QCD Thermodynamics from the lattice

Introduction

I Equation of State

II EoS at finite density

III Some spatial correlations

Summary and Outlook

mostly based on work of the

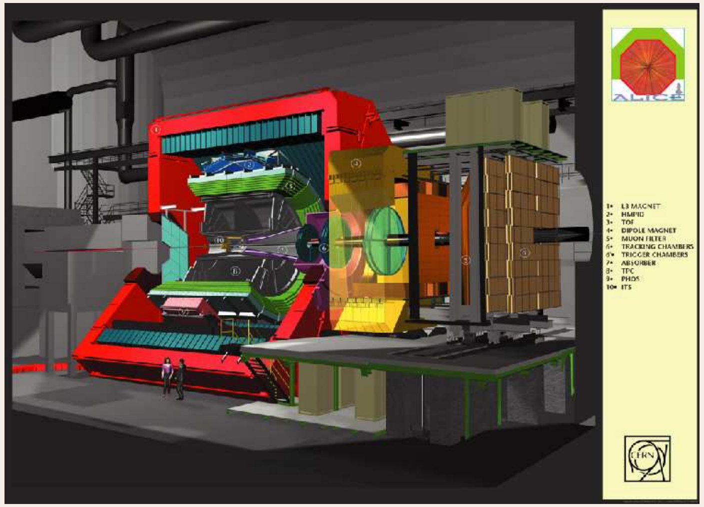
RIKEN –
BNL –
Columbia –
Bielefeld – Collaboration

Introduction

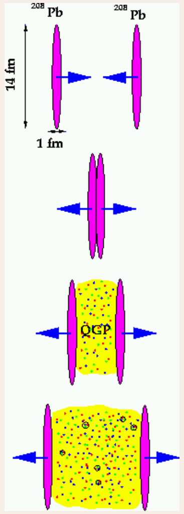
(part of the) Motivation:

Experiment

Nucleus-Nucleus Collisions at CERN
ALICE at LHC starting NOW



Generation of a QGP in Pb-Pb Collisions



beam energy: **2.8 TeV/A** (for lead)
$\sim$ **10000** particles/collision

initial energy density
$\sim$ **200 times nuclear density**

initial temperature; QGP life time
$\sim (4-5)\ \mathbf{T_c}$; $\tau \sim 10^{-22}$ **sec.**

phase transition at $\mathbf{T_c} \simeq \mathbf{10^{12}}\ {}^0\mathrm{K}$
back to "normal" vacuum

Lattice QCD may help to understand the experiment $\longrightarrow$ the early universe

Quantum Statistics in equilibrium :

partition function $Z = \mathrm{tr}\left\{e^{-\hat{H}/T}\right\}$

→ Feynman path integral

$$Z(T,V) = \int \mathcal{D}\phi(\vec{x},\tau) \exp\left\{-\int_0^{1/T} d\tau \int_0^V d^3\vec{x}\, \mathcal{L}_E[\phi(\vec{x},\tau)]\right\}$$

- integral over all configurations $\phi(\vec{x},\tau)$
- weighted by Boltzmann factor $\exp(-S_E)$
- euclidean "time" $\tau = it$
- (anti-) periodic boundary conditions in τ

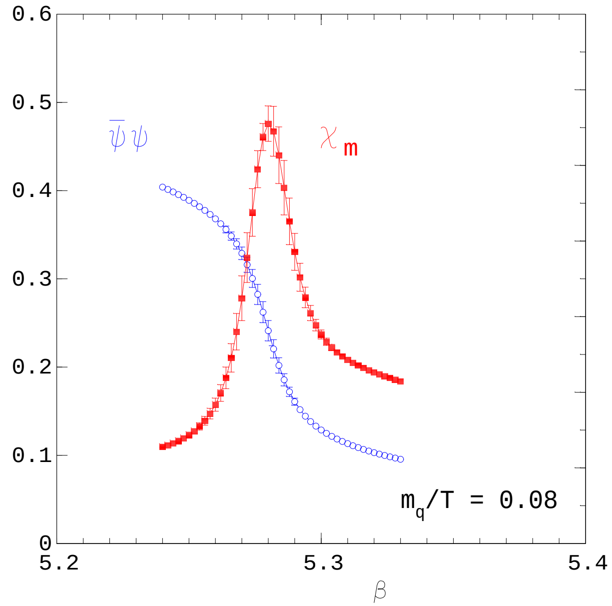
apply standard thermodynamic relations, e.g.

energy density $\epsilon = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T}\Big|_V$

specific heat $c_V = \frac{1}{VT^2}\frac{\partial^2 \ln Z}{\partial(1/T)^2}\Big|_V$

in general

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\mathrm{tr}\left\{\hat{\mathcal{O}}e^{-\hat{H}/T}\right\} = \frac{1}{Z}\int \mathcal{D}\phi\, \mathcal{O}[\phi] e^{-S_E[\phi]}$$

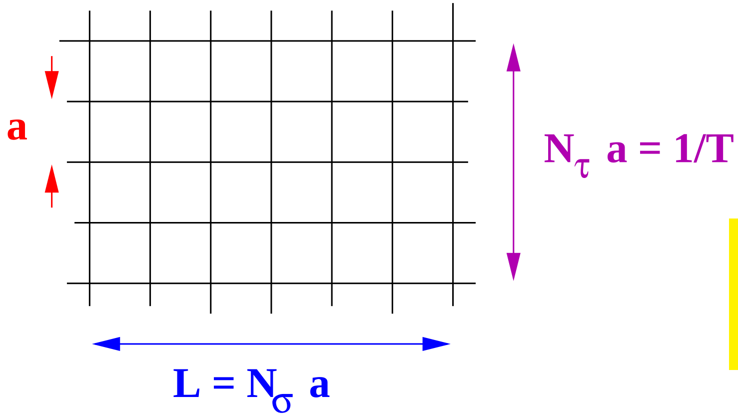


also : starting point of perturbation theory i.e. expansion in coupling strength g

numerical treatment of QCD $\Rightarrow$ discretize (Euclidean) space-time

$\Rightarrow$ **lattice**

$N_\sigma^3 \times N_\tau$



$$Z(T,V) = \int \prod_{i=1}^{N_\tau N_\sigma^3} d\phi(x_i) \exp\{-S[\phi(x_i)]\}$$

finite yet high-dimensional path integral

$\longrightarrow$ **Monte Carlo**

- thermodynamic limit, IR - cut-off effects $\quad LT = \frac{N_\sigma}{N_\tau} \longrightarrow \infty \quad$ (finite size scaling)
- continuum limit, UV - cut-off effects $\quad aT = \frac{1}{N_\tau} \longrightarrow 0 \quad$ improved actions
- chiral limit $\quad m \longrightarrow m_{\rm phys} \simeq 0$

  numerical effort $\sim (1/m)^p$

Choice of fermions

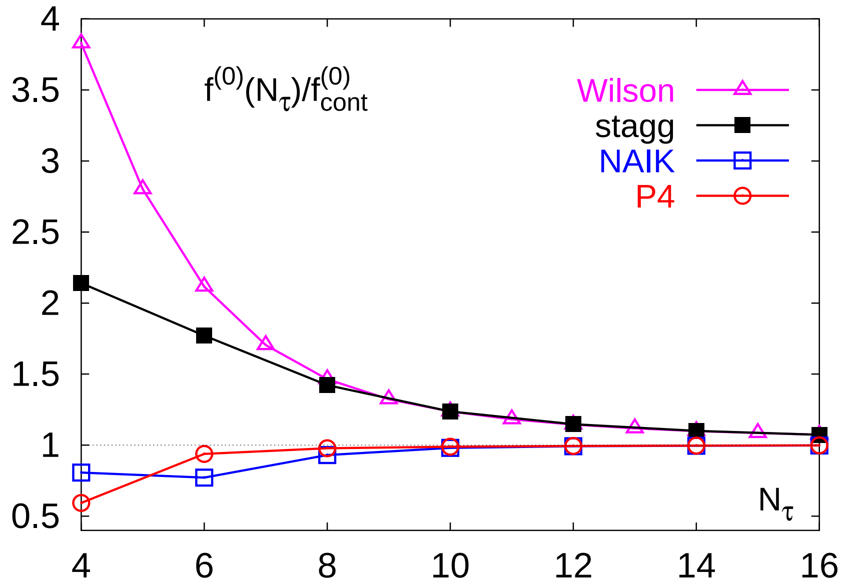
- free energy density, for instance (see later): $f/T^4 \;\sim\; N_\tau^4 \times \text{signal}$

$\Rightarrow$ signal $\sim 1/N_\tau^4$

$\Rightarrow$ keep N_τ small

$\Rightarrow$ coarse lattices $a = 1/N_\tau T$

$\Rightarrow$ improved actions



- Wilson-like fermions have turned out to be notoriously difficult to simulate at small quark masses

⋆ in the following: `p4` (to improve thermodynamics) and `fat3` (to improve flavor symmetry)

Simulation parameters

- $N_F = 2 + 1$: two degenerate u/d quarks + strange quark
- RHMC algorithm, exact to machine precision
  - polynomial approximation: 16/10 for light/strange quarks in molecular dynamics
    20/16 for light/strange quarks in heatbath/ Metropolis
  - Sexton/Weingarten, Hasenbusch
- lattice sizes $16^3 \times 4$, $24^3 \times 6$, $32^3 \times 8$ (prelim.) $(T > 0)$
  $16^3 \times 32, 24^3 \times 32, 32^3 \times 32, 24^2 \times 32 \times 48$ $(T = 0$, for scales and normalization$)$
- statistics $\mathcal{O}(10k - 60k)$ for $N_\tau = 4$, each $(\beta, \hat{m}_q, \hat{m}_s)$
  $\mathcal{O}(8k - 20k)$ for $N_\tau = 6$, each $(\beta, \hat{m}_q, \hat{m}_s)$
  $\mathcal{O}(5k - 20k)$ for $N_\tau = 8$, each $(\beta, \hat{m}_q, \hat{m}_s)$
  $\mathcal{O}(\geq 5k)$ for $T = 0$, each $(\beta, \hat{m}_q, \hat{m}_s)$
- along "line of constant physics" i.e. constant physical $m_K = 500$MeV, $m_\pi \simeq 220$MeV

I. Equation of State

start from energy-momentum tensor $\frac{\Theta^\mu_\mu(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T\frac{d}{dT}(p/T^4)$

where $p = \frac{T}{V}\ln Z(T,V) - \lim_{T\to 0}\frac{T}{V}\ln Z(T,V)$ subtracting T = 0 normalization

thus $$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{1}{T'^5}\Theta^\mu_\mu(T')$$

now $Z(T,V;g,m_l,m_s) = Z(N_\tau,N_\sigma,a;\beta,\hat{m}_l,\hat{m}_s) \longrightarrow Z(N_\tau,N_\sigma,a;\beta,m_\pi,m_K)$

and tune bare lattice parameters $\hat{m}_l, \hat{m}_s$ with β such that $m_{\pi,K} = \text{const} \Rightarrow \hat{m}_{l,s}(\beta), a(\beta)$

$$\Rightarrow \qquad \frac{\Theta^\mu_\mu(T)}{T^4} = -R_\beta(\beta) N_\tau^4 \left(\left\langle \frac{d\bar{S}}{d\beta} \right\rangle_T - \left\langle \frac{d\bar{S}}{d\beta} \right\rangle_{T=0} \right)$$

with $$R_\beta(\beta). = T\frac{d\beta}{dT} = -a\frac{d\beta}{da}$$

furthermore, will need $(\hat{m}_s(\beta) = \hat{m}_l(\beta) \times h(\beta))$

$$R_m(\beta). = \frac{1}{\hat{m}_l(\beta)}\frac{d\hat{m}_l(\beta)}{d\beta} \qquad\qquad R_h(\beta). = \frac{1}{h(\beta)}\frac{dh(\beta)}{dh}$$

action $S = \beta\, S_G \,+\, 2\,\hat{m}_l(\beta)\;\bar{\psi}_l\psi_l \,+\, \hat{m}_s(\beta)\;\bar{\psi}_s\psi_s \,+\, \beta$ independent

such that Θ^μ_μ consists of three pieces

$$\frac{\Theta^{\mu\mu}_G(T)}{T^4} = R_\beta\; N_\tau^4\; \Delta\langle\bar{S}_G\rangle$$

where $\Delta\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_{T=0} - \langle\mathcal{O}\rangle_T$

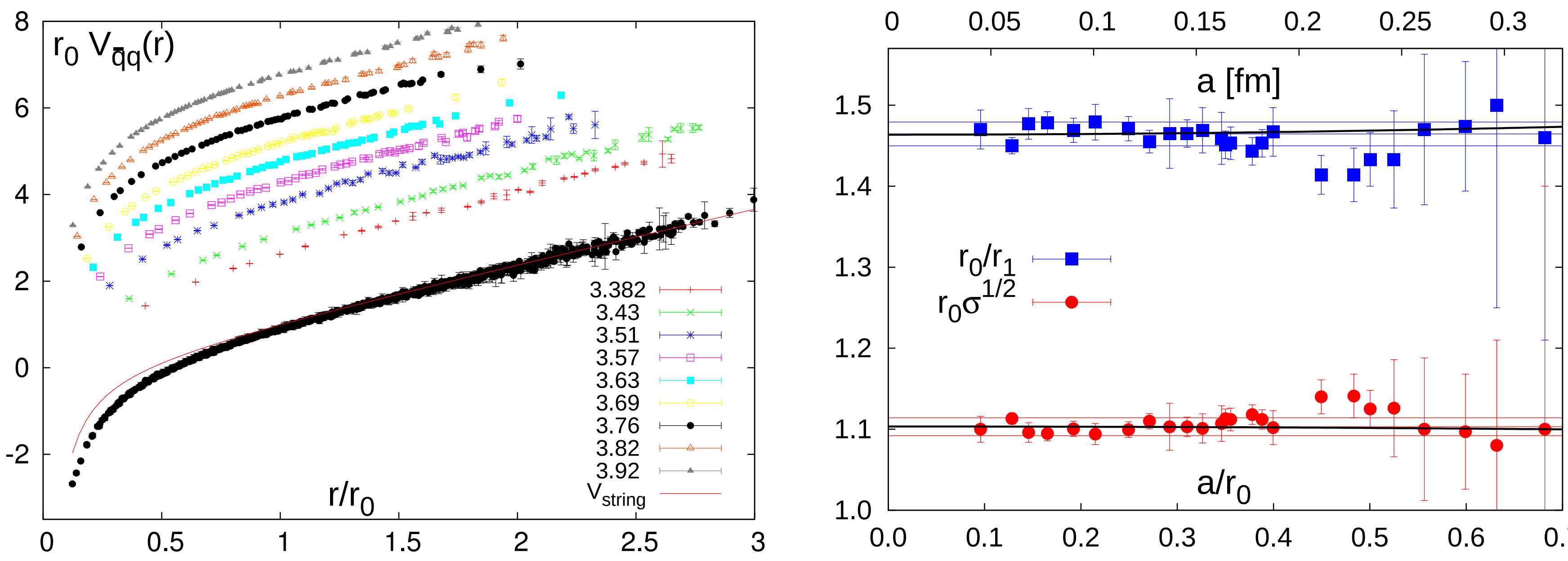
$$\frac{\Theta^{\mu\mu}_F(T)}{T^4} = -R_\beta\, R_m\; N_\tau^4\; \{\, 2\,\hat{m}_l\,\Delta\langle\bar{\psi}\psi\rangle_l \,+\, \hat{m}_s\,\Delta\langle\bar{\psi}\psi\rangle_s \,\}$$

$$\frac{\Theta^{\mu\mu}_h(T)}{T^4} = -R_\beta\, R_h\; N_\tau^4\; \hat{m}_s\,\Delta\langle\bar{\psi}\psi\rangle_s$$

need: β functions $R_\beta(\beta), R_m(\beta), R_h(\beta)$

"action differences" $\Delta\bar{S}_G, \Delta\langle\bar{\psi}\psi\rangle_{l,s}$

⋆ T = 0 scale taken from $\Upsilon\, 2S - 1S$ splitting [A. Gray et al.] via the heavy quark potential $V(r)$



potential well decribed by

$$V(r) = c_0 - \frac{\alpha}{r_{imp}} + \sigma r_{imp} \qquad \text{with} \qquad \frac{a}{r_{imp}} = 4\pi \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\vec{r}} \left[4 \sum_i^3 \sin^2(ak_i/2) + \frac{1}{3}\sin^4(ak_i/2)\right]^{-1}$$

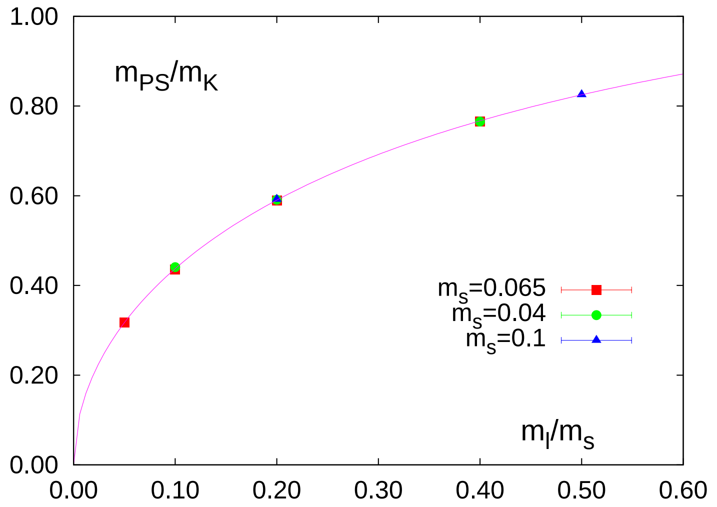
ar_0 from $\left. r^2 \frac{dV(r)}{dr} \right|_{r=r_0} = 1.65$ together with $r_0 = 0.469(7)$ fm [A. Gray et al.] $\Rightarrow$ $a(\beta)$

$m_{\pi,K}$ = const: **Line of Constant Physics (LoCP)**

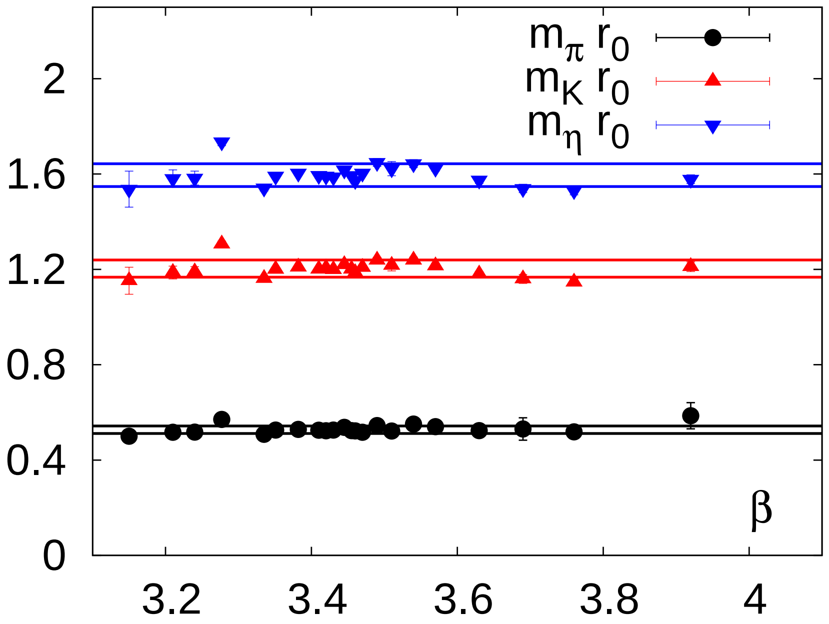
- to sufficient precision,
  m_π/m_K depends on $h = \hat{m}_s/\hat{m}_l$ only

  $\Rightarrow$ fix $h = 10$

  $\Rightarrow$ $R_h(\beta) = 0$

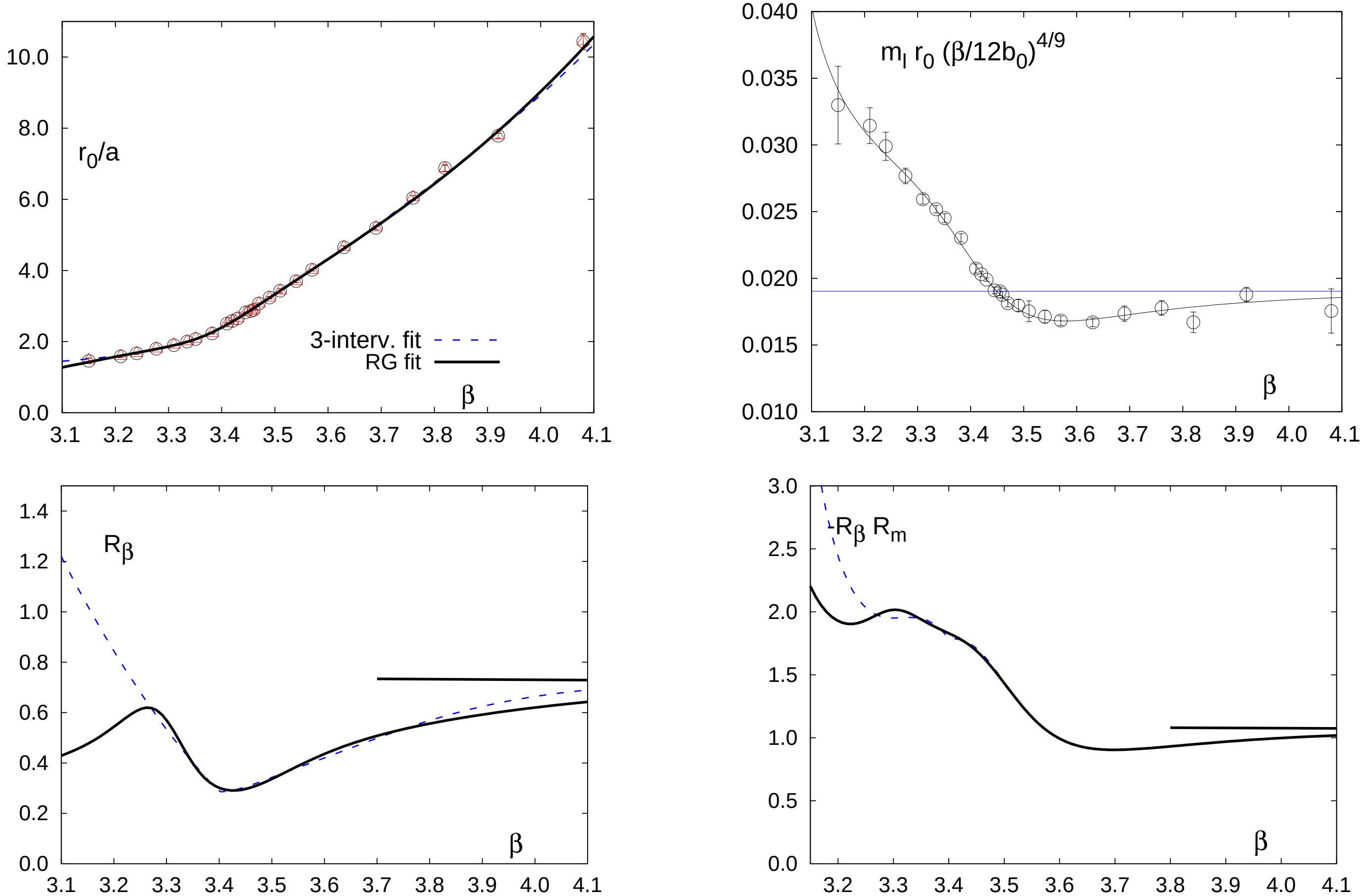


- fine tune $\hat{m}_l(\beta)$



$m_K \simeq m_K^{\text{phys}}$

$m_\pi \simeq 220\,\text{MeV}$



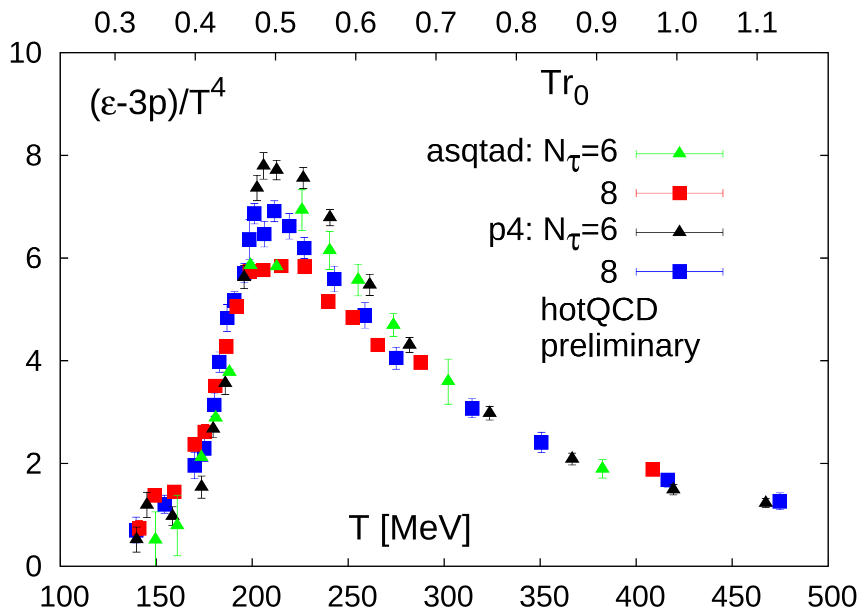
Allton inspired parametrization with rational fct. in $\hat{a}(\beta) = R_\beta^{(2-loop)}(\beta)/R_\beta^{(2-loop)}(\beta = 3.4)$

$$\frac{a}{r_0} = a_r R_\beta^{(2-loop)} \frac{1 + b_r \hat{a}^2 + c_r \hat{a}^4 + d_r \hat{a}^6}{1 + e_r \hat{a}^2 + f_r \hat{a}^4} \qquad \Rightarrow \quad R_\beta = \frac{r_0}{a} \left(\frac{dr_0/a}{d\beta} \right)^{-1}$$

$$\hat{m}_l = a_m R_\beta^{(2-loop)} \left(\frac{12b_0}{\beta} \right)^{4/9} \frac{1 + b_m \hat{a}^2 + c_m \hat{a}^4 + d_m \hat{a}^6}{1 + e_m \hat{a}^2 + f_m \hat{a}^4 + g_m \hat{a}^6} \qquad \Rightarrow \quad R_m$$

the results:

$\Theta^{\mu}_{\mu}(T)/T^4$ (the "raw" data)

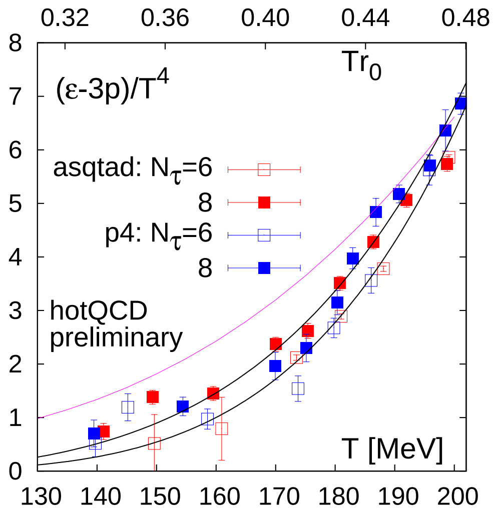


⋆ small discretization effects

⋆ $N_\tau = 8$ data corroborating

⋆ agreement with [MILC]
i.e. different, asqtad action

<u>low T</u>



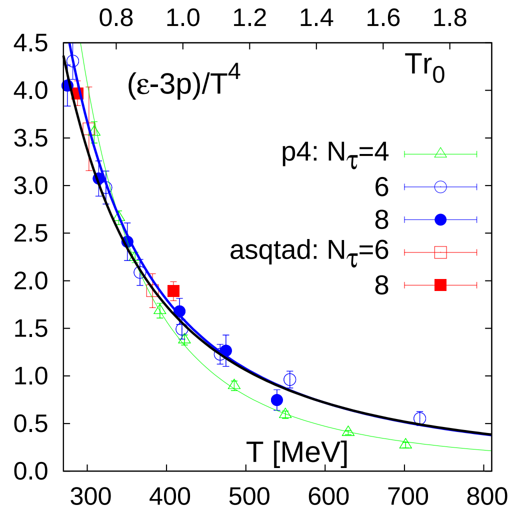
⋆ shift in T_c

⋆ pink line:
hadron gas

<u>high T</u>

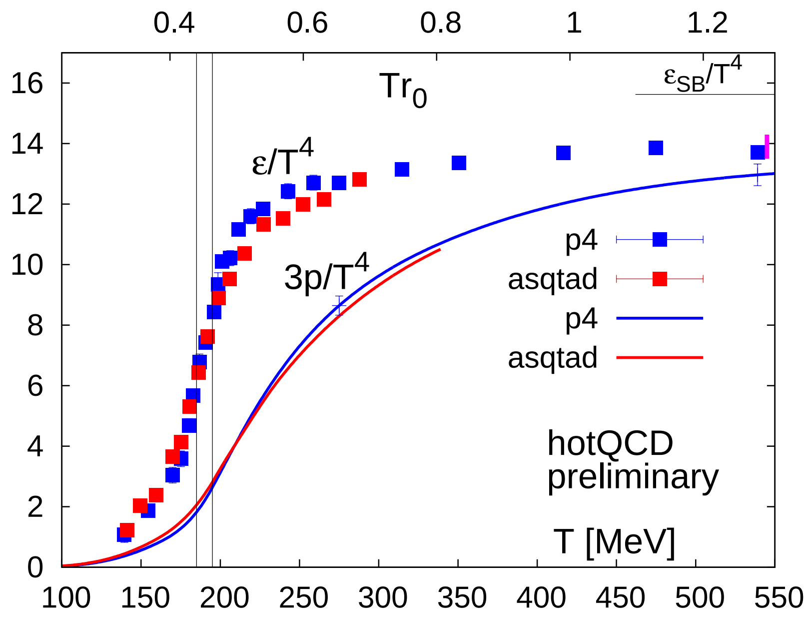
⋆ deviations from
conformal symm.

⋆ get discretization
effects controlled

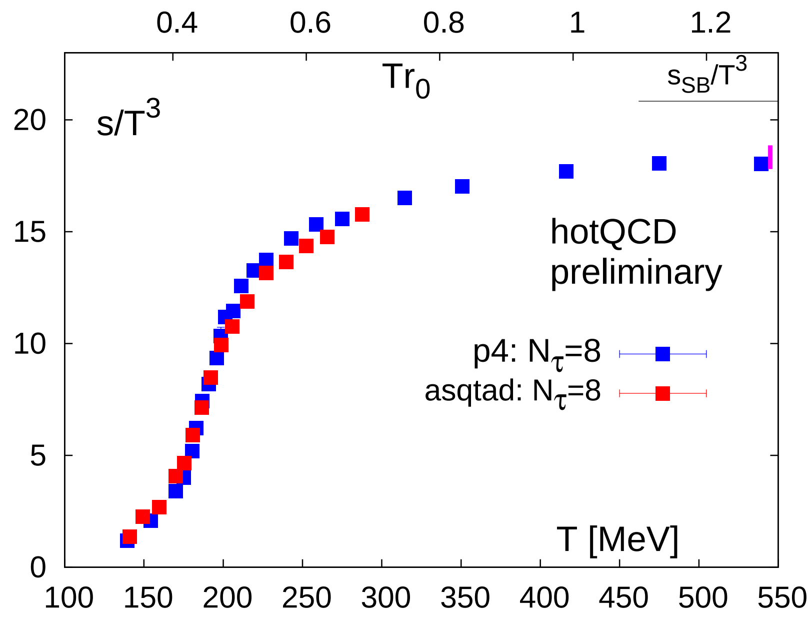


the results:

pressure and energy density



entropy density $s/T^3 = (\epsilon + p)/T^4$



⋆ discretization errors small, good agreement with `asqtad` action

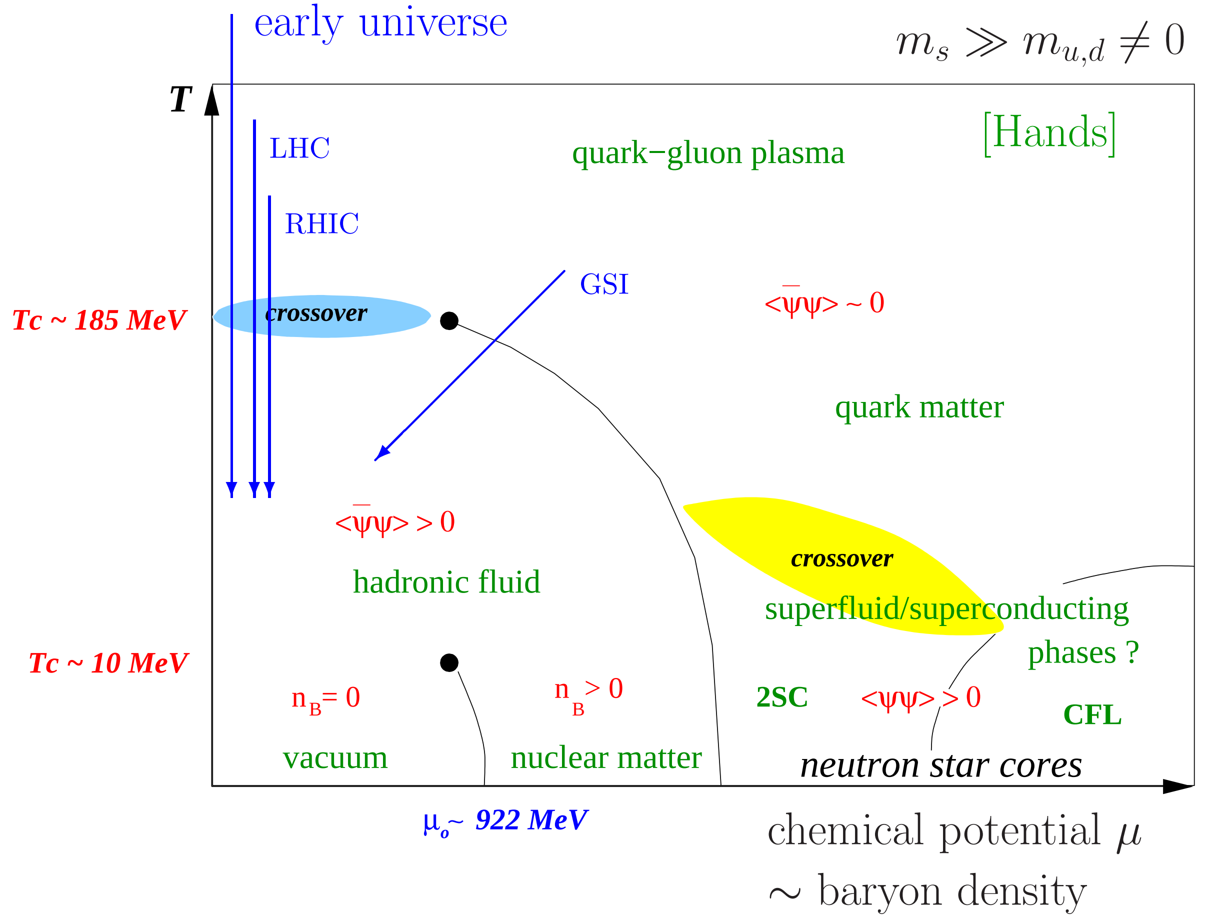
⋆ integration error: see little bar to the right

⋆ in comparison with **Stefan-Boltzmann**: 10 % below at 2 - 3 T_c

→ compare with dim.red. [Kajantie et al.]

II. EoS at finite density

expected properties of the phase diagram (standard scenario):



in detail dependent on masses of light flavors

$m_{u,d} \ll m_s \quad N_F = 2$

$m_{u,d} < m_s \quad N_F = 2+1$

$m_{u,d} \simeq m_s \quad N_F = 3$

[see e.g. Rajagopal, Wilczek]

⋆ for RHIC, LHC, universe: μ small

Taylor expansion

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(T, \mu_u, \mu_d, \mu_s) = \sum_{i,j,k} c_{ijk}(T) \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

with ($\hat{\mu} = \mu/T$)

$$c_{ijk} = \frac{1}{i!j!k!} \frac{\partial^i}{\partial \hat{\mu}_u^i} \frac{\partial^j}{\partial \hat{\mu}_d^j} \frac{\partial^k}{\partial \hat{\mu}_s^k} \left(\frac{p}{T^4}\right) |_{\vec{\mu}=0}$$

for instance

$$c_{200} = \frac{N_\tau}{2N_\sigma^3} \left(\frac{1}{4} \left\langle \frac{\partial^2 \ln \det M}{\partial \hat{\mu}_u^2} \right\rangle + \frac{1}{16} \left\langle \left(\frac{\partial \ln \det M}{\partial \hat{\mu}_u} \right)^2 \right\rangle \right)$$

with

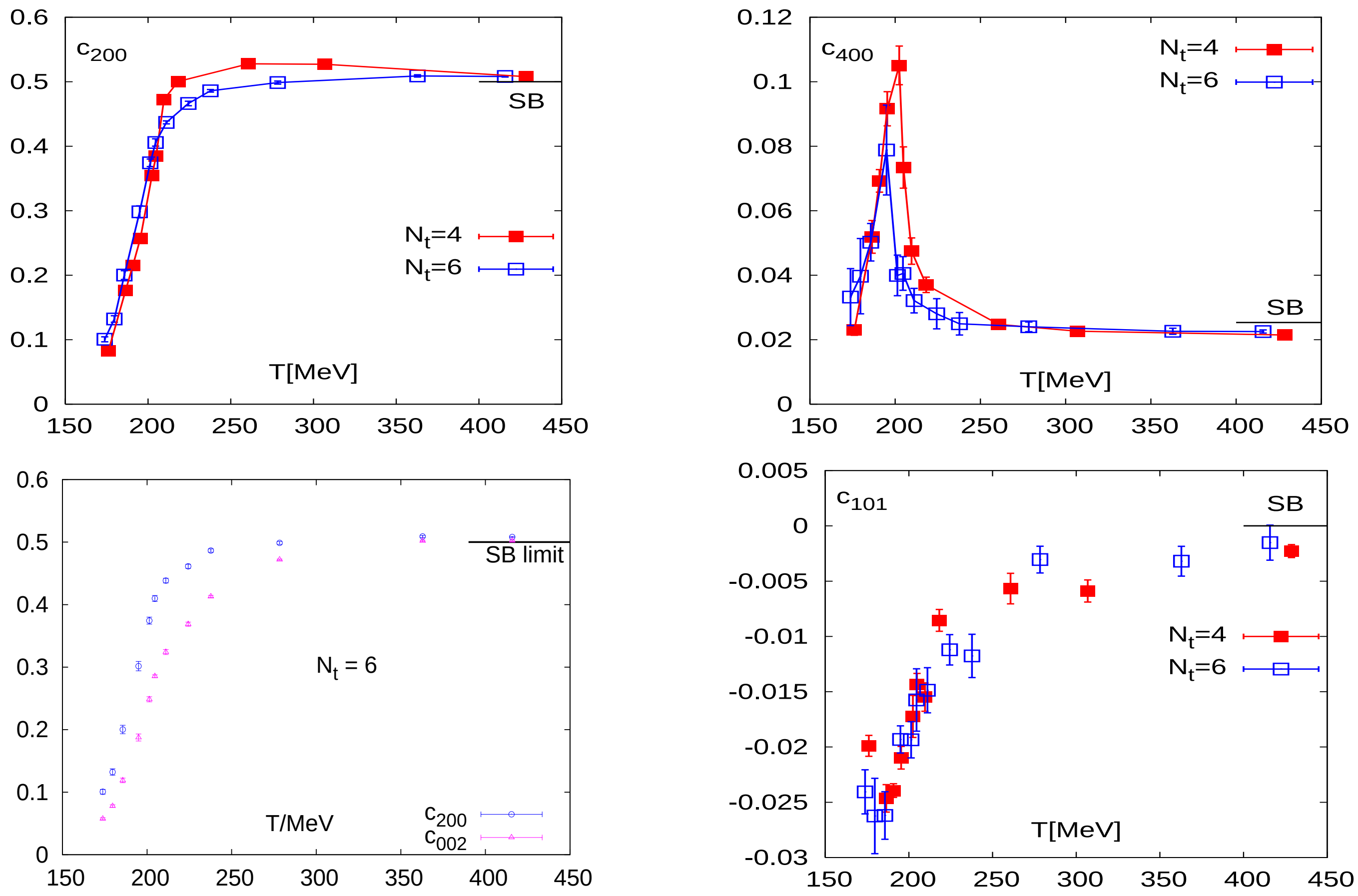
$$\frac{\partial^2 \ln \det M}{\partial \hat{\mu}_u^2} = \mathrm{tr} \left(M^{-1} \frac{\partial^2 M}{\partial \hat{\mu}_u^2} \right) - \mathrm{tr} \left(M^{-1} \frac{\partial M}{\partial \hat{\mu}_u} M^{-1} \frac{\partial M}{\partial \hat{\mu}_u} \right)$$

For degenerate u and d quarks: $\mu_u = \mu_d \equiv \mu_q \Rightarrow$ e.g.

$$c_{20}^{qs} = c_{200} + c_{020} + c_{110}$$

note: $c_{ijk} = 0$ for $i + j + k$ odd because of charge symmetry

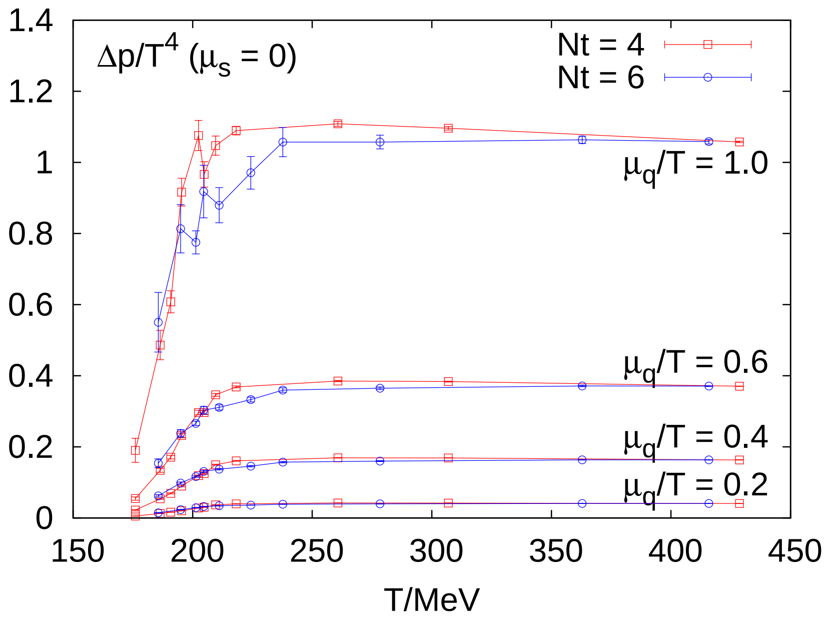
some examples of the coefficients



- ⋆ some but not large discretization effects
- ⋆ approaching SB limit quickly
- ⋆ rapid rise in quadratic coeff.
- ⋆ peaks in quartic coeff.
- ⋆ small but non-vanishing off diagonal coeff.

⇒ **pressure** difference

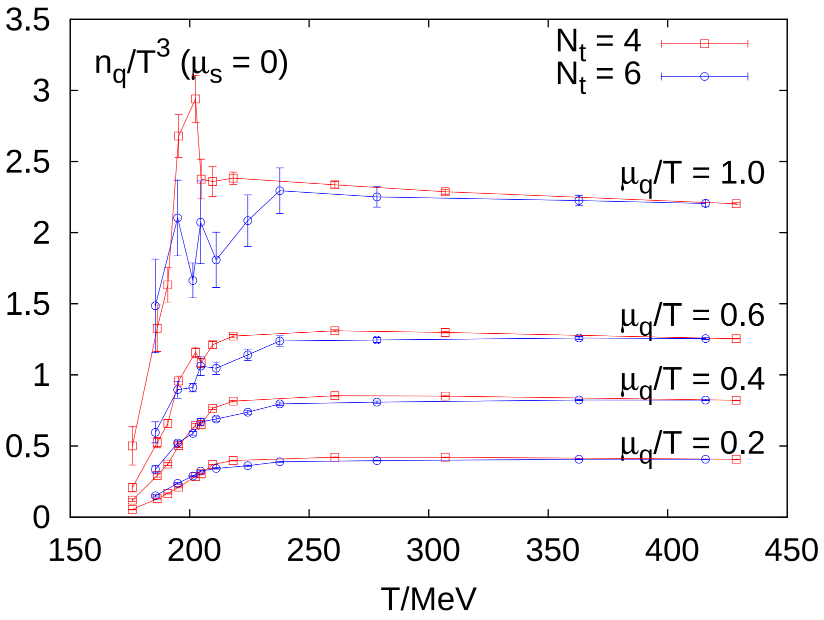
$$\Delta p = p(\vec{\mu}) - p(\vec{\mu} = 0)$$



⋆ small discretization effects

⋆ rapid rise at $T \simeq 200$ MeV

⋆ small contribution compared to $\mu = 0$



⇒ **number density**

$$\frac{n_q}{T^3} = 2\, c_{20}^{qs}\, \hat{\mu}_q + \ldots$$

⋆ vanishing at $\mu_q = 0$

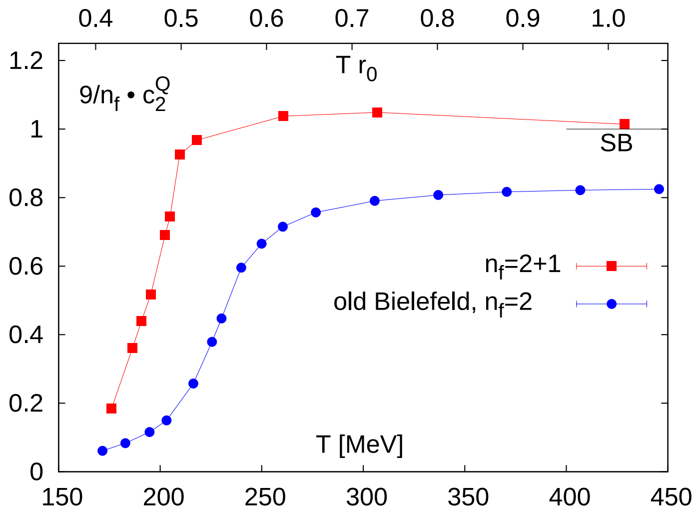
⋆ with rising μ_q , developing a peak more physical in terms of B, S, Q quantum numbers

$$\frac{p}{T^4} = \sum_{i,j,k} c_{ijk}^{BSQ}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k$$

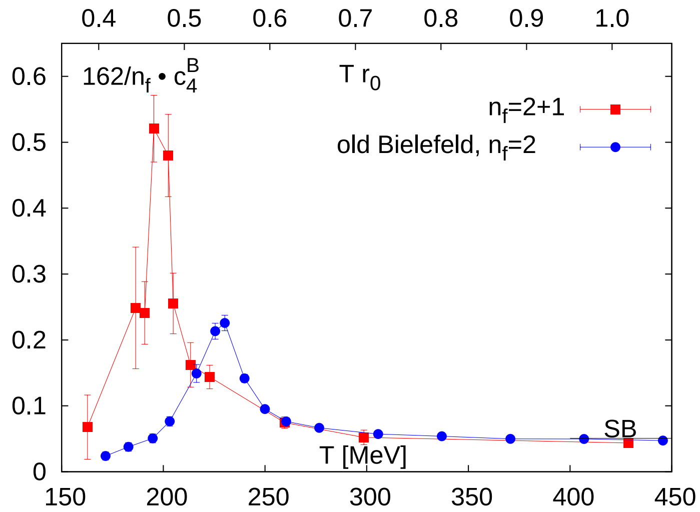
where $\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$

for instance $c_{400}^{BSQ} = \frac{1}{81}\left(c_{40}^{qs} + c_{31}^{qs} + c_{22}^{qs} + c_{13}^{qs} + c_{04}^{qs}\right)$

leading to



$c_2^Q = c_{002}^{BSQ}$



$c_4^B = c_{400}^{BSQ}$

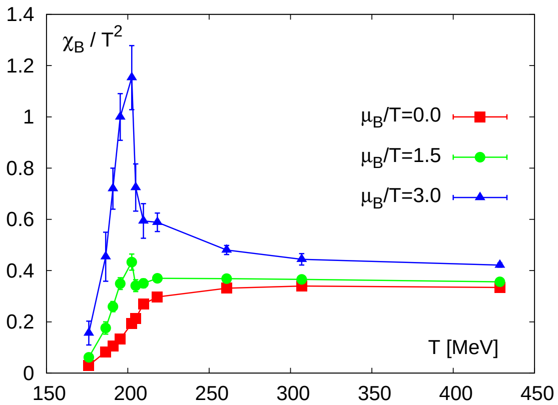
old Bielefeld: $m_{PS} \simeq 700$ MeV

$\Rightarrow$ investigate **quantum number fluctuations** $\longrightarrow$ event-to-event fluctuations in HIC

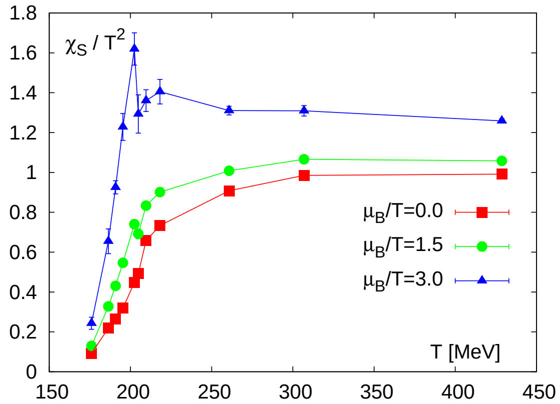
at $\mu_S = 0$: (note: at $\mu_u = \mu_d$ follows $\mu_Q = 0$)

$$\chi_X \sim \langle n_X^2 \rangle - \langle n_X \rangle^2$$

$$\frac{\chi_B}{T^2} = 2\, c_{200}^{BSQ} + 12\, c_{400}^{BSQ} \left(\frac{\mu_B}{T}\right)^2 + \ldots$$



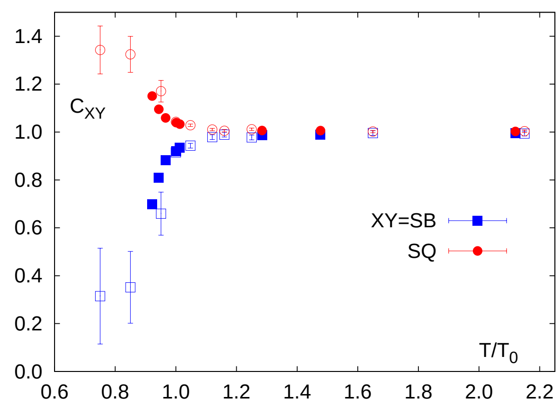
$$\frac{\chi_S}{T^2} = 2\, c_{020}^{BSQ} + 2\, c_{220}^{BSQ} \left(\frac{\mu_B}{T}\right)^2 + \ldots$$



⋆ peaks developing

suggest approaching the critical endpoint

$3\langle n_S n_Y \rangle / \langle n_S^2 \rangle$ [Gavai, Gupta]



⋆ $C_{SY} \simeq 1$ signals

$S = 1$ carried by $B, Q = \pm 1/3$

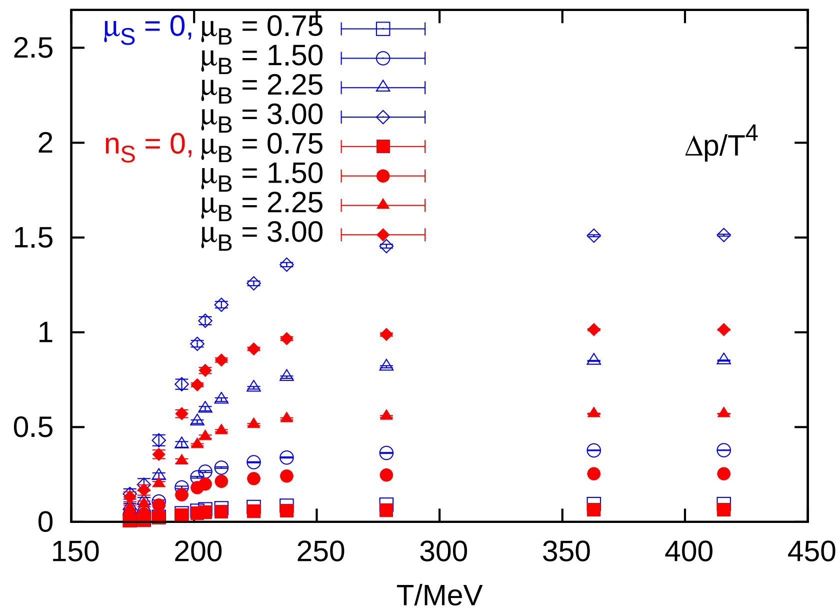
so far $\mu_S = 0$ $\rightsquigarrow$ $n_S = 0$ more appropriate for heavy ion collisions

i.e.

$$n_S(\hat{\mu}_B, \hat{\mu}_S) = c_{110}^{BSQ}\,\hat{\mu}_B + 2\,c_{020}^{BSQ}\,\hat{\mu}_S + \ldots \equiv 0 \quad \Rightarrow \quad \hat{\mu}_S(\hat{\mu}_B) = \left(-\frac{c_{110}^{BSQ}}{2c_{020}^{BSQ}}\right)\hat{\mu}_B + \ldots$$

may lead to sizeable differences, e.g.

$$\frac{\Delta p}{T^4} = \left(c_{200}^{BSQ} - \frac{\left[c_{110}^{BSQ}\right]^2}{4c_{020}^{BSQ}}\right)\hat{\mu}_B^2 + \ldots$$



III. Some spatial correlations

(A) aim at **analyzing existence and properties of hadronic excitations at** $T > 0$

$\rightsquigarrow$ Correlator in momentum space defined at (boson) Matsubara frequencies $\omega_n = 2\pi T n$

$$\begin{aligned} \Delta(i\omega_n, \vec{p}) &= \oint \frac{dp_0}{2\pi i} \frac{\Delta(p_o, \vec{p})}{p_0 - i\omega_n} \\ &= \int \frac{dp_0}{2\pi} \frac{1}{p_0 - i\omega_n} \underbrace{\frac{1}{i}\left[\Delta(p_0 + i\epsilon, \vec{p}) - \Delta(p_0 - i\epsilon, \vec{p})\right]}_{\textbf{spectral density } \sigma(p_0, \vec{p})} + \ Subtr. \end{aligned}$$

because of limited physical extension $1/T$ in the temporal direction

$\rightsquigarrow$ spatial correlator

$$G^S(z) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \Delta(0, p) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma(p_0, p)}{p_0}$$

$\rightsquigarrow$ screening masses

$$G^S(z) \sim e^{-M^{screen} z}$$

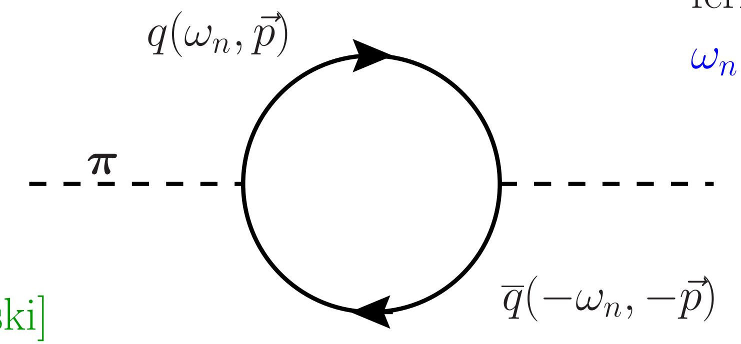
in particular

- symmetries
- comparison with free quark propagation

fermion Matsubara

$\omega_n = \pi T(2n+1)$

At T large: expect free quarks



- free Continuum: [Friman, Florkowski]

$$G^S_\pi(z) = \frac{N_c T}{2\pi z^2 \sinh(2\pi T z)} \left[1 + 2\pi T z \coth(2\pi T z)\right]$$

define effective (z-dependent) screening mass

$$m^{\text{eff}}_{\text{screen}}(z) = -\frac{1}{G(z)} \frac{\partial G(z)}{\partial z} \simeq 2\pi T \left\{1 + \frac{1}{2\pi T z} + \ldots\right\}$$

- lowest order correction: [Laine, Vepsäläinen]

$$m_{\text{screen}} = 2\pi T (1 + g^2 \times \left\{ \begin{array}{l} 0.022 (N_F = 0) \\ 0.033 (N_F = 3) \end{array} \right\})$$

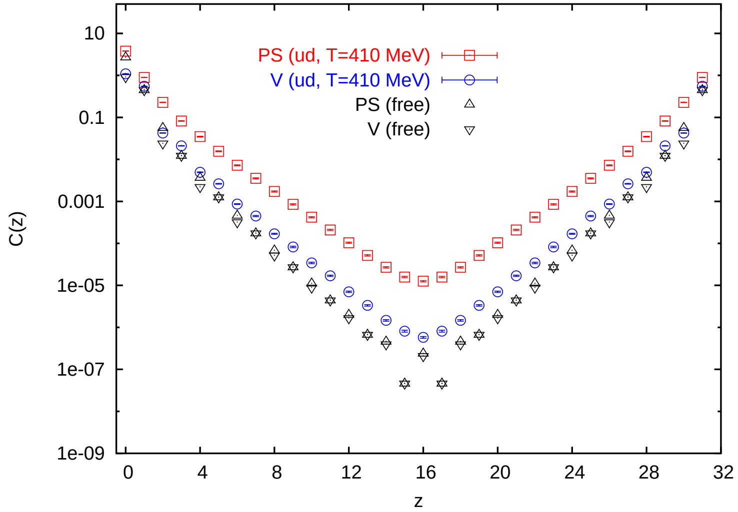
free lattice staggered fermions

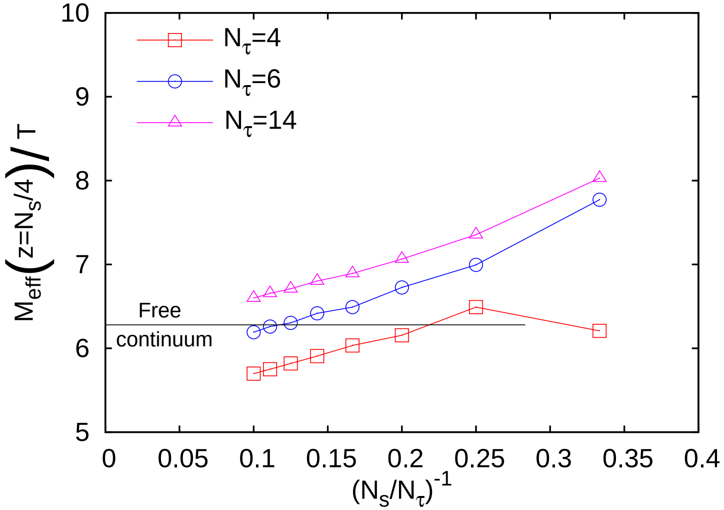
$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1,k_2,k_4} \frac{1}{\cosh^2 E} \frac{1}{\sinh^2(E N_\sigma/2)} \left\{ b_M \cosh\left[2E\left(\frac{N_\sigma}{2} - z\right)\right] + d_M \right\}$$

where $\quad \sinh^2 E = m^2 + \sum_{1,2,4} \sin^2 k_i$

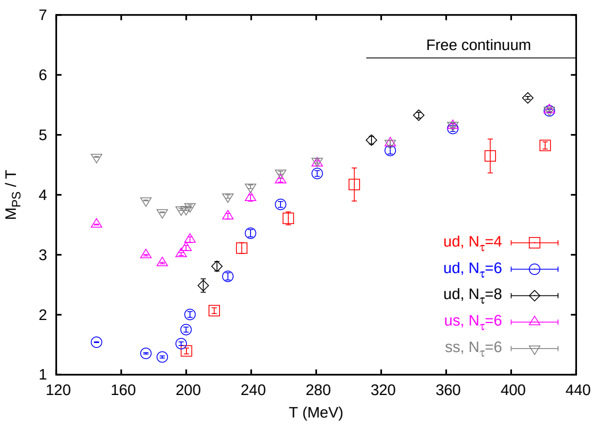
	b^{odd}	d^{odd}	b^{even}	d^{even}
P	1	-1	1	+1
$\frac{1}{2}(V_1+V_2)$	1	-1	$\frac{m^2+\sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2+\sin^2 k_4}{\sinh^2(E)}$
S	1	-1	$\frac{2m^2}{\sinh^2(E)} - 1$	$\frac{2m^2}{\sinh^2(E)} - 1$
$\frac{1}{2}(A_1+A_2)$	1	-1	$\frac{m^2-\sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2-\sin^2 k_4}{\sinh^2(E)}$

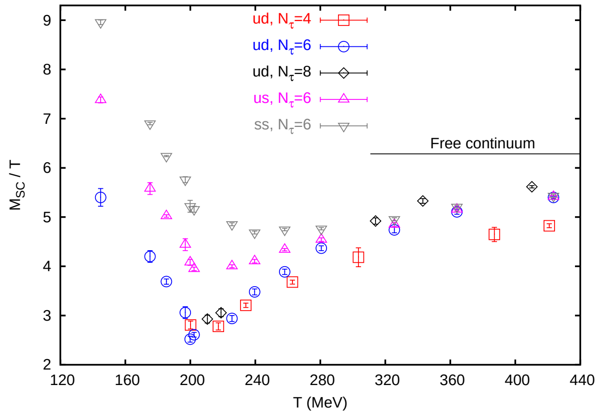
standard staggered; similar for p4

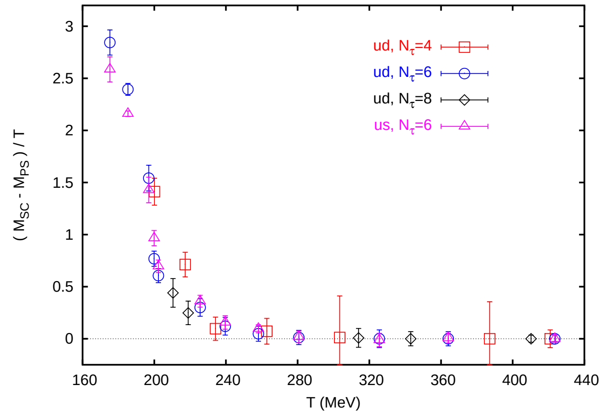




Pseudoscalar - Scalar (connected a_0/δ) sector

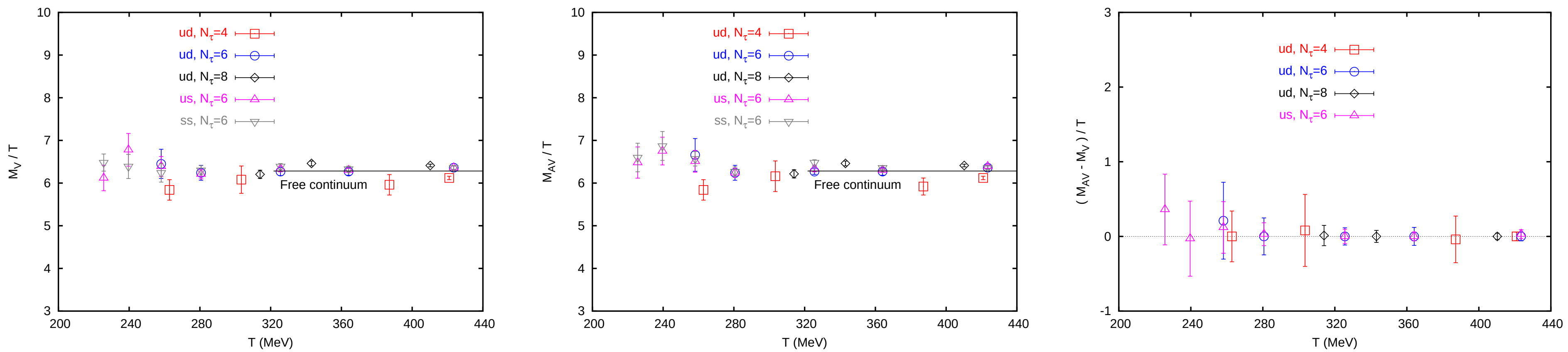






- all results at fixed $N_\sigma/N_\tau = 4$
- π and a_0 do not become degenerate up to $T \simeq 240$ MeV

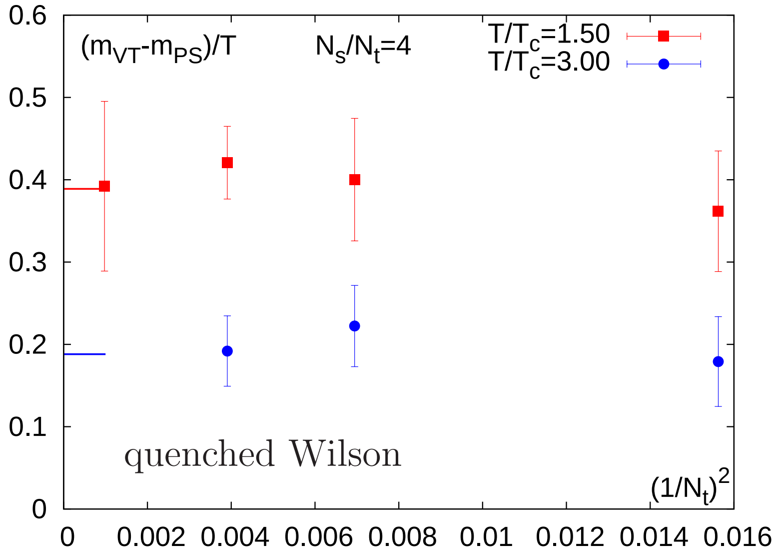
(transverse) Vector - Axialvector channel



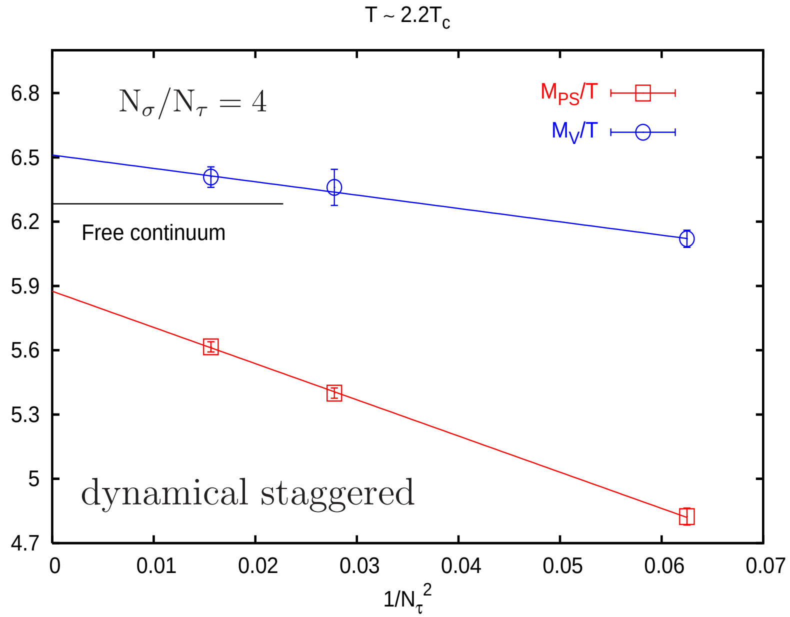
- all results at fixed $N_\sigma/N_\tau = 4$: coincidence with 2π is presumably accidental
- V_T and A_T appear degenerate even in the us channel at $T > T_c$

- compare with quenched non-pert. improved Wilson on large lattices (up to $128^3 \times 16$)

- up to $T \simeq$ few times T_c: π and ρ do not become degenerate ...



lines are $V \to \infty, a \to 0$



... as opposed to free $+ \mathcal{O}(g^2)$

- remains to be done: $V \to \infty$ for (dynamical) staggered

(B) spatial pseudo potentials and dimensional reduction

at large T: hierarchy of scales $T \gg gT \gg g^2T$ suggests

hierarchy of effective theories: 4d QCD $\longrightarrow$ 3d EQCD $\longrightarrow$ 3d YM

(i) integrate out non-static ($n \neq 0$) boson modes $2\pi nT$ and fermion modes $\pi(2n+1)T \;\Rightarrow$

$$S_3^E = \int d^3x \, \frac{1}{g_E^2} \mathrm{Tr}\, \mathrm{F_{ij}}(\mathbf{x})\, \mathrm{F_{ij}}(\mathbf{x}) + \mathrm{Tr}\, [\mathrm{D_i}, \mathrm{A_0}(\mathbf{x})]^2 + \mathrm{m_D^2} \mathrm{Tr} \mathrm{A_0}(\mathbf{x})^2 + \lambda_\mathrm{A} (\mathrm{Tr} \mathrm{A_0}(\mathbf{x})^2)^2$$

with A_0 static temporal gluon field. Parameters of the effective theory have been computed in perturbation theory [Kajantie et al., Laine and Schröder]

to leading order: $g_E^2 = g^2 T, \quad \lambda_A = (9 - n_f) g^4 T / (24\pi^2), \quad m_D^2 = g^2\, T^2\, (1 + n_f/6)$

confining nature of this theory reflects non-perturbative physics in magnetic sector of QCD

(ii) When $m_D \gg g^2T$: integrate out heavy $A_0 \Rightarrow$

$$S_3 = \int d^3x \, \frac{1}{g_3^2} \mathrm{Tr}\, \mathrm{F_{ij}}(\mathbf{x})\, \mathrm{F_{ij}}(\mathbf{x}$$

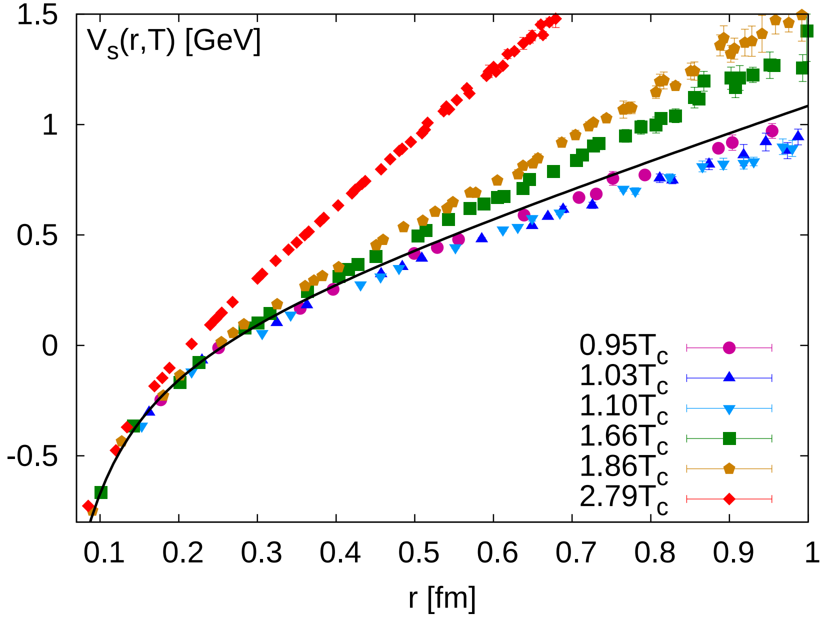
to leading order $g_3 = g_E$, at 2-loops $\longrightarrow$ [Giovannangeli]

in 3d YM: $\sqrt{\sigma_3} = c g_3^2 \quad \Rightarrow \quad$ spatial string tension $\sqrt{\sigma_s} = c g^2(T) T$

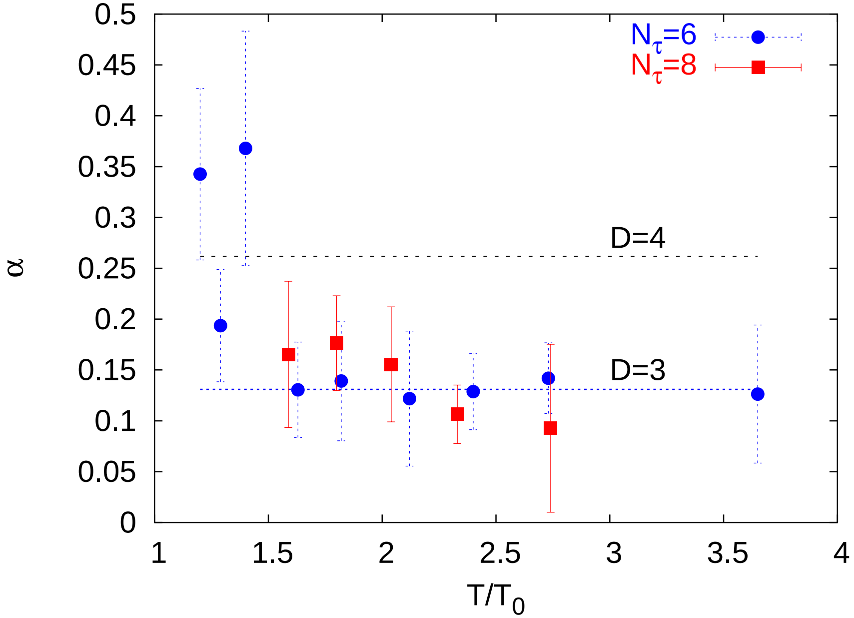
with c determined non-perturbatively in a 3d YM lattice simulation questions: ⋆ at T a few times T_c: hierarchy valid ?

⋆ dynamical quarks impeding dimensional reduction ? [Gavai, Gupta]

spatial potentials at $N_\tau = 6$



$\alpha_{\rm eff}$



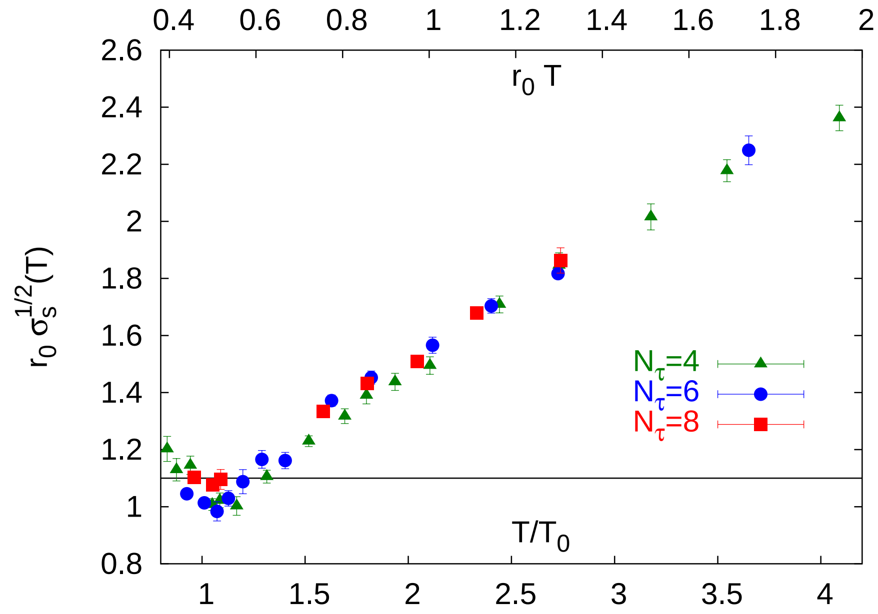
$$V_s = V_0 - \frac{\alpha_{\rm eff}}{r} + \sigma_s\, r$$

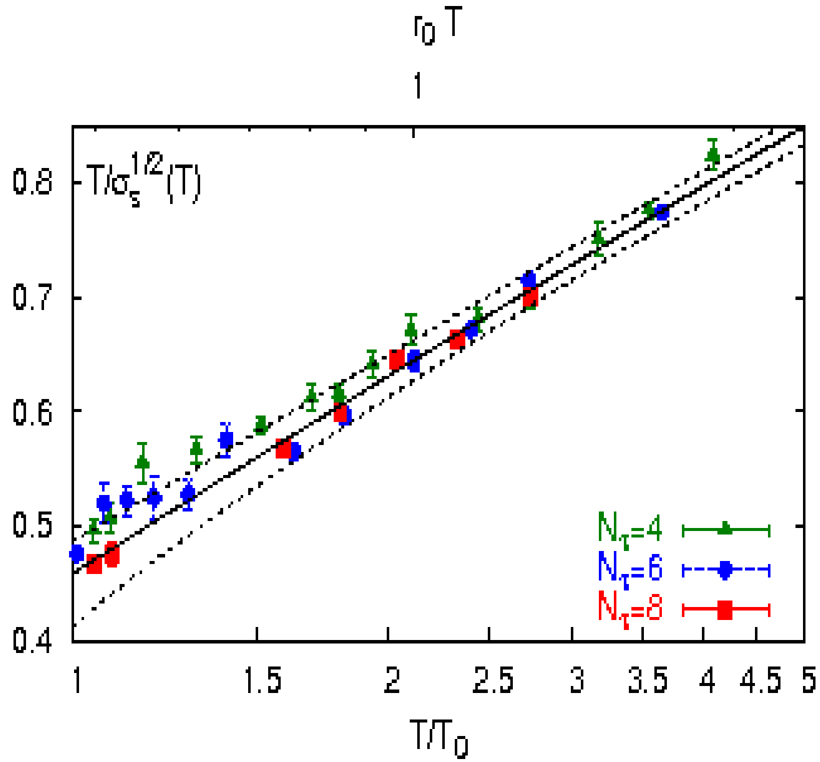
string fluctuations predict [Lüscher]

$$\alpha_L = \frac{(D-2)\,\pi}{24}$$

$\alpha_{\rm eff}$: gluon exchange ↔ string fluctuations ?

results for the spatial string tension





$\sqrt{\sigma_s}$ rising with T approx. linear

above $\sqrt{\sigma(T=0)}$ (horizontal line)

except small dip at $T \simeq T_c$

$\rightarrow$ discretization effect ?

compare with prediction by dim.red.

$$\frac{T}{\sqrt{\sigma_s}} = \frac{1}{cg^2(T)} \sim \ln(T/\Lambda_{\overline{MS}})$$

with matching/running at 2-loop [Laine,Schröder]
starting from $\alpha_V(7.5GeV)$ [Mason et al.]
transferred to $\overline{MS}$ at 3-loops [Schröder]

dimensional reduction may work down to T as low as $1.5T_c$ also in 2+1 flavor QCD

Summary and Outlook

- For the EoS: we are getting to robust numbers and are discussing errors at percent level !
  - $T < T_c$: make contact with hadron gas $\longrightarrow$ physical light quark mass $m_q = m_s/20$
  - $T < 2T_c$: conformal symmetry strongly broken
  - $T > 2T_c$: make contact with resummed perturbation theory and minimize errors

- at (small) finite density (relevant for RHIC, LHC, early universe):
  - interesting qualitative insights
  - get systematics under better control ⋆ higher orders in Taylor
    ⋆ comprehensive attack of the various approaches

- screening masses show
  - expected (?) symmetry restoration pattern
  - substantial non-perturbative effects

- spatial string tension supports dimensional reduction to work down to quite low T

- in the long run
  - ⋆ comparison with Wilson and chiral fermion discretizations
  - ⋆ addressing the critical end point (?) in the $T - \mu$ phase diagram ($\longrightarrow$ FAIR)