OBSERVING THE DECAY OF UNSTABLE STRINGS IN SU(2) YANG-MILLS THEORY

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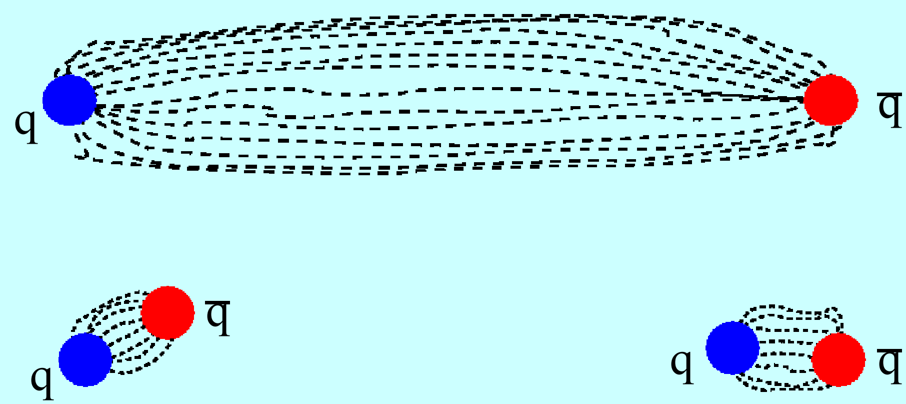
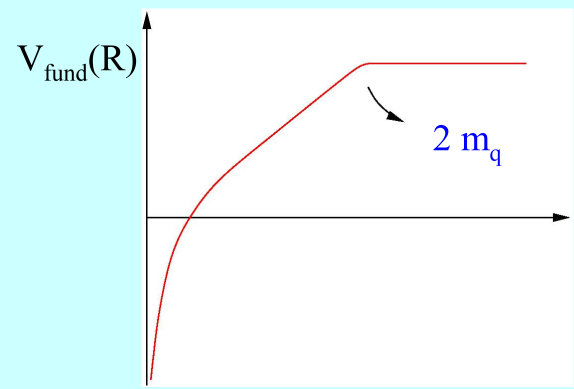
PLAN OF THE TALK

- Introduction
- String decay: a toy model
- Numerical results
- Conclusions

Introduction

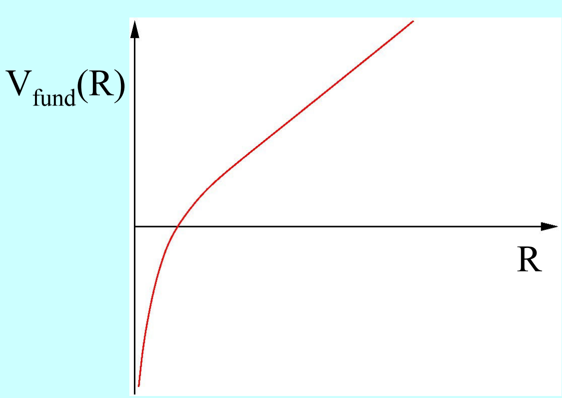
At low temperature quarks are confined inside baryons

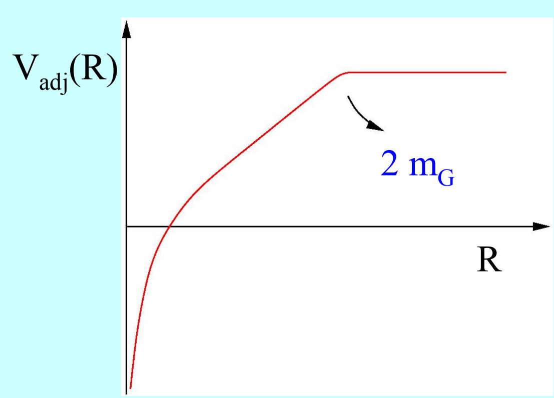


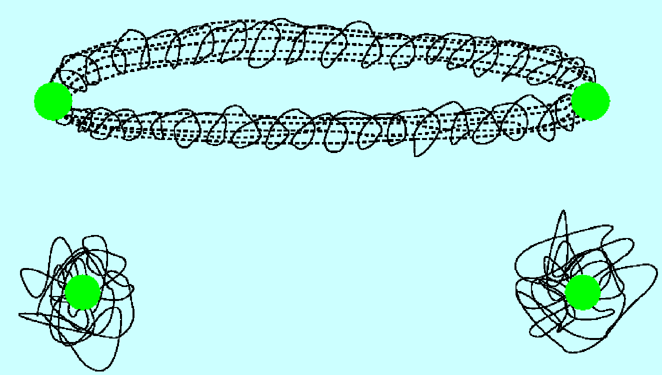


If $m_q \to \infty$ the string between two fundamental charges becomes stable

the string between two adjoint charges is still unstable by gluon emission







Many groups:
C. Michael et al., Bali et al., Sommer, Stephenson, Philipsen et al., de Forcrand et al, Gliozzi et al., Vicari et al., …

- Observing the decay of unstable strings is an important step in studying the phenomenology of confinement

- SU(N) Yang-Mills: a simplified framework where these effects can be studied with higher accuracy

- SU(N) representations branch in N sectors (N-ality sectors)

$$\mathcal{R}_k \otimes \{\mathrm{adj}\} \otimes \{\mathrm{adj}\} \ldots = \mathcal{R}'_k \oplus \ldots \Longrightarrow V_{\mathcal{R}_k} \sim V_{\mathcal{R}'_k}$$ by gluon emission

- For every N-ality sector there is a stable string; all other strings in that sector are unstable and decay into the stable as $R \to \infty$

- SU(2): 2 sectors
  - → {1} sources of integer spin
  - → {-1} sources of half-integer spin

- Very challenging numerically: only adjoint sources

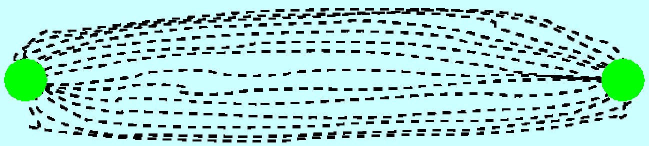
String decay: a toy model

Consider a two-state quantum system described by the Hamiltonian

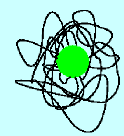
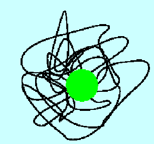
$$H_2 = \begin{pmatrix} E1 & \epsilon \\ \epsilon & E_2 \end{pmatrix} \quad \text{in the basis} \quad |S\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |L\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

ground state: $V_0 \Longrightarrow |0\rangle = a_0|S\rangle + b_0|L\rangle \qquad a_0^2 + b_0^2 = 1$

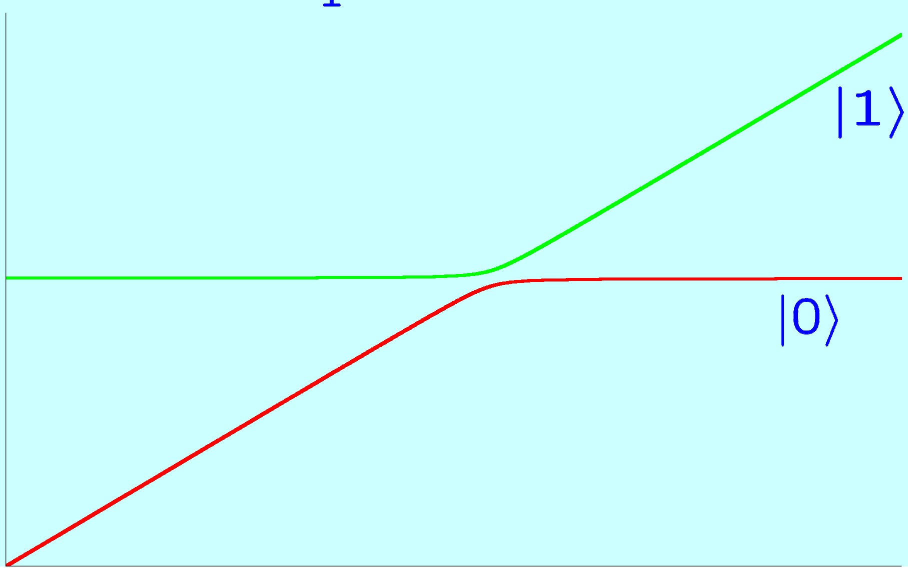
excited state: $V_1 \Longrightarrow |1\rangle = a_1|S\rangle + b_1|L\rangle \qquad a_1^2 + b_1^2 = 1$

$|S\rangle \simeq$ 

$E_1 = \sigma R$

$|L\rangle \simeq$  

$E_2 = 2m$



A different two-state quantum system described by a similar Hamiltonian

$$H_2 = \begin{pmatrix} E1 & \epsilon \\ \epsilon & E_2 \end{pmatrix} \quad \text{in the basis} \quad |S\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |L\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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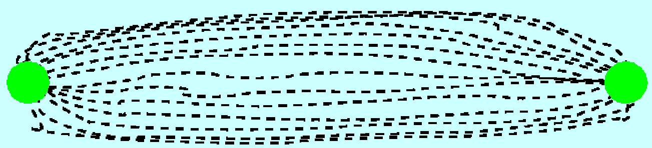
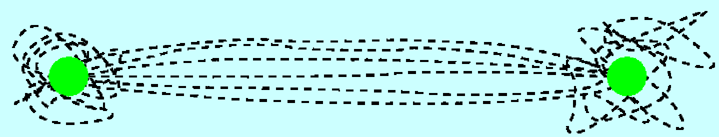
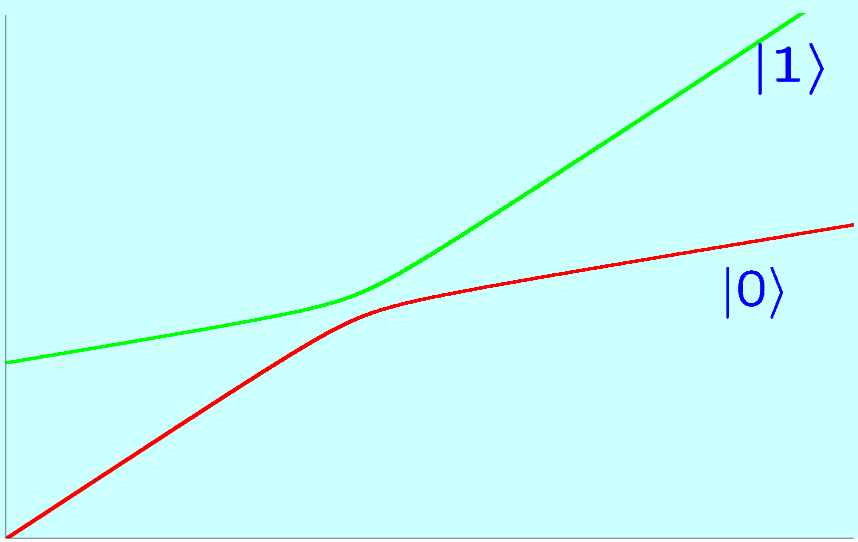
$|S\rangle \simeq$ 

$E_1 = \sigma_0 R$

$|L\rangle \simeq$ 

$E_2 = \sigma_1 R + 2m$



string decaying to another string

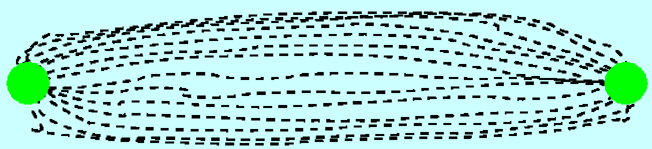
Consider a three-state quantum system described by the Hamiltonian

$$H_3 = \begin{pmatrix} E1 & \epsilon_1 & \epsilon_2 \\ \epsilon & E_2 & \epsilon_3 \\ \epsilon_2 & \epsilon_3 & E_3 \end{pmatrix}$$ in the basis $|S\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $|I\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $|L\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

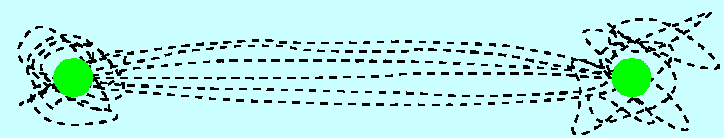
ground state: $V_0 \Rightarrow |0\rangle = a_0|S\rangle + b_0|I\rangle + c_0|L\rangle \quad a_0^2 + b_0^2 + c_0^2 = 1$

1st excited state: $V_1 \Rightarrow |1\rangle = a_1|S\rangle + b_1|I\rangle + c_1|L\rangle \quad a_1^2 + b_1^2 + c_1^2 = 1$

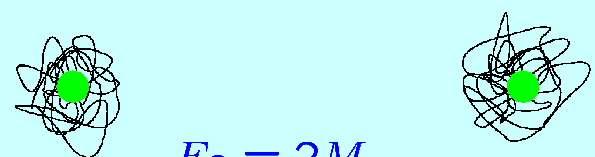
2nd excited state: $V_2 \Rightarrow |2\rangle = a_2|S\rangle + b_2|I\rangle + c_2|L\rangle \quad a_2^2 + b_2^2 + c_2^2 = 1$

$|S\rangle \simeq$ 

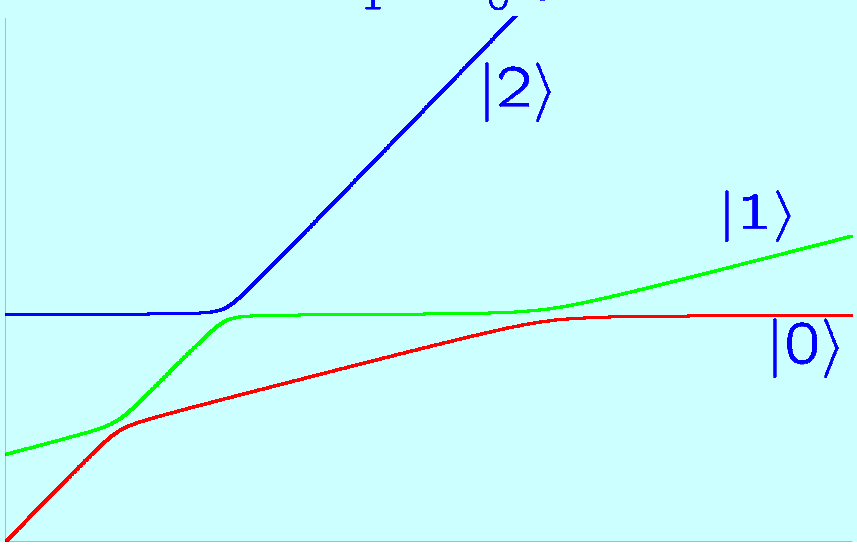
$E_1 = \sigma_0 R$

$|I\rangle \simeq$ 

$E_2 = \sigma_1 R + 2m$

$|L\rangle \simeq$ 

$E_3 = 2M$



string decaying and then breaking

"explicit mixing method"

We have a multi-state quantum system

$$|X_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle + \gamma_1|2\rangle + \ldots$$

$$|X_2\rangle = \alpha_2|S\rangle + \beta_2|I\rangle + \gamma_2|L\rangle + \ldots$$

$$\ldots$$

$$\begin{pmatrix} \langle X_1|\mathrm{e}^{-HT}|X_1\rangle & \langle X_1|\mathrm{e}^{-HT}|X_2\rangle & \ldots \\ \langle X_2|\mathrm{e}^{-HT}|X_1\rangle & \langle X_2|\mathrm{e}^{-HT}|X_2\rangle & \ldots \\ \ldots & \ldots & \end{pmatrix}$$

$|X_1\rangle, |X_2\rangle, \ldots$ chosen to enhance the overlap with |0> at every distance (small T)

"implicit mixing method"

$$\langle X|\mathrm{e}^{-HT}|X\rangle = \alpha^2\mathrm{e}^{-V_0T} + \beta^2\mathrm{e}^{-V_1T} + \gamma^2\mathrm{e}^{-V_2T} + \ldots$$

$$= \alpha^2\mathrm{e}^{-V_0T}\left(1 + \frac{\beta^2}{\alpha^2}\mathrm{e}^{-(V_1-V_0)T} + \frac{\gamma^2}{\alpha^2}\mathrm{e}^{-(V_2-V_0)T} + \ldots\right)$$

$$\xrightarrow[T\to\infty]{} \alpha^2\mathrm{e}^{-V_0T}$$

large T: numerically challenging

Numerical results

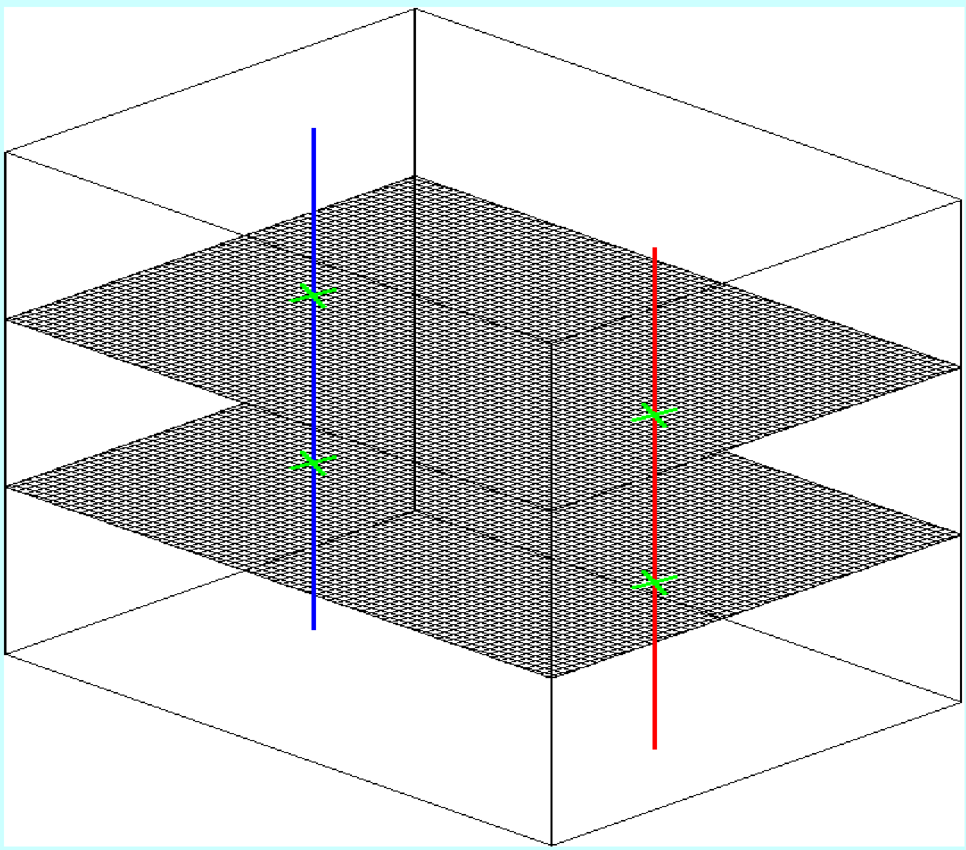
We consider the SU(2) Yang-Mills theory on the lattice in (2+1)-d and we use the "implicit method" to observe the string decay

Wilson action: $\mathcal{S}_{YM}[U] = -\beta \sum_P \mathrm{Tr}\,(U_P)$

$32^2 \times 64$ at $\beta = 6.0$

$$\mathcal{O} = \frac{\int \mathcal{D}U \; \phi_{\mathcal{R}}(R)\, \phi_{\mathcal{R}}(0)\; \mathrm{e}^{-\mathcal{S}_{YM}}}{\int \mathcal{D}U \;\; \mathrm{e}^{-\mathcal{S}_{YM}}}$$

$$\phi_{\mathcal{R}}(\vec{x}) = \mathrm{Tr}_{\mathcal{R}}[\prod_t U_4(\vec{x}, t)]$$



$$\mathcal{O} \sim \mathrm{e}^{-V_0 T} \sim \left(\mathrm{e}^{-V_0 \tau}\right)^{T/\tau}$$

$\mathcal{O} \rightarrow$ tensor product of the two segments

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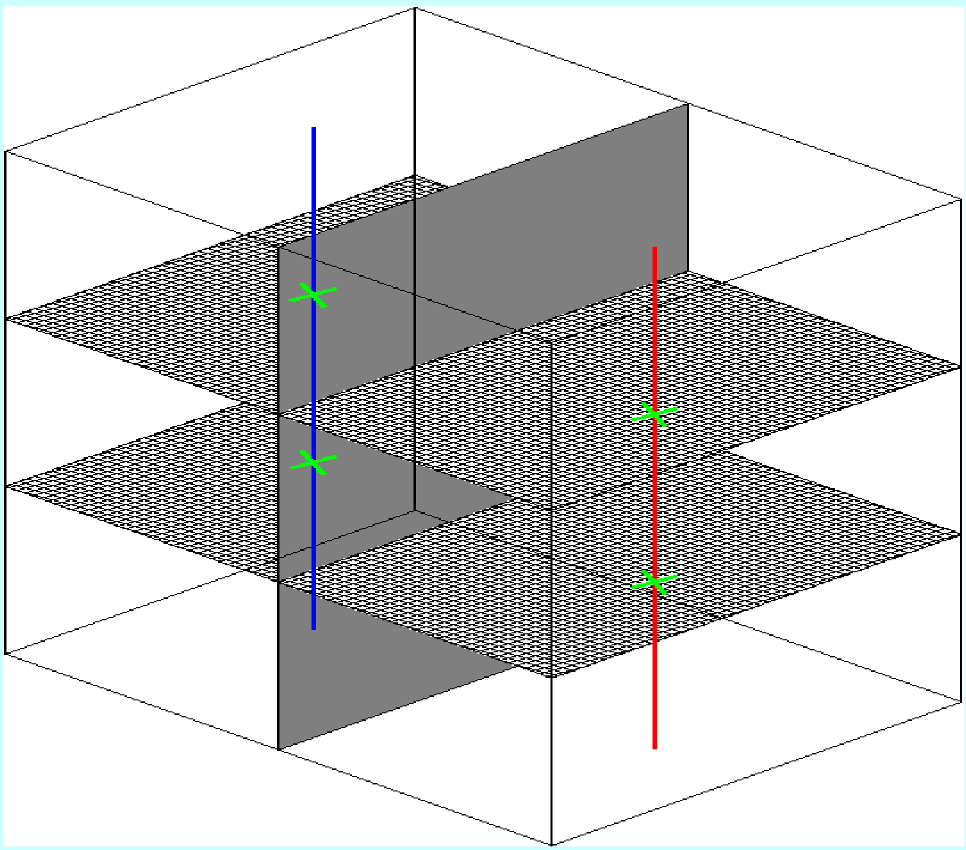


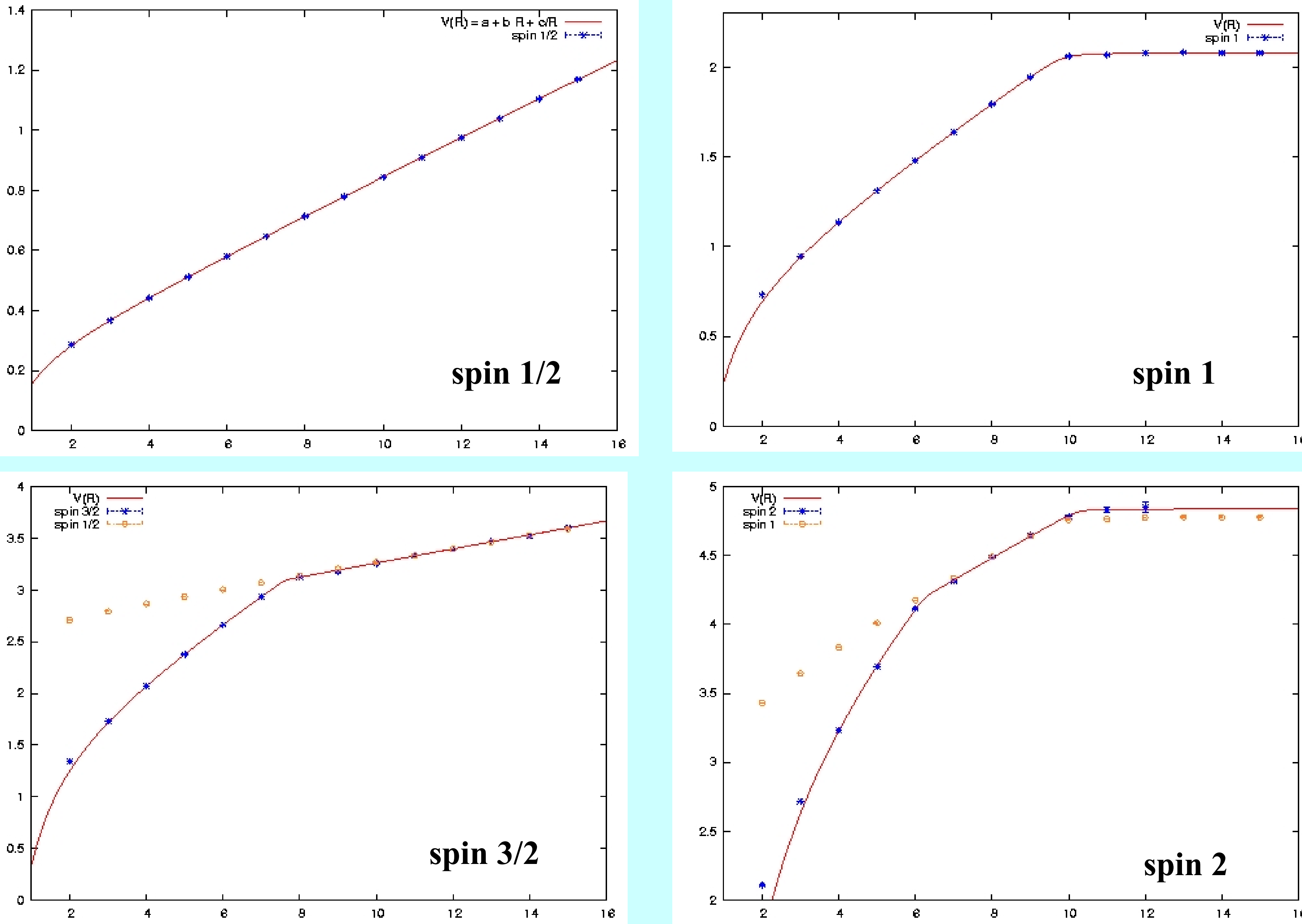
$$\mathcal{O} \sim \mathrm{e}^{-V_0 T} \sim \left(\mathrm{e}^{-V_0 \tau}\right)^{T/\tau}$$

$\mathcal{O} \rightarrow$ tensor product of the two segments

$$\mathrm{e}^{-V_0 \tau} \sim \left(\mathrm{e}^{-V_0 \tau/2}\right)^2$$

Potential

N-ality = {-1} | **N-ality = {1}**

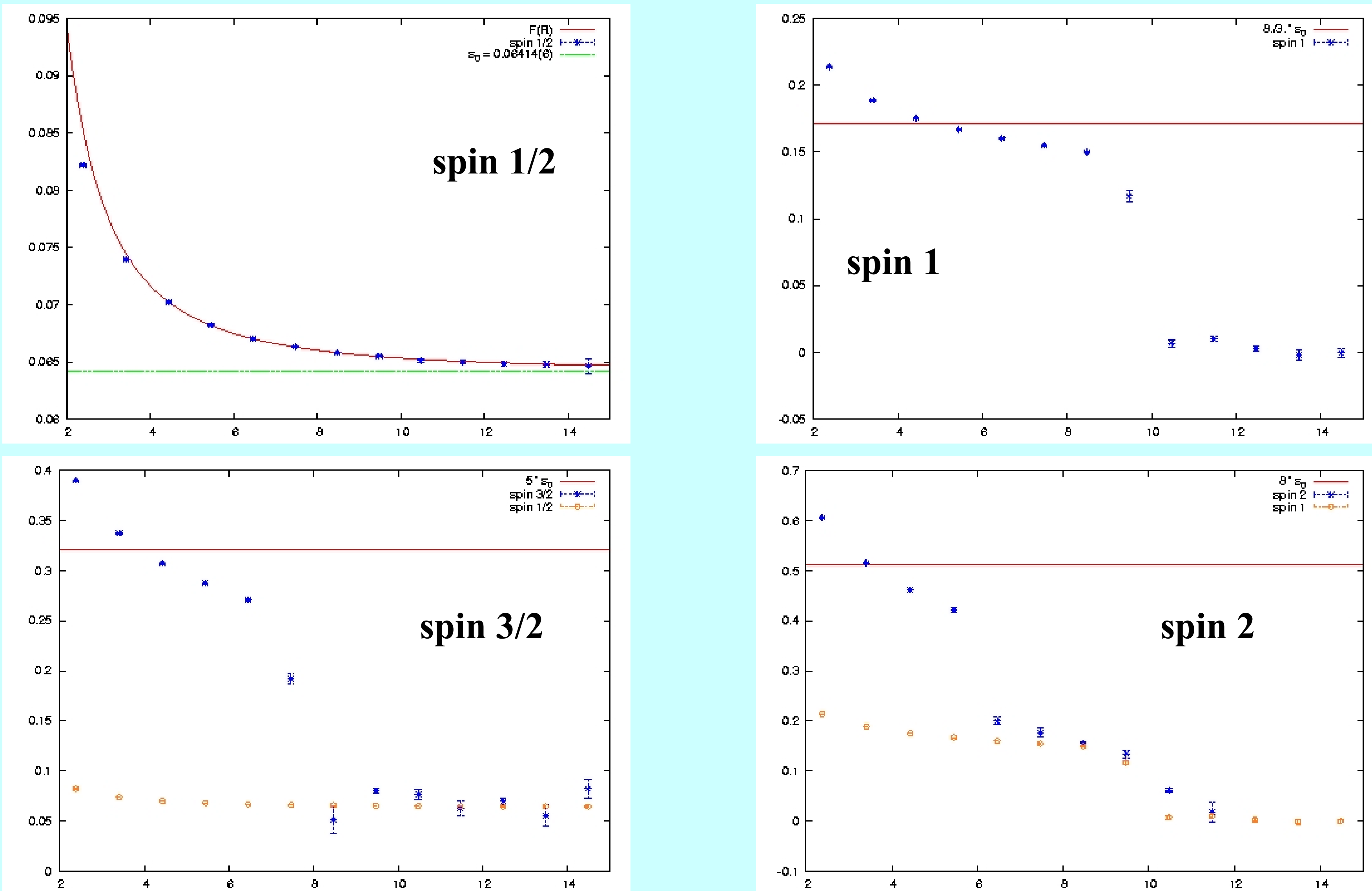


Force

$$V(R) \sim \sigma R - \frac{c}{R} + k \quad \Longrightarrow \quad F(R) \sim \sigma + \frac{c}{R^2}$$

N-ality = {-1}

N-ality = {1}

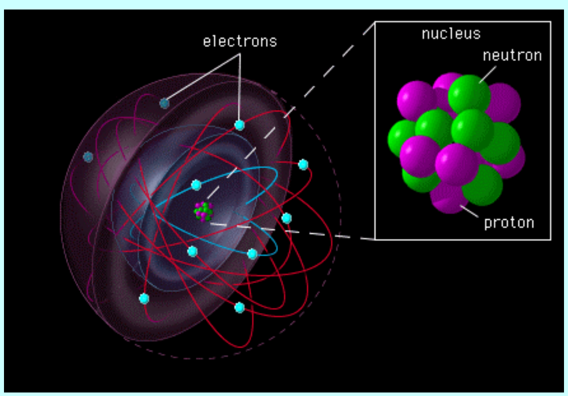


Conclusions

- We have observed the decay of unstable strings using a single observable in SU(2) Yang-Mills theory
- We have measured the 2-point function for the reps {2}, {3}, {4}, {5}: it is an important step in the study of the phenomenology of confinement

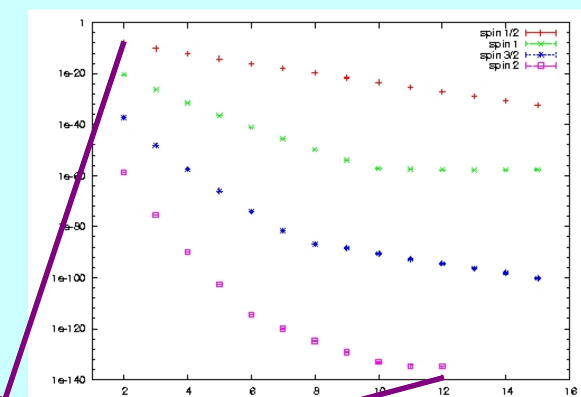
  decay of $\{4\} \rightarrow \{2\}$ double decay $\{5\} \rightarrow \{3\} \rightarrow \{1\}$

- Casimir scaling for the string tensions is ruled out ((2+1)-d)
- Numerically very challenging: used the multilevel Lüscher-Weisz algorithm



nucleon radius ⟷ universe

40 orders of magnitude



$10^{-8} \rightarrow 10^{-135}$: ~130 orders of magnitude!