The monopole mass in the random percolation gauge theory

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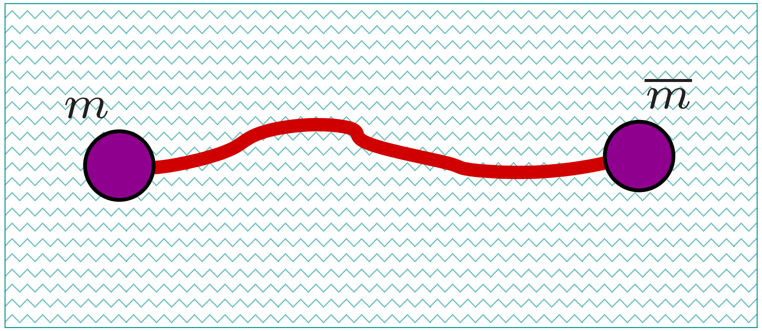
We study the behaviour of the monopole at finite temperature in the (2+1)-dimensional lattice gauge theory dual to the percolation model; by exploiting the correspondences to statistical systems, we possess powerful tools to evaluate the monopole mass both above and below the critical temperature with high-precision Monte Carlo simulations.

QCD vacuum as a dual superconductor

One of the oldest and most trusted proposals for quark confinement is the dual superconductor picture [Polyakov '75; 't Hooft '78, Mandelstam '76]:

type II superconductor

Meissner effect



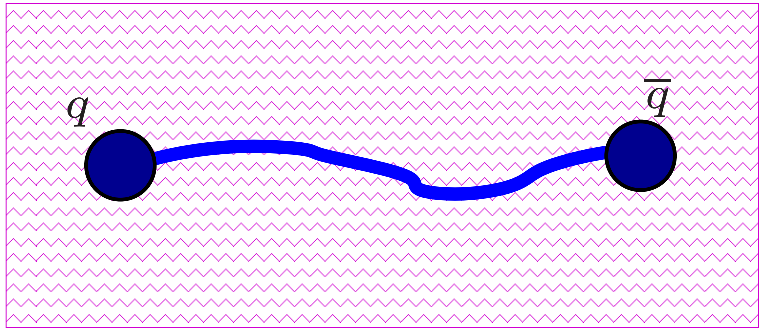
$$\langle \Phi_e \rangle \neq 0$$
$$V_{m\overline{m}}(R) \propto R$$

duality

$$\Longleftrightarrow$$

QCD vacuum

dual Meissner effect



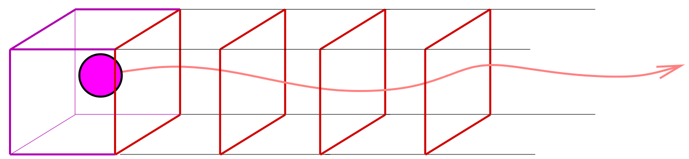
$$\langle \Phi_m \rangle \neq 0$$
$$V_{q\overline{q}}(R) \simeq \sigma R$$

Electric and magnetic lines

In three dimensions, an electric (magnetic) static source is inserted in $\mathbf{x}$ via a nonlocal operator which also places an electric (magnetic) flux-line joining $\mathbf{x}$ to ∞:

$\downarrow$

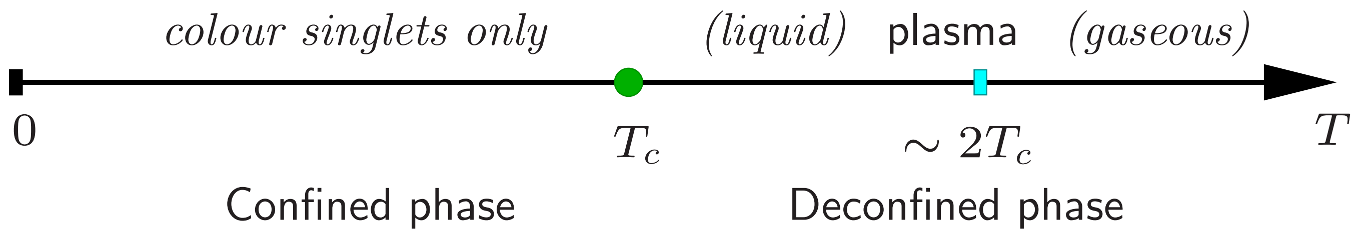
On the lattice the flux lies on dual links (i. e. plaquettes)



(magnetic flux through red plaquettes)

Confinement, order and disorder parameters

While the Wilson loop $\langle W \rangle$ (therefore σ as well) is an order parameter for confinement, $\langle \Phi_m \rangle$ is a *disorder parameter*:



Symmetric phase	Broken phase
Percolation of magnetic strings: (gauge field disordered)	Percolation of electric strings: (order in gauge configurations)
$\langle \Phi_m \rangle \neq 0$ $\langle \Phi_e \rangle = 0$	$\langle \Phi_m \rangle = 0$ $\langle \Phi_e \rangle \neq 0$

The operator Φ_m can be put in relation to the *monopole condensate*.

The deconfinement transition

Confinement is due to magnetic degrees of freedom.

At $T < T_c$ they form the monopole condensate.

At the critical point, the condensate melts down (only lines wrapped locally around imaginary time survive): its leftovers will be real, thermal monopoles [Chernodub, Zakharov '06].

$\Rightarrow$ The plasma must exhibit a magnetic component (i. e. monopoles)!

Some remarks

- "Abelian vs. non-Abelian"
  - Monopoles are well defined and understood in Abelian theories. To approach non-Abelian theories, one relies on *Abelian projections*, thanks to the *Abelian/monopole dominance* phenomenon ['t Hooft '81].
  - In discrete Abelian theories there are no *dynamical* monopoles: they need to be inserted as external sources.
- The typical investigation is carried on in terms of an operator ρ ($\sim$ finite-temperature monopole density), and the corresponding correlators examined are $\rho(x)\rho(y)$. We will instead possess a microscopic quantum monopole creation operator.

The case for percolation

We will study the monopole *mass* and *condensate* behaviour in the (2+1)-dimensional percolation theory both below and above the transition temperature.

Percolation is a well-defined pure gauge theory, despite its apparently trivial construction [Gliozzi, S. L., Panero, Rago, '04]. Among its properties:

- string effects in loops up to the NNLO;
- glueball spectrum in the confined phase;
- finite-temperature confinement/deconfinement second-order transition, with a "proper" universal ratio $\frac{T_C}{\sqrt{\sigma}}$.

The percolation model in short

- Each link of an empty (2+1)- or 3-dimensional lattice (dual to the gauge one) is *switched on* with a probability $p \in [0, 1]$ independently.
- At p_c an infinite connected cluster appears: second-order critical point.
- The expectation value of a loop $W(\mathcal{C})$: **zero** if there are clusters with nonzero winding around $\mathcal{C}$; **one** otherwise (hence: "on" links $\simeq$ magnetic flux lines).
- This implies: confined phase $\Longleftrightarrow p > p_c$.
- This framework is suggested by the chain of maps: Z_2-gauge (S_q-gauge) $\Rightarrow$ Ising (q-state Potts) model $\Rightarrow$ Fortuin-Kasteleyn cluster reformulation $\Rightarrow$ $\lim_{q\to 1}$ of the theory.
- As a guideline, notice that $\beta_{\text{gauge}} = -\log(p)$ ... were it possible to explicitly formulate the theory instead of its dual!

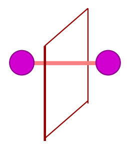
Monopoles & percolation - I

Pedigree of the theory:

gauge	3D Kramers-Wannier duality $\Longleftrightarrow$	spin
S_q	$\longleftrightarrow$	q-state Potts
$\downarrow$ $(S_2 \simeq Z_2)$	$\longleftrightarrow$	$\downarrow$ (Ising model)
$\downarrow$ **Our model** $q \longrightarrow 1$	$\longleftrightarrow$	$\downarrow$ Random percolation (clusters $\simeq$ F-K clusters $\simeq$ magn. strings)

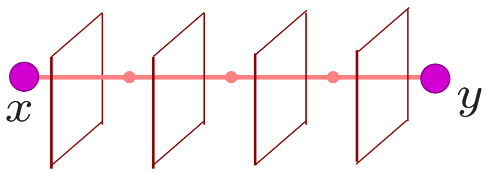
Monopoles & percolation - II

- In Z_2-gauge, a monopole ($\equiv$ antimonopole) is a cube with total outgoing flux $= -1$.
- To insert a monopole (and its 't Hooft string) as external source, change $\beta \longrightarrow -\beta$ along an infinite line of plaquettes (but two superimposed lines are equivalent to nothing!).
- Under duality, a frustrated plaquette $(x; j, k)$ becomes $\sigma_x \sigma_{x+\hat{i}}$ $\Rightarrow$ a monopole in $\mathbf{x}$ amounts to just the spin operator σ_x.
- *Example*: a single plaquette flip means a monopole couple at distance 1:



$$\Rightarrow \quad \langle \sigma_x \sigma_{x+1} \rangle_{\text{Ising}}$$

- *Example #2:* A flip on a finite segment of plaquettes:



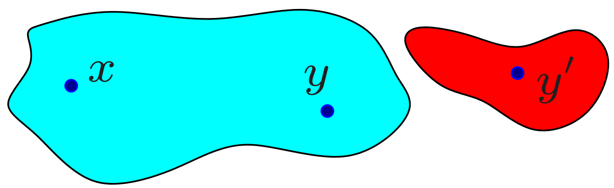
$$\Rightarrow \quad \langle \sigma_x \cdot 1 \cdot 1 \cdots 1 \cdot \sigma_y \rangle_{\text{Ising}}$$

Monopoles & percolation - III

The Ising (as well as the generic Potts) model admits F-K representation: the functional measure $\sum_{\{\sigma_x\}}$ becomes a sum over *all partitions of the lattice into clusters of aligned sites*.

Averaging over **cluster sign** variables one gets:

$$\langle \sigma_x \sigma_y \rangle \longmapsto \begin{cases} 1 & \text{if } x \text{ is connected to } y \\ 0 & \text{otherwise} \end{cases}$$



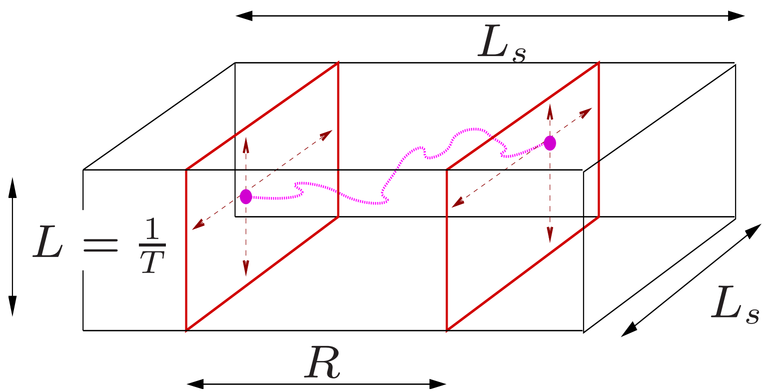
This holds for all values of q, including percolation: the correlation function $C(\mathbf{x}, \mathbf{y})$ measures whether $\mathbf{x}$ and $\mathbf{y}$ belong to the same connected component.

Plan of the numerical investigation

1. Probe the zero-momentum projected correlation function

$$C(R) \equiv \sum_{y_1=x_1+R} C(\mathbf{x},\mathbf{y})$$

to extract monopole mass(es) via its exponential decay.



2. Probe, in the confined phase, the monopole condensate with the magnetisation operator

$$\langle \sigma \rangle$$

corresponding, in percolation, to the *strength of the infinite cluster* $\langle s \rangle$.

Some numbers and details

- In the deconfined phase:

  at $p_c = 0.265615$ ($L_c = \frac{1}{T_c} = 8$), we studied $L = 7, 6, 5, 4, 3, 2, 1$ (i. e. $1.14\, T_c \leq T \leq 8\, T_c$);

  at $p_c = 0.268459$ ($L_c = \frac{1}{T_c} = 7$), we studied $L = 6, 5, 4, 3, 2, 1$ (i. e. $1.17\, T_c \leq T \leq 7\, T_c$)

- In the confined phase and at criticality:

  at $p_c(1/8)$, we studied $L = 8, 9, 10, 11, 12, 13, 14, 15, 17, \infty$ (the last being actually 48)

- Spatial size and statistics [current data are still rather preliminary!]:

  we inspected about 300.000 to 10^6 realisations, with a spatial size L_s ranging from 64 to 256.

  $C(R)$ in the confined phase: $R = 1, \ldots, L_s/2$ in an uncorrelated fashion.

  $C(R)$ in the deconfined phase: $R = L_s/4, \ldots, L_s/2$ (no strong correlation issues).

- Expectations and functional forms for $C(R)$:

  deconfined : $C(R) = Ae^{-mR}$

  confined : $C(R) = Ae^{-mR} + \langle \Phi_m \rangle^2$

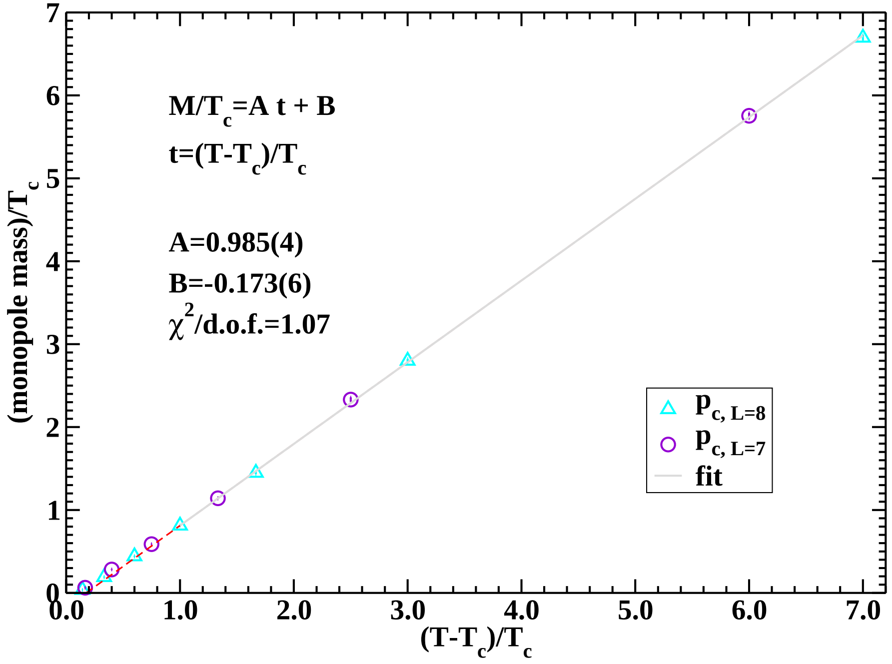
  in case of more than one mass : $C(R) = A_1 e^{-m_1 R} + A_2 e^{-m_2 R} + \cdots$

Results, deconfined phase

- The background constant scales well to zero for large systems.
- The correlator clearly shows a single-mass behaviour.
- Mass scaling with L_s is ok.
- Linear behaviour from slightly above T_c:

$$\frac{m}{T_c} = A \cdot \frac{T - T_c}{T_c} - B$$

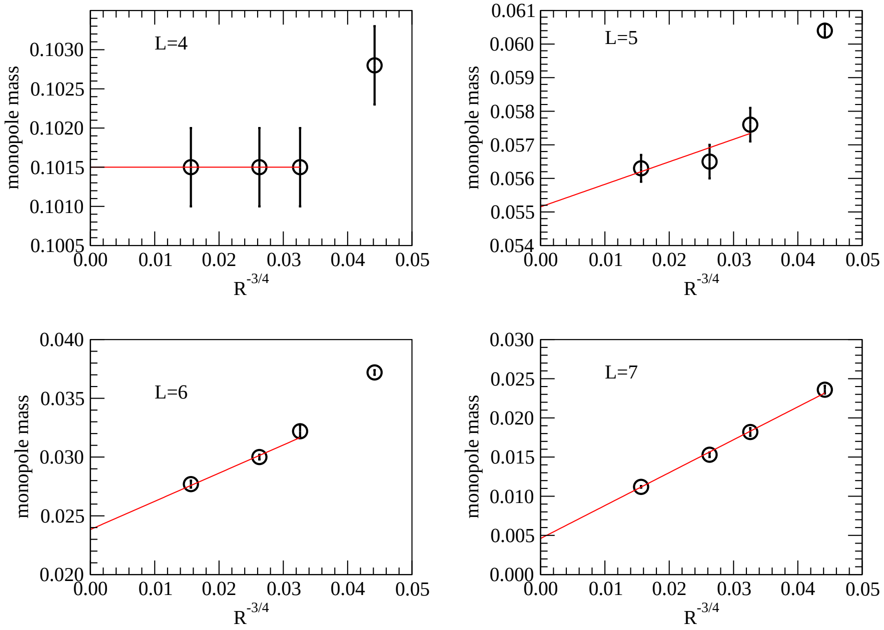
. . . but it is broken *just above* the critical point!



The intercept is at $T^* \sim 1.11\ T_c$: is the monopole mass zero in $[T_c\ ,\ T^*]$? Does it rise initially as $\left(\frac{T}{T_c}\right)^{\nu}$? (more statistics and sampling needed for an answer)

System size dependence

$m(L)$ vs. $L^{-\frac{3}{4}}$ in the deconfined phase; here, critical point is at $L_c = 8$.



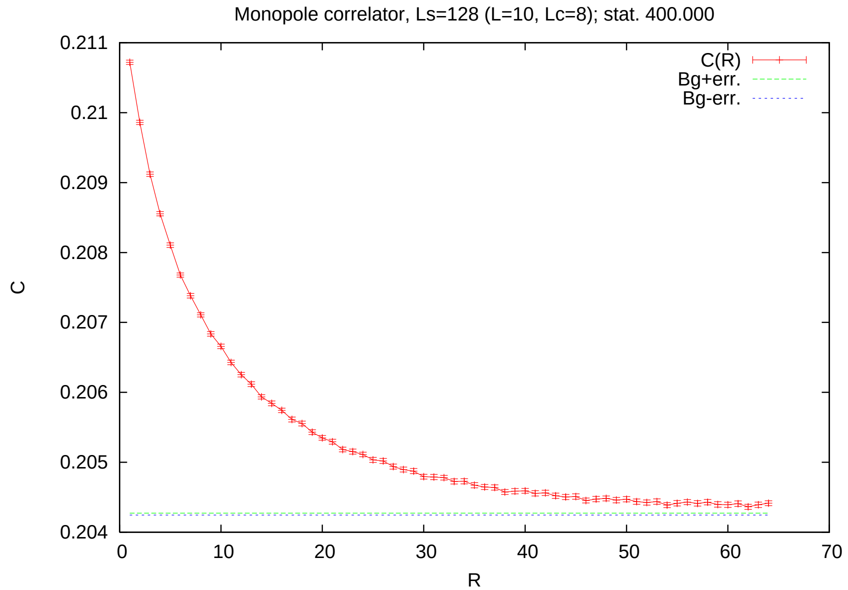
When sensible, the scaling is:

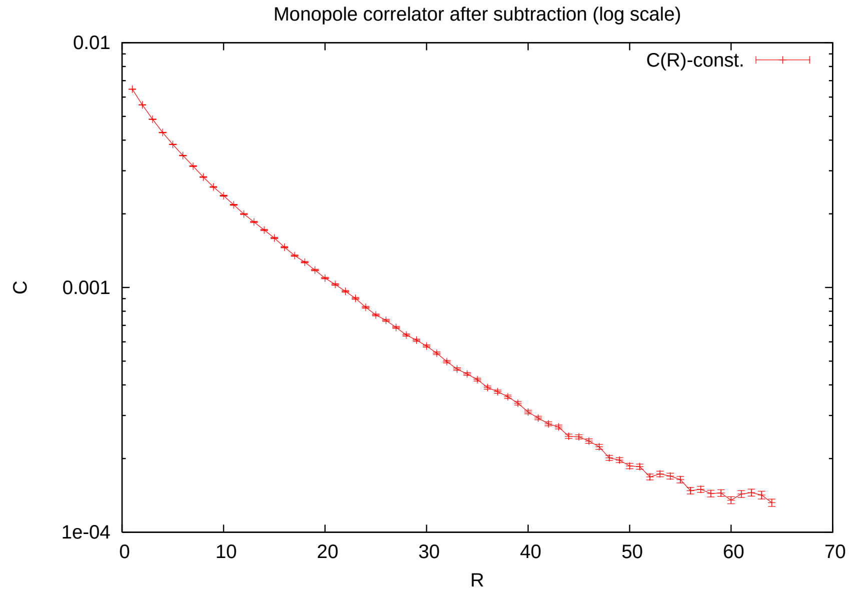
$$m(L) = m(\infty) + a \cdot L^{-\frac{1}{\nu}}$$

with $\nu = \frac{4}{3}$ (2D percolation).

(Confined phase scaling seems much more noisy due to the nonzero background, with an apparent power-law $\sim L^{-1.3(2)}$)

Results: monopoles at $T \leq T_c$





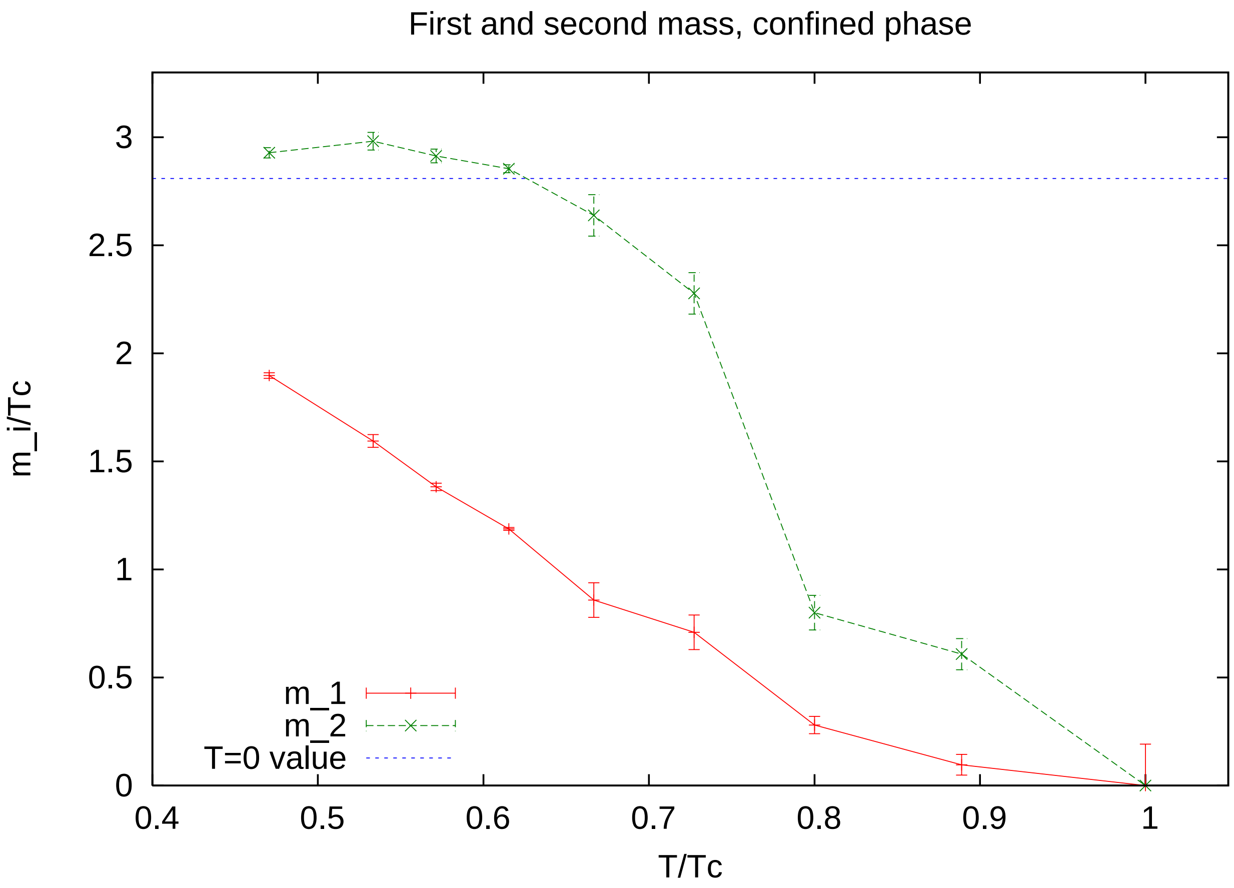
The constant background $\langle \Phi_m \rangle^2$ has to be subtracted to data before looking for masses.

For every spatial size and temperature, a double-mass signal is visible.

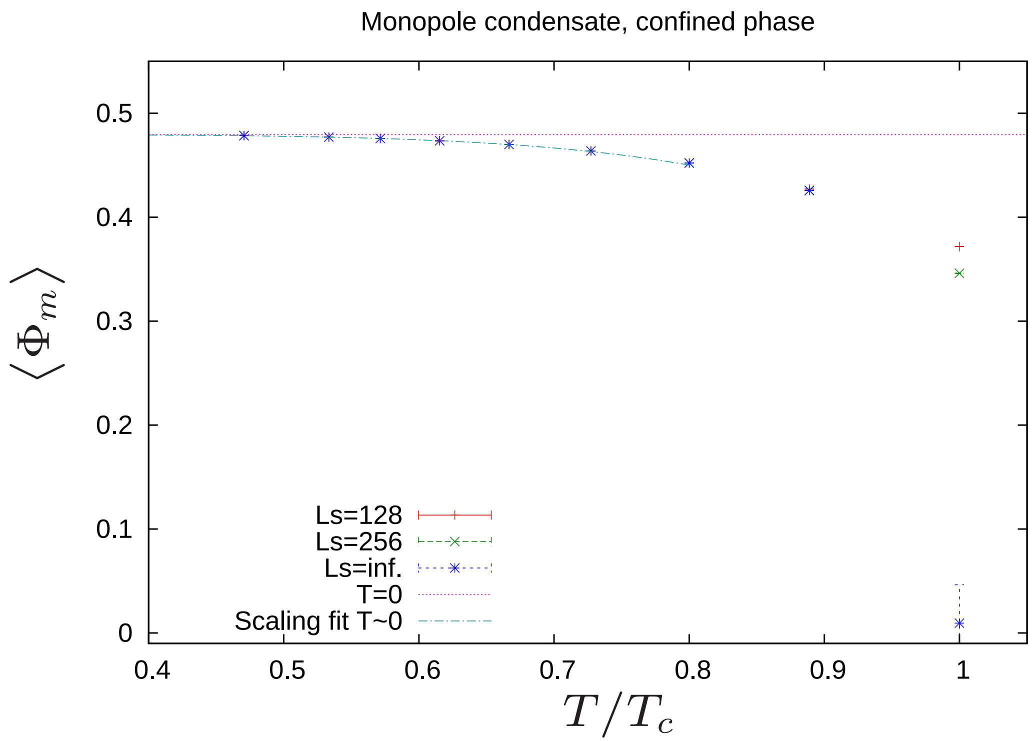
At zero temperature there is only one mass (coinciding with the lightest scalar glueball): the two finite-T values apparently flow both to it (at different temperature scales).

At criticality, we have a single mass again: its non-nullity is only a finite-size effect, which vanishes for large systems.

Results, confined phase



Results, monopole condensate under T_c



The condensate reaches its $T = 0$ value as:

$$\langle \Phi_m \rangle_{(T)} = \langle \Phi_m \rangle_{(0)} - B \cdot \left(\frac{T}{Tc}\right)^{3.00(3)}$$

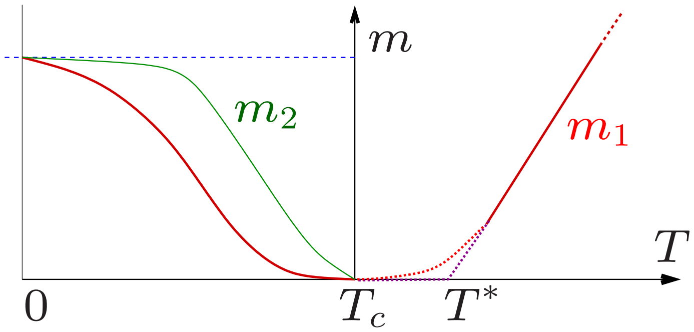
Still not enough data to attempt a near-T_c scaling.

$\Leftarrow$ *note that* $\langle \Phi_m \rangle_{(T_c)}$ *is zero!*

Conclusions & open issues

We could show that, in the confined phase, there are at least two monopole states, which fall onto each other at confinement and at zero temperature. In the latter case, their mass coincides with that of the lightest scalar glueball as was known.

A linearly rising behaviour, with the temperature, of the only mass in the deconfined phase is observed, but the situation is still puzzling near T_c.



A higher statistics is under production, to help clarify the behaviour just above deconfinement and to better define under-T_c curves (which suffer from major systematics due to the presence of the background).