Convergence scenarios in an opinion-dependent communication framework

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Different communication frameworks have been developed to implement mutual interactions and opinion dynamics on social networks. Opinion formation is a complex process, and a realistic description should also take into account the feedback that the agents' opinion has on the structure of the network and on the opinion dynamics itself. We propose a model in which different kinds of interconnections and interacting behaviours are associated to the agents depending on their opinion: extremists tend to self-segregate whilst having a stronger convincing power toward other agents. The system is fully described by a static and a dynamical parameter; in this space of parameters, a curve delimiting two different final-state scenarios can be drawn: a continuous phase transition appears, separating an ordered consensus phase from a pluralistic situation. Pluralism can only be achieved when extremists are not too much self-segregating and their tolerance threshold is high.

Socio-physics

Idea: apply statistical mechanics to social phenomena (mostly non-equilibrium SM).

Microscopic interaction between individuals (social psychology)

$\Downarrow$

collective behaviours (markets, fashion, politics, . . .)

(if large number of individuals)

Phase transitions between qualitatively different behaviours.

At the transition: critical exponents, power-laws, finite-size scaling, universality.

Opinion formation

Opinion formation models: will the simulated society reach a consensus over a given topic or not?

The society as a network: nodes (agents) are people, links are friendships (people who talk to each other).

Starting from some initial state, let the system evolve, according to some update rule, up to a fixed point in the dynamics (nothing more can happen).

Note that *usually there is no Hamiltonian and no detailed balance*: cannot escape from the final state! (and a stochastic approach is mandatory to obtain meaningful results).

We will present a modification of the Deffuant model
that focuses on the role of extremism

Deffuant model for opinion formation

Each agent i holds an opinion $\theta_i \in [-1;+1]$ (initially assigned at random).

Agents interact according to some *social topology*, built before starting the dynamics (not a regular lattice!).

Update step: take two neighbours at random, and if $|\theta_i - \theta_j| < \epsilon$ set both opinions to $\frac{\theta_i+\theta_j}{2}$; otherwise, do nothing.

$\epsilon =$ **tolerance** (global parameter)

The final state can exhibit either *total consensus* or *opinion fragmentation*: Second-order critical point at $\epsilon_c = 1$, regardless of network topology.

Main shortcoming: *everybody behaves the same !*

Modeling extremism and opinion-dependent behaviours

When do minorities have influence? (cf. Asche experiment)

People's primary fear is to be different and isolated $\Rightarrow$ neutrality: $\theta \simeq 0$

Radicals, heretics, outsiders . . . $\Rightarrow$ extreme opinions, $|\theta| \sim 1$

Extremists hardly change their opinion; conversely, neutrals are easily persuaded.

Will organised minorities survive in society? Are they absorbed by the mainstream?

How to implement opinion-dependence in the model? A suitable *communication framework* is mandatory.

Communication framework I: network topology

A realistic social topology is well described by a random network of N agents with power-law degree distribution: $P(k) \propto k^{-\gamma}$.

In Deffuant model, the recipe for building the network (Barabasi-Albert construction) is opinion-independent. Real people, however, choose their friends according to their ideas (especially if they are located near the extreme opinions).

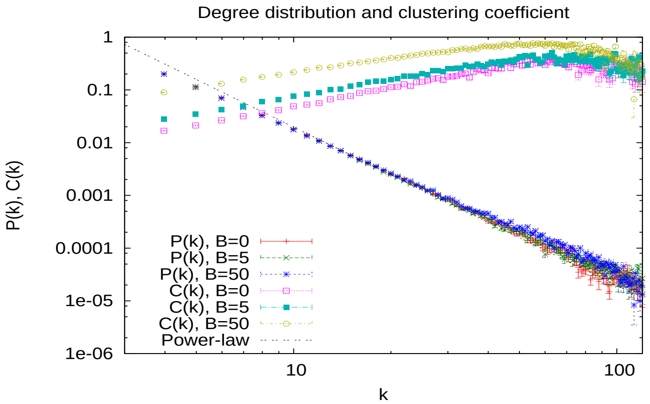
Our recipe: the probability for the node i to establish an acquaintance with node j is given by:

$$P^{(i)}(j) \propto k_j e^{-\beta|\theta_i|\cdot|\theta_i-\theta_j|} \quad ; \quad \beta \geq 0 \quad \textit{homophily parameter}$$

β controls how much selective are extremists in choosing acquaintances, while preserving the scale-freedom of the network.

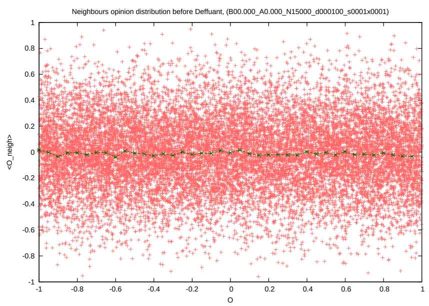
High β leads to higher clustering coefficient and to a *segregation* of extremists from society.

Homophily and segregation

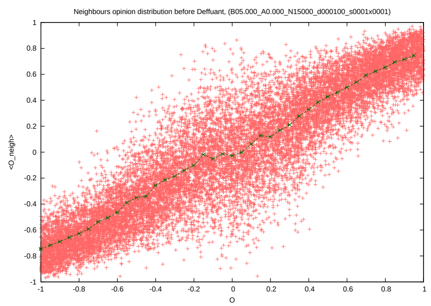


Degree distribution and clustering coefficient for $\beta = 0, 5, 50$ (and $N = 5000$ agents).

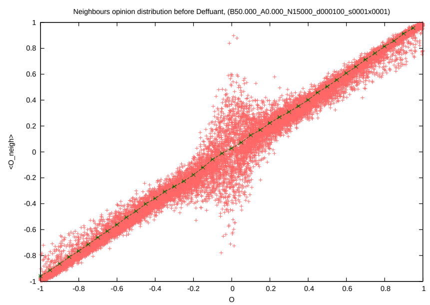
Agents' θ vs. neighbours' θ and average neighbours' θ $(N = 15000)$



$\beta = 0$



$\beta = 5$



$\beta = 50$

Communication framework II: social interaction

Let's assign an individual tolerance to each agent according to his/her opinion:

$$t_i = 1 - \alpha \cdot |\theta_i| \quad ; \quad \alpha \in [0;1] \quad \text{dynamic parameter}$$

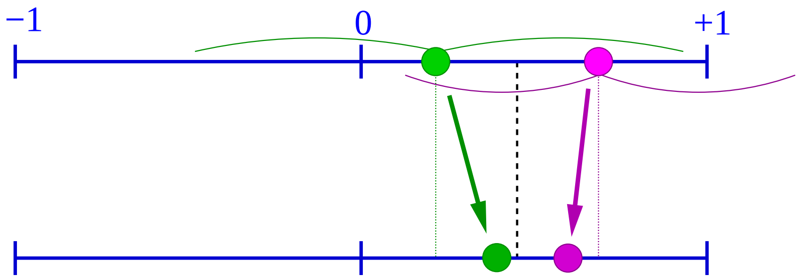
so that extremists are the less tolerant. The update step yields a successful communication if:

$$|\theta_i - \theta_j| \leq \text{min}[t_i, t_j]$$

In that case, an *asymmetric drift* of opinions is performed:

$$\theta_i \mapsto \theta_i + t_i \frac{\theta_j - \theta_i}{2}$$

$$\theta_j \mapsto \theta_j + t_j \frac{\theta_i - \theta_j}{2}$$



Neutrals change their opinion easily, while extremists are much difficult to convince (especially for high α)

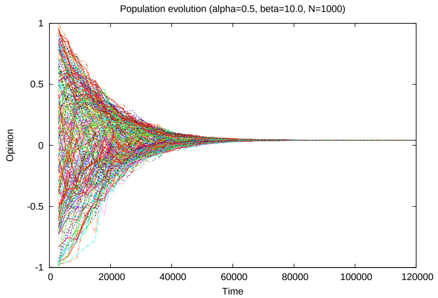
Qualitative results: consensus vs. fragmentation

Intuitive expectations:

(1) At fixed β, low α leads to consensus, high α disrupts it (isolated mini-communities near opinion extremes): threshold at α_c.

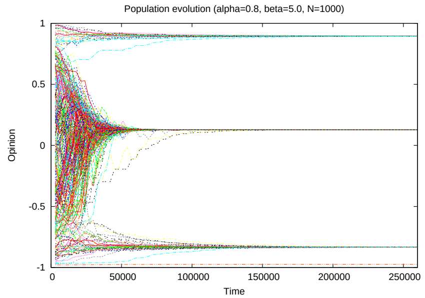
(2) $\beta > \beta' \Rightarrow \alpha_c \geq \alpha'_c$, since the network is better suited to opinions assignment.

Evolution with time in the uniformity and fragmented cases:



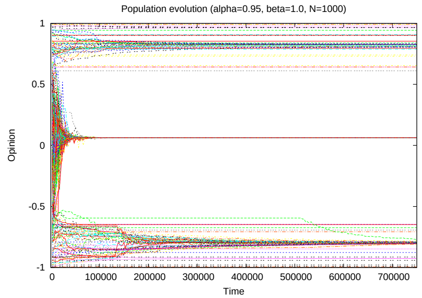
$\beta = 10$

$\alpha = 0.50$



$\beta = 5$

$\alpha = 0.80$



$\beta = 1$

$\alpha = 0.95$

The third case is *heavily fragmented*: another regime change beyond fragmentation (**few** $\longrightarrow$ **many** opinion clusters).

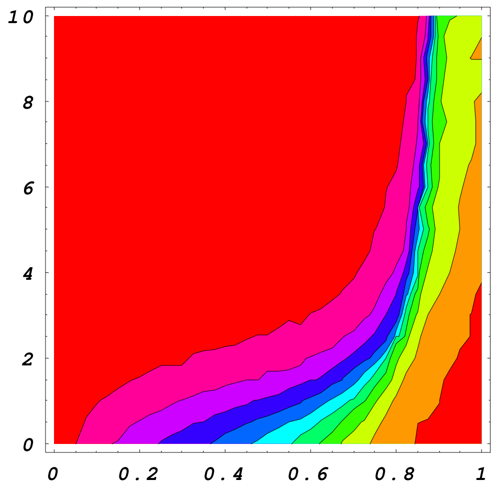
Fragmentation transition

We define the order parameter (evaluated on the final state)

$$\phi \equiv \sqrt{\frac{\sum_i (\theta_i - \langle\theta\rangle)^2}{N}} \qquad \text{"susceptibility"}$$

so that $\phi \neq 0 \Leftrightarrow$ total consensus.
ϕ increases with α and decreases with β.

- At $\beta = 0$: transition at $\alpha_c(0) = 0$ (the system is always fragmented)
- The threshold increases up to $\alpha_c(\beta = 5) \simeq 0.85$, then stays constant.
  (on the right: ϕ in the α–β plane)



Finite-size scaling analysis

The transition is **second order**: determination of the critical indices.

At fixed β we expect a scaling relation in the form:

$$\phi = N^{-\nu} F[N^{-\sigma}(\alpha - \alpha_c)] \ ,$$

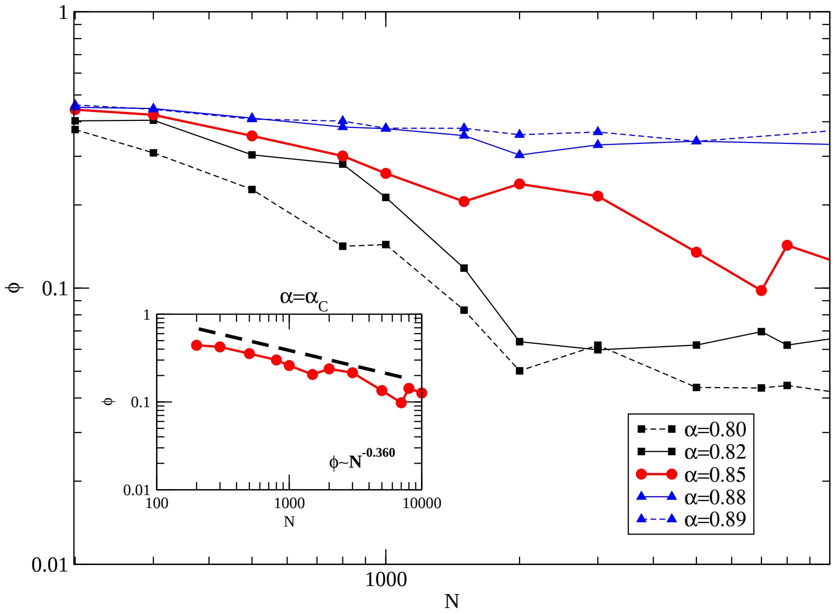
with F a universal function.

This implies:

1. $\phi \sim N^{-\nu}$ at $\alpha \equiv \alpha_c$
2. $\alpha_c^{(N)} = \alpha_c + \mathscr{O}(N^{-\sigma})$

Finite-size scaling at $\beta = 6$

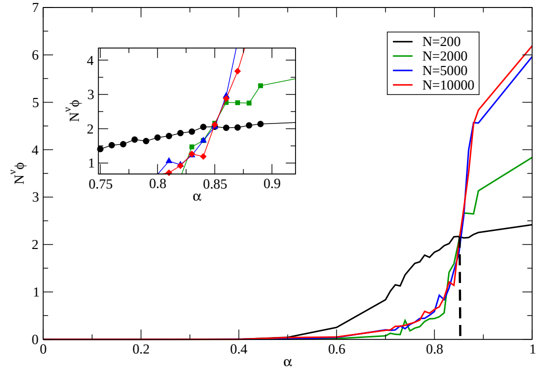
Inspect different values of α looking for a power-law: $\phi(\alpha_c; N) \propto N^{-\nu}$



Results ($\beta = 6.0$):

$\alpha_c \simeq 0.85$
$\nu \simeq 0.36$

Rescaling & collapsing window at $\beta = 6$



Crossing point in α at various system sizes:

$N = 200, 2000, 5000, 10000$

left: $N^\nu \phi$ vs. α

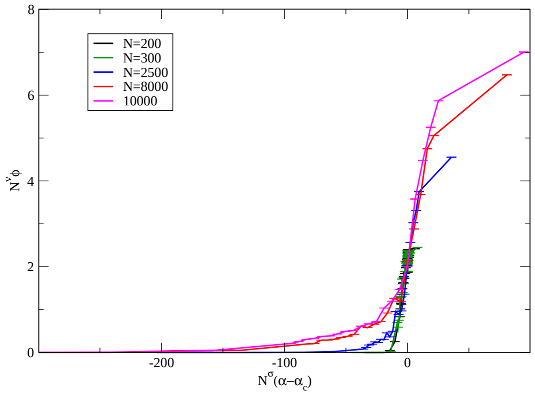
Once α_c and ν are known, find a collapsing window by tuning σ in:

$$N^\nu \phi = F[N^{-\sigma}(\alpha - \alpha_c)]$$

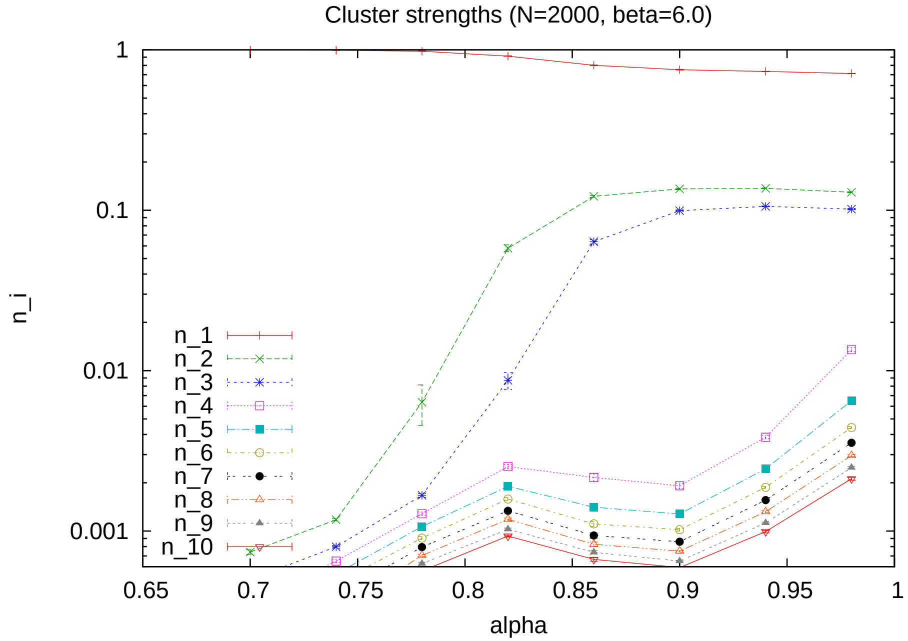
(here $N = 200, 300, 2500, 8000, 10000$)

Result:

$$\sigma \simeq 0.7$$



The "extremely broken" phase ($\beta = 6.0$)



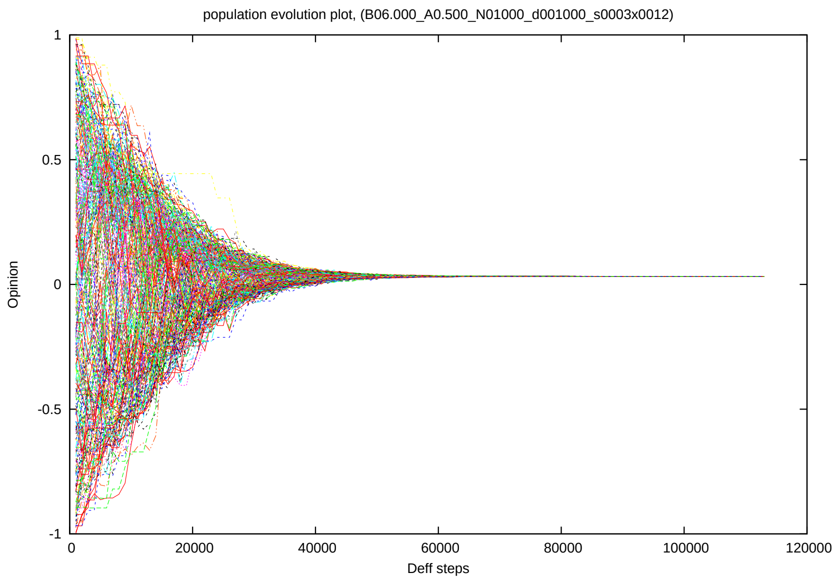
Critical point at $\alpha_c \simeq 0.85$: signalled by 2nd and 3rd cluster strengths (although blurred by finite-size effects).

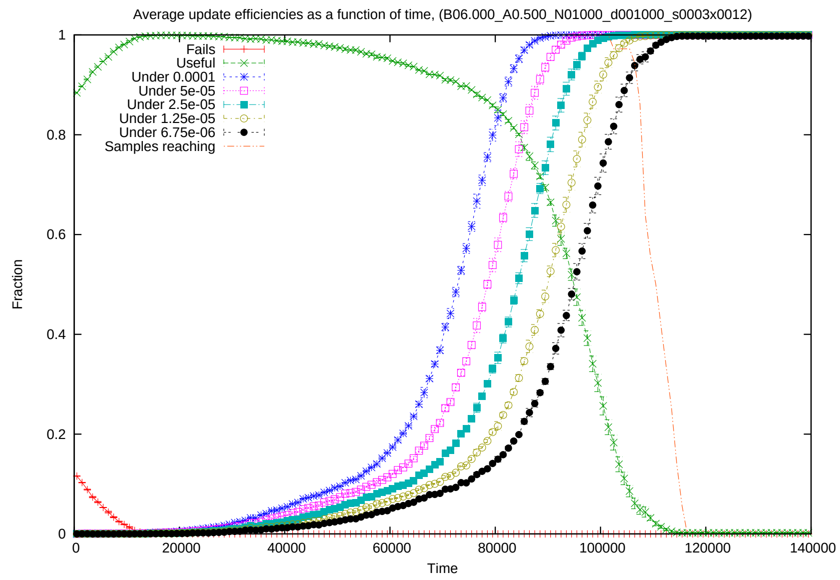
Around $\alpha \simeq 0.90 - 0.95$ another regime change (apparently not a PT):

"Deep pluralism" regime

Convergence times and timescales - total consensus

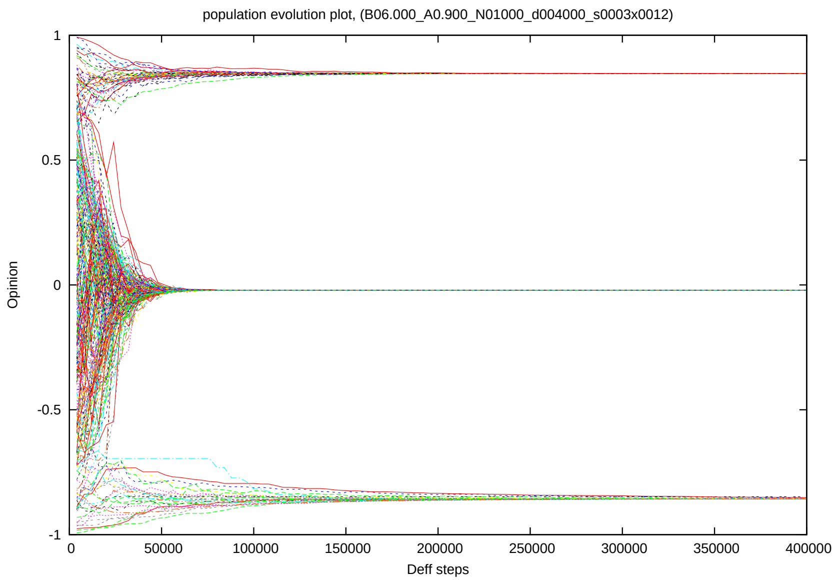
Algorithm performance as a function of Monte Carlo time:

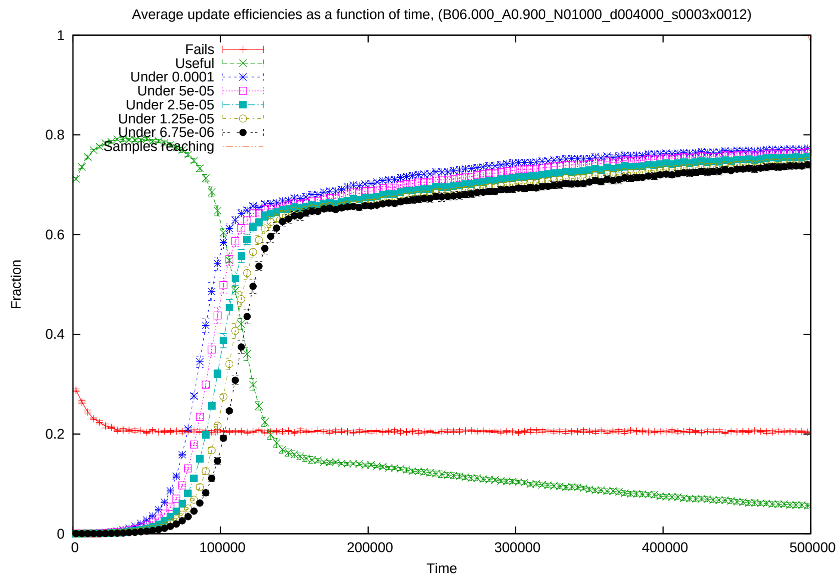




Total consensus: short convergence times, only one timescale
(here: $N = 1000, \alpha = 0.5, \beta = 6.0$).

Convergence times and timescales - broken phase





Two (or more) timescales for different kind of clusters.

Extremists take longer to organise!

(here: $N = 1000, \alpha = 0.9, \beta = 6.0$)

Intermezzo: media influence

Suppose an official media promotes an extreme opinion $(+1)$.

Will people react by forming an antagonist cluster or not? It depends on the strenght of the media imposition, but. . .

Media influence modeled as a "Big-Agent" that does not change its opinion but affects all other nodes in the dynamics.

- If media is weak, the usual dynamics takes place: $\langle \theta_{\text{fin}} \rangle \sim 0$.
- If media is moderately strong, everybody accepts the proposed idea: $\langle \theta_{\text{fin}} \rangle \sim 1$.
- If media is *too strong*, a huge **antagonist cluster** arises!

see References for more info.

Conclusions

- Extremist minorities are quite influent if:
  - they maintain their viewpoint over time (i. e. α large);
  - they are integrated enough in society (i. e. β not too high).
- Radical minorities are cohesive if:
  - they are tolerant enough (α not too high), otherwise heavily fragmented!
- In any case, lateral groups take *longer* to settle than centrists!

More statistics needed for a precise determination of the critical indices . . .

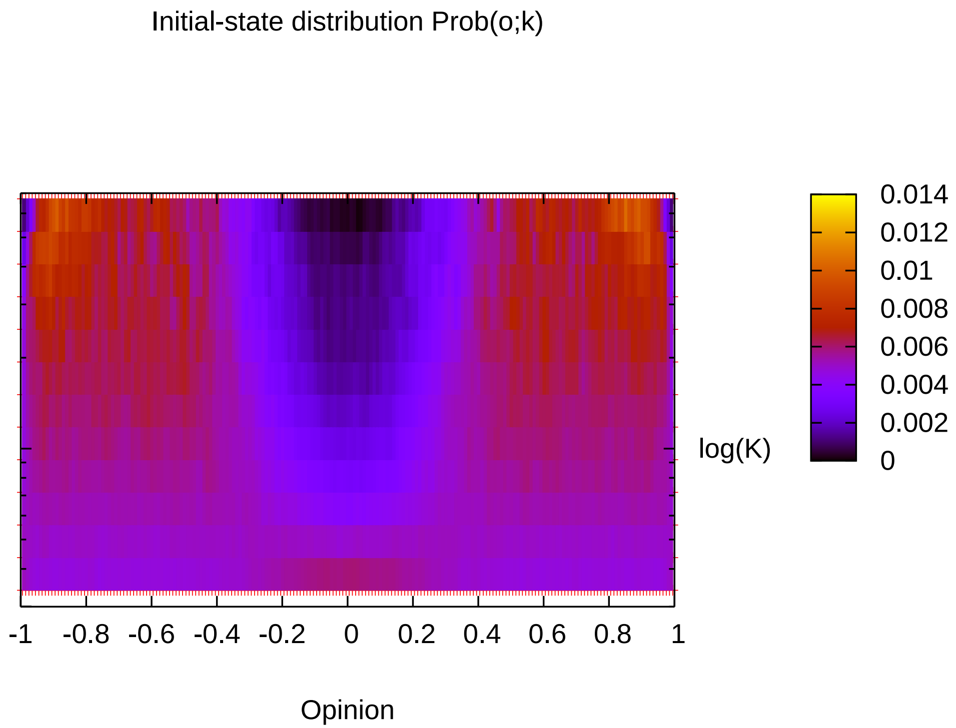
References

♠ **Can extremism guarantee pluralism?**, Floriana Gargiulo, Alberto Mazzoni
(`arXiv:0803.3879; submitted to JASS`)

♢ **Convergence scenarios in an opinion-dependent communication framework**, Floriana Gargiulo, S. L., José J. Ramasco
(`in preparation for PRE`)

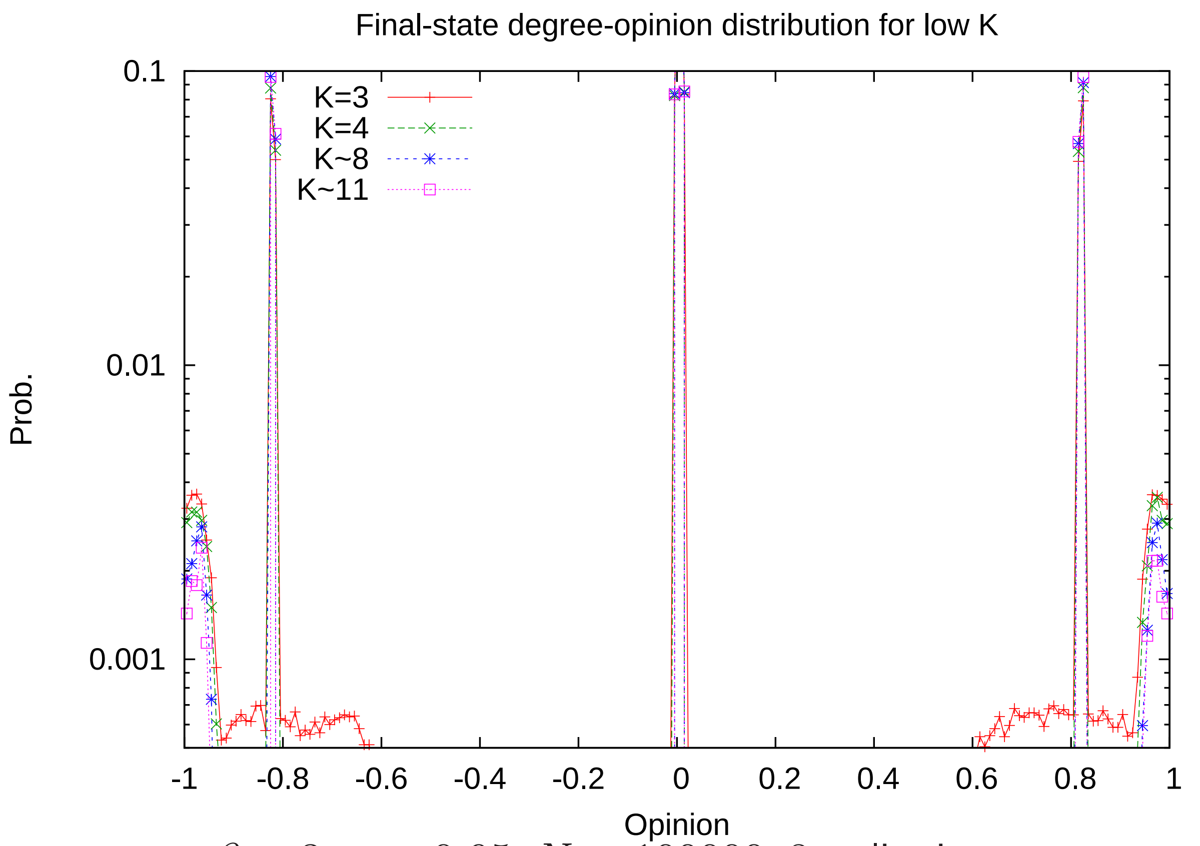
♣ **The saturation threshold of public opinion: are aggressive media campaigns always effective?**, Floriana Gargiulo, S. L., Alberto Mazzoni
(`ESSA2008 Proceeding; arXiv:0807.3937`)

Supplementary plots I: initial state network structure



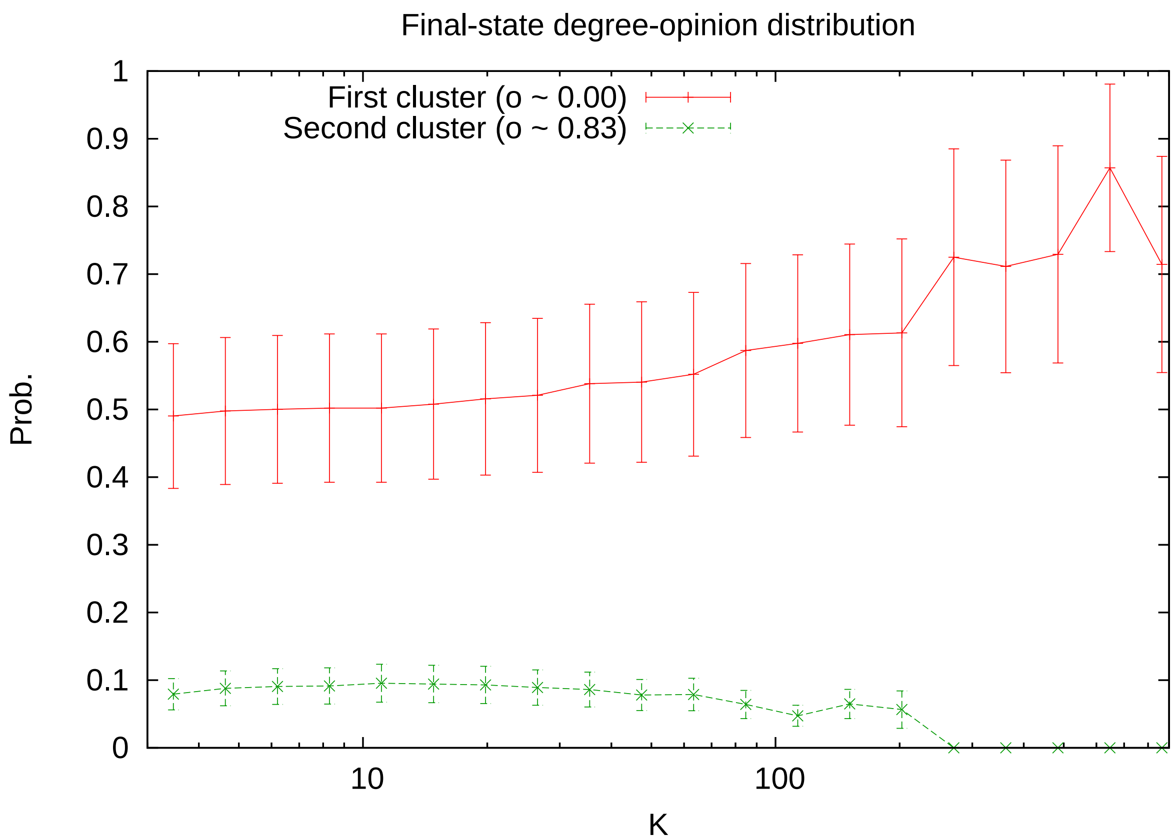
$\beta = 20,\ N = 2000,\ 1200$ realisations

Supplementary plots II: final state network structure



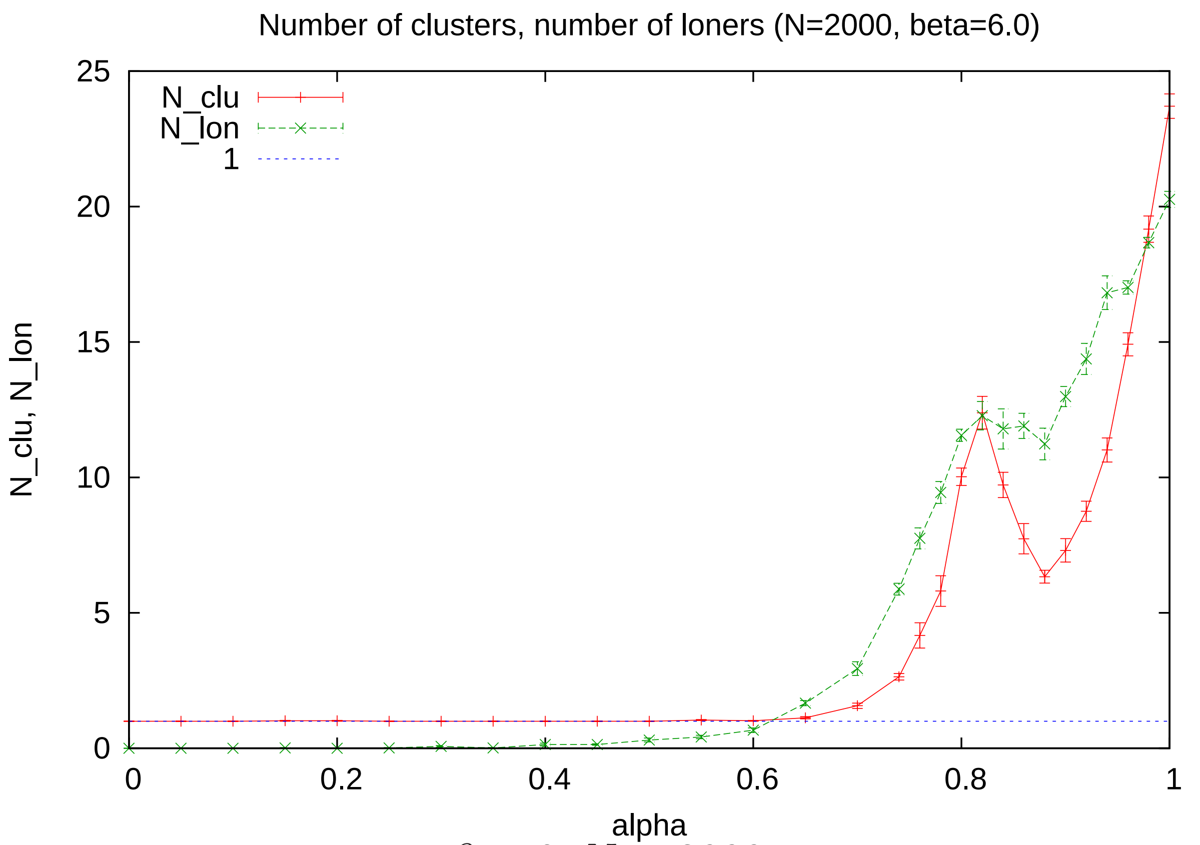
$\beta = 3$, $\alpha = 0.95$, $N = 100000$, 8 realisations

Supplementary plots III: final state network structure



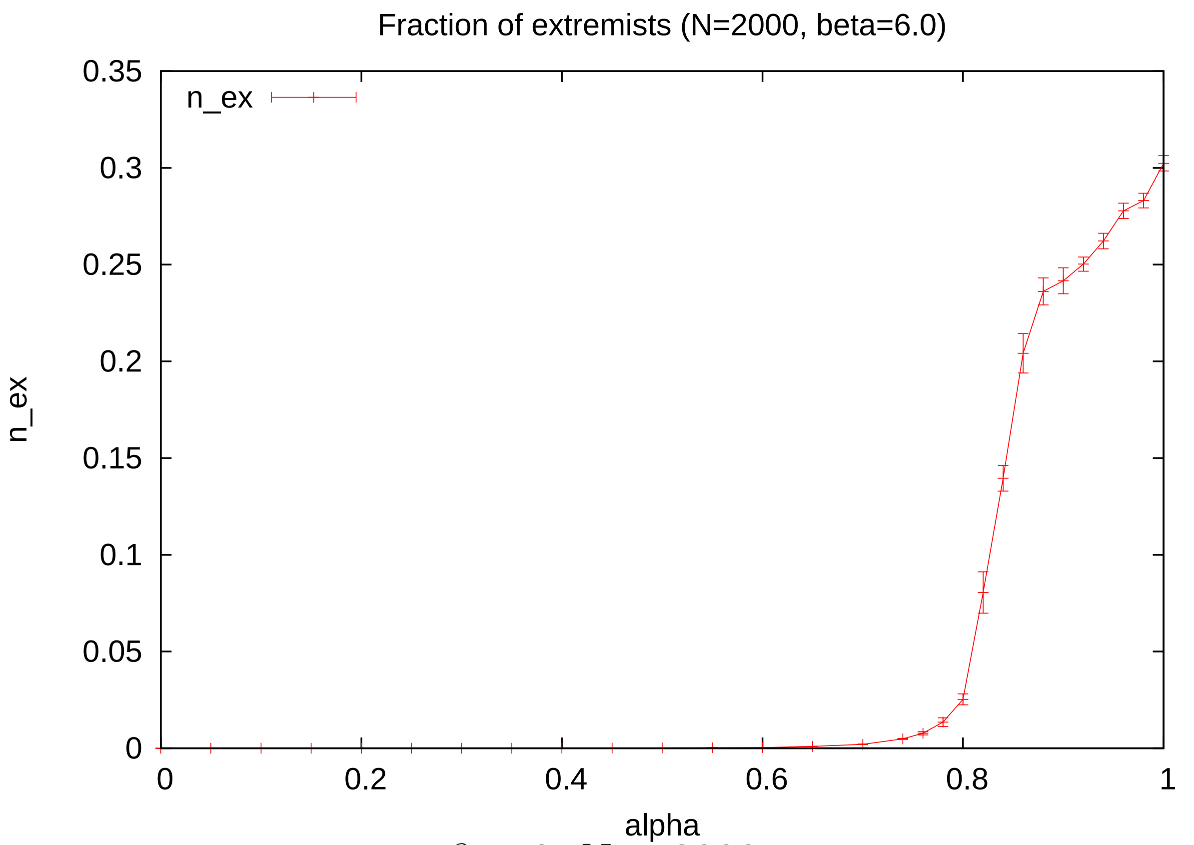
$\beta = 3$, $\alpha = 0.95$, $N = 100000$, 8 realisations

Supplementary plots IV: loners and clusters



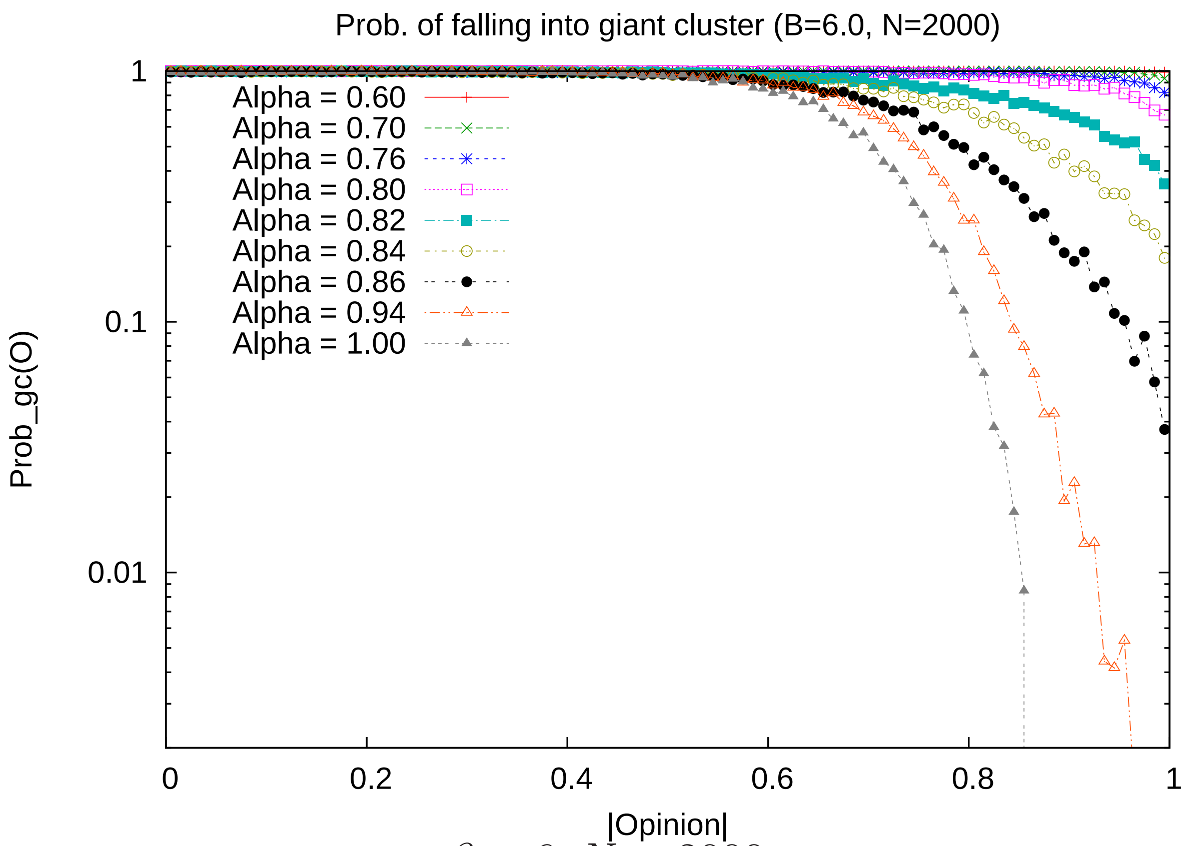
$\beta = 6, \; N = 2000$

Supplementary plots V: extremists



$\beta = 6, \ N = 2000$

Supplementary plots VI: giant-cluster attraction



$\beta = 6,\ N = 2000$