function of $R\sqrt{\sigma}$ shows a very good χ^2 with an angular coef. 0.150(5) to be compared with $1/2\pi \sim 0.15915...$

- **High** T we studied the model at $\beta = 0.75180$ which corresponds to $aT_c = 1/8$ and $a^2\sigma = 0.0105(2)$ and tested lattice sizes $9 \leq L \leq 16$ i.e. temperatures ranging from $T = \frac{T_c}{2}$ to $T = \frac{8}{9}T_c$ For all the values $L \geq 10$ the linear fit has very good χ^2, but the ang.coef. shows deviations with respect to the expected value $1/(4\sigma L)$.

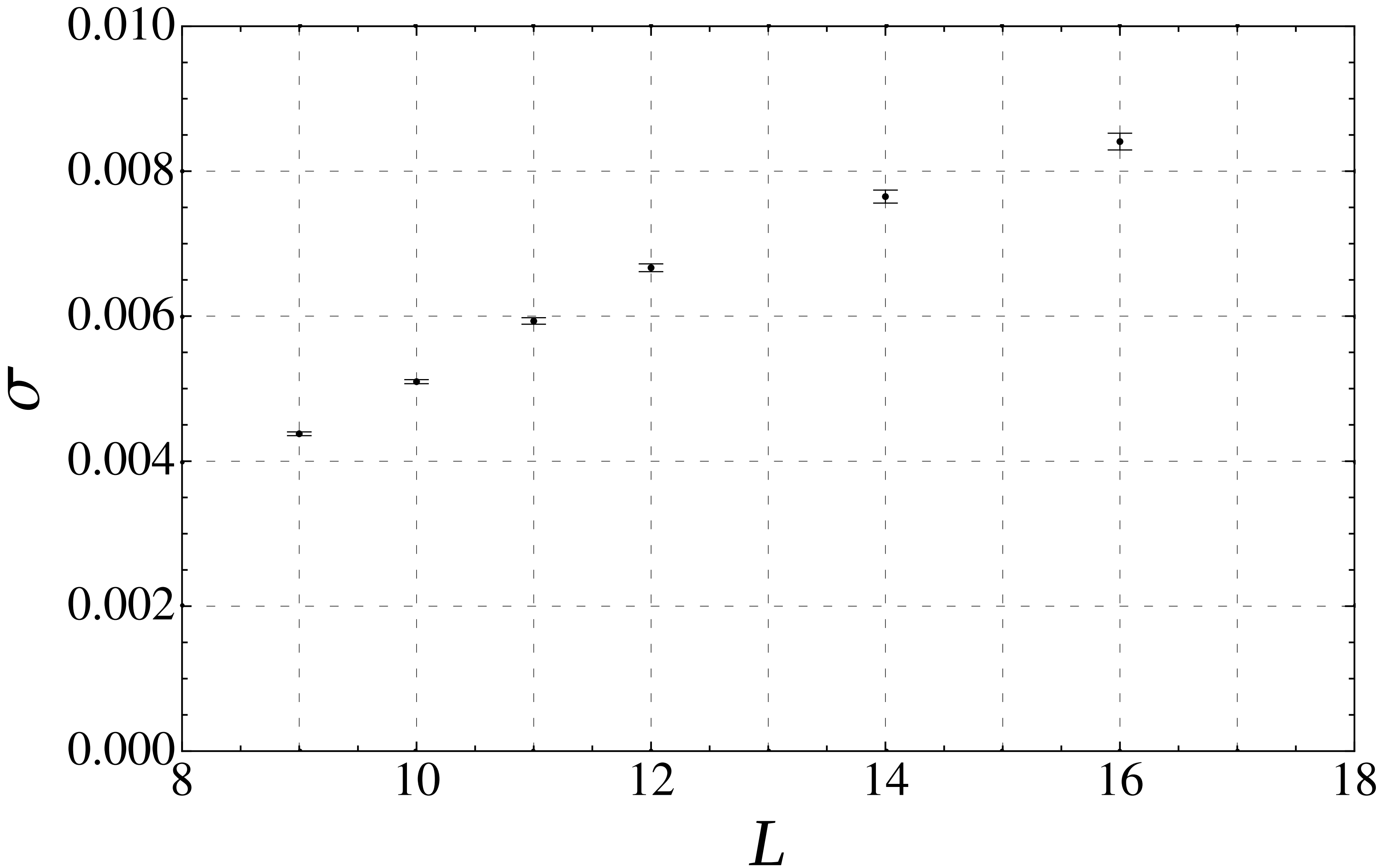
Squared width of the flux tube in units of sigma for the $\mathbb{Z}_2$ gauge theory. The open symbols are Wilson loop data while the black circles refer to the (dual) Ising interface.

fig. 3





Finite size
Lattice 80×80×L
J=0.75180
L_cr=8
L=9
L=10
L=11
L=12
L=14
L=16
w^2
R
0
100
200
300
400
0
10
20
30
40
50
60
70


0.010
0.008
0.006
0.004
0.002
0.000
σ
8
10
12
14
16
18
L

Dimensional reduction and the Svetitsky-Yaffe conjecture

- "weak form of the S-Y conjecture:"

  The high temperature behaviour of a (d+1) LGT with gauge group G can be effectively described by a spin model in d dimensions with (global) symmetry group C (the Center of G).

- "strong form of the S-Y conjecture:"

  If both the spin model and the LGT have continuous phase transitions then they share the same universality class.

The mapping between the LGT and the effective spin model is based on the following identifications

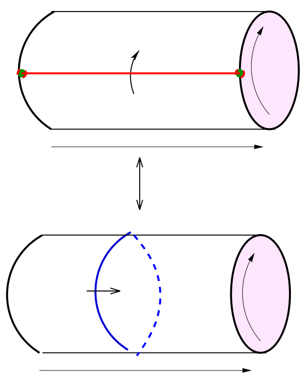
LGT	spin model
Low T confining phase	High T symmetric phase
Polyakov loop (“C-odd”)	spin operator
Plaquette operator (“C-even”)	energy operator
Thermal perturbation	energy perturbation
string tension (σ/T)	mass of the theory
Polyakov loop correlator	spin-spin correlator

These mappings should be intended in a “renormalization group sense” i.e. the Polyakov loop operator is mapped into a linear combination of all the (C-odd) operators in the spin model. For instance, in the 2d Ising case the whole conformal family of the spin operator. This combination will be dominated by the relavant(s) operator(s). In the Ising case only one: the spin operator.

In the case of the plaquette operator the mapping will be in general a linear combination of the energy and the identity families.

Duality and the closed string interpretation

It is interesting to look at this result from the "dual channel", i.e. perform a modular transformation $t \longrightarrow 1/t$ of the open string channel 1-loop free energy.



The result of the transformation is

$$\mathcal{F} = 2\pi L \left(\frac{\pi}{\sigma}\right)^{\frac{d}{2}-2} \sum_k c_k \, G\left(R; M(k)\right)$$

where $G\left(R; M\right) =$ propagator of a scalar field of mass M over the spatial distance $\vec{R}$ between the two D0-branes:

$$G(R; M) = \frac{1}{2\pi}\left(\frac{M}{2\pi R}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(MR) \quad ,$$

the mass $M(k)$ is that of a closed string state with k representing the total oscillator number:

$$M^2(k) = (\sigma L)^2 \left[1 + \frac{8\pi}{\sigma L^2}\left(k - \frac{d-2}{24}\right)\right]$$

This expression agrees with that obtained by Lüscher and Weisz (2005) with a different approach.

Two important remarks

- From the lowest mass ($k = 0$, $m = 1$) in the closed string channel:

$$M^2(1,0) = (\sigma L)^2 \left[1 - \frac{\pi(d-2)}{3\sigma L^2} \right]$$

one can obtain an estimate for the deconfinement temperature (recall that $T = 1/L$) which in this framework appears as a consequence of the tachionic state present in the NG string (Olesen, 1985):

$$\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{3}{\pi(d-2)}}$$

This estimate turns out to be in very good agreement with MC simulations for several LGT's.

- In the large R and small L limit the NG partition function reduces to a single Bessel function. This means that the the NG effective string predicts the following behaviour for the Polyakov loop correlator:

$$\langle P(0)P^\dagger(R)\rangle \sim K_{\frac{d-3}{2}}(MR) \quad ,$$

with $M \sim \sigma L$. This is exactly the limit in which dimensional reduction occurs ("Svetitsky-Yaffe" conjecture). From the QFT approach to spin models we know that at high temperature and large distance whatever model we study the spin spin correlator will be dominated by the state of lowest mass whose propagator in d' dimensions is given by

$$G(R) \sim K_{\frac{d'-2}{2}}(mR)$$

Since $d' = d - 1$ this result exactly coincides with what we obtain with the NG string.

Agreement between 2d spin model estimates and effective string predictions.

With the Nambu-Goto effective string we obtain for the Polyakov loop correlator:

$$\langle P(0,0)P(0,R)^{+}\rangle = \sum_{n=0}^{\infty} |v_n|^2 \left(\frac{\tilde{E}_n}{\pi}\right) K_0(\tilde{E}_n R).$$

This expression is expected to be reliable in the large distance limit. In this limit only the lowest state $(n=0)$ survives and we end up with a single K_0 function:

$$\lim_{R\to\infty} \langle P(0,0)P(0,R)^{+}\rangle \sim K_0(\tilde{E}_0 R).$$

The spin-spin correlator of any 2d spin model in the symmetric phase is given by

$$\lim_{R\to\infty} \langle \sigma(0,0)\sigma(0,R)^{+}\rangle \sim K_0(mR).$$

The two expression coincide, they are universal (no dependence on the symmetry group) and allow to identify m with $\tilde{E}_0 = \sigma L \left\{1 - \frac{\pi}{3\sigma L^2}\right\}^{1/2}$. which, at the first order in $1/L$ becomes

$$m \leftrightarrow \sigma L = \sigma / T$$

Effective spin model description of the the flux tube thickness.

Following the S-Y mapping the flux density in the Ising LGT in (2+1) dimensions becomes the ratio of *connected* correlators:

$$\frac{\langle \sigma(x_1)\epsilon(x_2)\sigma(x_3)\rangle}{\langle \sigma(x_1)\sigma(x_3)\rangle}$$

in the high temperature phase of the 2d Ising model in zero magnetic field.

The large distance behaviour of this correlator can be evaluated using the Form Factor approach.

Width of the flux tube

The width of the flux tube evaluated at the midpoint between the two spins is given by

$$w^2(r) \;=\; \frac{r^2}{2K_0(2mr)} \int_{-\infty}^{\infty} dx \, \frac{x^2}{1+x^2} e^{-2mr\sqrt{1+x^2}}.$$

where $R \equiv 2r$ is the distance between the two spins.

In the large R limit we thus obtain

$$w^2 \simeq \frac{1}{2\pi}\left(\frac{\pi R}{2m} - \frac{\pi}{2m^2} + \ldots\right) \qquad (L \sim 1)$$

to be compared with the effective string result:

$$w^2 \sim \frac{1}{2\pi}\left(\frac{\pi R}{2\sigma L} + \log(\frac{L}{2\pi}) + \ldots\right) \qquad (R >> L)$$

linear increase of the width in both cases but with a different T dependence of the coefficent $m = \tilde{E}_0 = \frac{\sigma}{T}\sqrt{1 - \frac{\pi T^2}{3\sigma}} = \sigma L\sqrt{1 - \frac{\pi}{3\sigma L^2}}$

This discrepancy is most likely due to the free bosonic approx in the string calculation.

Conclusions

- The effective string approach predicts a logarithmic increase of the flux tube width at low temperature and a linear increase at high T (but still in the confining regime)

- This scenario is confirmed by MC simulations in the 3d gauge Ising model both at low and at high T, but an increasing discrepancy in the coefficient of the linear term appears as T_c is approached

- Also dimensional reduction predicts a linear increase of the flux tube thickness at high temperature, but with a T dependence of the coefficient wich could explain the MC results.

The snake algorithm.

Goal: compute the ratio $G(R)/G(R+1)$.

Proposal: Use duality and factorize the ratio of partition functions in such a way that for each factor the partition functions differ just by the value of $J_{\langle ij \rangle}$ at a single link

$$\frac{Z_{L\times R}}{Z_{L\times(R+1)}} = \frac{Z_{L\times R,0}}{Z_{L\times R,1}} \cdots \frac{Z_{L\times R,M}}{Z_{L\times R,M+1}} \cdots \frac{Z_{L\times R,L-1}}{Z_{L\times R,L}} ,$$

where $L\times R, M$ denotes a surface that consists of a $L\times R$ rectangle with a $M\times 1$ column attached.

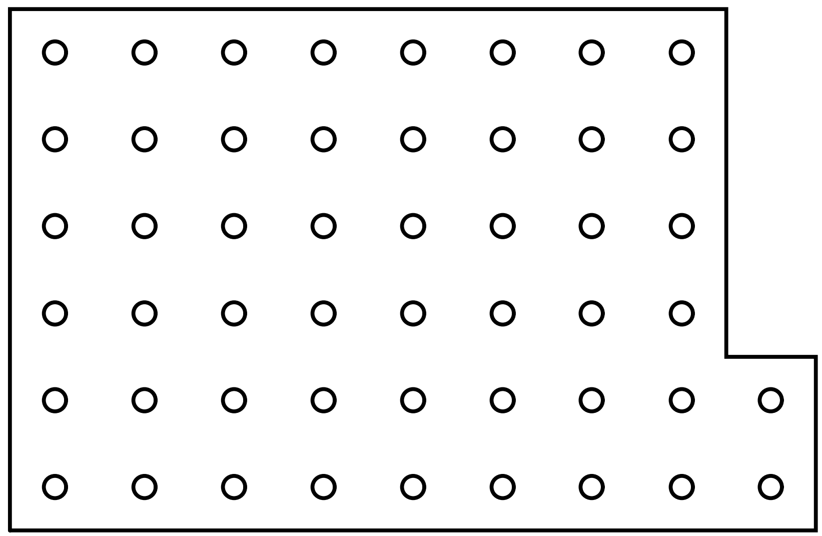


Figure 1: Sketch of the surface denoted by $L \times R, M$. In the example, $L = 6$, $R = 8$ and $M = 2$. The circles indicate the links that intersect the surface.

Each of the factors can be written as expectation value in one of the two ensembles:

$$\frac{Z_{L\times R,M+1}}{Z_{L\times R,M}} = \frac{\sum_{s_i=\pm 1} \exp(-\tilde{\beta} H_{L\times R,M}(s))\ \exp(-2\tilde{\beta} s_k s_l)}{Z_{L\times R,M}} \ ,$$

where $< k, l >$ is the link that is added going from $L\times R, M$ to $L\times R, M+1$.

Further improvement: hierarchical updates.

Result: the error show no dependence on R !!