QCD thermodynamics and the large-$N$ limit – A review

Marco Panero

Department of Physics and Helsinki Institute of Physics
University of Helsinki, Finland

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Outline

1. Physical motivation
2. The large-$N$ limit
3. Lattice QCD
4. Equation of state in $D = 3 + 1$ dimensions
5. Equation of state in $D = 2 + 1$ dimensions
6. Conclusions

Based on:
- M. Caselle et al., JHEP 1106 (2011) 142
- M. Caselle et al., in preparation

See also the talk by A. Mykkänen
Outline

1. Physical motivation
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Due to asymptotic freedom in non-Abelian gauge theories [Gross and Wilczek, 1973; Politzer, 1973], hadronic matter is expected to undergo a change of state to a deconfined phase at sufficiently high temperatures or densities [Cabibbo and Parisi, 1975; Collins and Perry, 1974].

Extensive experimental investigation through heavy ion collisions since the Eighties: first at AGS (BNL) and SPS (CERN), then at RHIC (BNL) and more recently at LHC (CERN)

Experimental evidence from SPS, RHIC and LHC: a ‘A new state of matter’ has been created [Heinz and Jacob, 2000, Arsene et al., 2004; Back et al., 2004; Adcox et al., 2004; Adams et al., 2005; Aad et al., 2010; Aamodt et al., 2010; Chatrchyan et al., 2011]
The physical problem - II

- The plasma behaves as an almost ideal fluid [Kolb and Heinz, 2003] (‘The most perfect liquid observed in Nature’)
- Further experiments at LHC, FAIR, NICA and J-PARC to provide a more detailed picture
- However, the theoretical understanding of the QCD plasma [Rischke, 2003; Shuryak, 2008] is still incomplete . . .
Relativistic fluidodynamics is a successful phenomenological description [Kolb, Heinz, Huovinen, Eskola and Tuominen, 2001]—see also [Romatschke, 2009] for an introductory review—, but is not derived from QCD first principles.

The perturbative approach in thermal gauge theory has a non-trivial mathematical structure, involving odd powers of the coupling [Kapusta, 1979], as well as contributions from diagrams involving arbitrarily large numbers of loops [Linde, 1980; Gross, Pisarski and Yaffe, 1980] . . .

. . . and shows poor convergence at the temperatures probed in experiments [Kajantie, Laine, Rummukainen and Schröder, 2002].

The long-wavelength modes of the plasma are strongly coupled even at high temperature [Blaizot, 2011].


In the large-$N$ limit, the Maldacena conjecture relates a strongly interacting gauge theory to the classical limit of a gravity model
Numerical approach: Computer simulations of QCD regularized on a lattice allow first-principle, non-perturbative studies of the finite-temperature plasma.

The lattice determination of equilibrium thermodynamic properties in SU(3) gauge theory is regarded as a solved problem [Boyd et al., 1996; S. Borsányi et al., 2011].

In recent years, finite-temperature lattice QCD has steadily progressed towards parameters corresponding to the physical point [Karsch et al., 2000; Ali Khan et al., 2001; Aoki et al., 2005; Bernard et al., 2006; Cheng et al., 2007; Bazavov et al., 2009; S. Borsányi et al., 2010].

Simulations at finite $\mu$ must cope with a NP-hard [Troyer and Wiese, 2004] sign problem [de Forcrand and Philipsen, 2002; D’Elia and Lombardo, 2002; Allton et al., 2002; Fodor and Katz, 2004; Cea, Cosmai, D’Elia, Manneschi and Papa, 2009].

Related lattice studies of the thermal properties of gauge theories in the large-$N$ limit: [Lucini, Teper and Wenger, 2003], [Bringoltz and Teper, 2005] and [Datta and Gupta, 2010].
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The large-$N$ limit

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The old perspective: QCD at large $N$

- ’t Hooft proposed to use $1/N$ ($N$ being the number of colors) as an expansion parameter [’t Hooft, 1974]
- Generically, a large-$N$ limit can be interpreted as a ‘classical limit’; identification of coherent states and construction of a classical Hamiltonian [Yaffe, 1982]
- In the large-$N$ limit at fixed ’t Hooft coupling $\lambda = g^2 N$ and fixed number of flavors $N_f$, certain non-trivial non-perturbative features of QCD can be easily explained in terms of combinatorics [Witten, 1979; Manohar, 1998]
- Planar diagrams’ dominance
- Formal connection to string theory: loop expansion in Riemann surfaces for closed string theory with coupling constant $g_{\text{string}} \sim 1/N$ [Aharony, Gubser, Maldacena, Ooguri and Oz, 1999; Mateos, 2007]

$$\mathcal{A} = \sum_{G=0}^{\infty} N^{2-2G} \sum_{n=0}^{\infty} c_{G,n} \lambda^n$$

- Interesting implications from strong coupling expansions on the lattice [Langelage and Philipsen, 2010]
Maldacena conjectured that the large-$N$ limit of the maximally supersymmetric $\mathcal{N} = 4$ supersymmetric YM (SYM) theory in four dimensions is dual to type IIB string theory in a $AdS_5 \times S^5$ space [Maldacena, 1997]

$$ds^2 = \frac{r^2}{R^2} \left( -dt^2 + dx^2 \right) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2$$

The conjecture arises from the observation that the low-energy dynamics of open strings ending on a stack of $N$ D3 branes in $AdS_5 \times S^5$ can be described in terms of $\mathcal{N} = 4$ SYM

- Geometric interpretation: There exists a correspondence of symmetries in the two theories
- A highly non-trivial correspondence, linking the strongly coupled regime of field theory to the weak-coupling limit of a gravity model
- Identification of the generating functional of connected Green’s functions in the gauge theory with the minimum of the supergravity action with given boundary conditions: correlation functions of gauge theory operators from perturbative calculations in the gravity theory [Gubser, Klebanov and Polyakov, 1998]
- A stringy realization of the holographic principle: the description of dynamics within a volume of space is “encoded on the boundary” ['t Hooft, 1993; Susskind, 1995]—see also [Bousso, 2002] for a review
- The large-$N$ limit of the $\mathcal{N} = 4$ SYM theory exhibits a phase transition which can be related to the thermodynamics of $AdS$ black holes [Witten, 1998]
The AdS/CFT correspondence

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  - \(R\)-symmetry in the gauge theory is \(SU(4) \sim SO(6)\) symmetry of \(S^5\)
  - The conformal invariance group in the gauge theory is isomorphic to \(SO(2, 4)\), the symmetry group of \(AdS_5\)
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\[
\begin{align*}
g^2 &= 4\pi g_s \\
g^2 N &= \frac{R^4}{l_s^4}
\end{align*}
\]

Identification of the generating functional of connected Green’s functions in the gauge theory with the minimum of the supergravity action with given boundary conditions: correlation functions of gauge theory operators from perturbative calculations in the gravity theory \[\text{[Gubser, Klebanov and Polyakov, 1998]}\]

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Non-perturbative predictions for QCD-like theories from holographic models

- ‘Top-down’ approach: break some symmetries of the $\mathcal{N} = 4$ theory explicitly, add fundamental matter fields to the gauge theory by including new branes in the string theory [Bertolini, Di Vecchia, Frau, Lerda, and Marotta, 2001; Graña and Polchinski, 2001; Karch and Katz, 2002] to get a non-trivial hadron sector with ‘mesons’ and $\chi_{SB}$ [Erdmenger, Evans, Kirsch and Threlfall, 2007]

- Derivation of hydrodynamic and thermodynamic properties for a strongly interacting system from gauge/gravity duality [Policastro, Son and Starinets, 2001]—see also [Son and Starinets, 2007; Mateos, 2007; Gubser and Karch, 2009] and references therein

- ‘Bottom-up’ approach: construct a 5D gravitational background reproducing the main features of QCD [Polchinski and Strassler, 2001; Erlich, Katz, Son and Stephanov, 2005; Da Rold and Pomarol, 2005; Karch, Katz, Son and Stephanov, 2006]

- Hard-wall versus soft-wall AdS/QCD, and related thermodynamic features [Herzog, 2007]
Kiritsis and collaborators [Gürsoy, Kiritsis, Mazzanti and Nitti, 2008] proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the SU($N$) gauge theory.

Field content on the gravity side: metric (dual to the SU($N$) energy-momentum tensor), the dilaton (dual to the trace of $F^2$) and the axion (dual to the trace of $F \tilde{F}$).

Gravity action:

$$S_{IHQCD} = -M_P^3 N^2 \int d^5 x \sqrt{g} \left[ R - \frac{4}{3} (\partial \Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4 x \sqrt{h} K$$

$\Phi$ is the dilaton field, $\lambda = \exp(\Phi)$ is identified with the running 't Hooft coupling of the dual SU($N$) YM theory.

The effective five-dimensional Newton constant $G_5 = 1 / (16\pi M_P^3 N^2)$ becomes small in the large-$N$ limit.
Dilaton potential $V(\lambda)$ defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory; a possible Ansatz is:

$$V(\lambda) = \frac{12}{\ell^2} \left[ 1 + V_0 \lambda + V_1 \lambda^{4/3} \sqrt{\log \left( 1 + V_2 \lambda^{4/3} + V_3 \lambda^2 \right)} \right],$$

where $\ell$ is the AdS scale (overall normalization), and two free parameters are fixed by imposing that the dual model reproduces the first two coefficients of the $SU(N)$ $\beta$-function.

Gauge/gravity duality expected to hold in the large-$N$ limit only, because calculations in the gravity model neglect string interactions which can become important above a scale $M_P N^{2/3} \simeq 2.5$ GeV in $SU(3)$.

First-order transition from a thermal-graviton- to a black-hole-dominated regime in the 5D gravity theory dual to the $SU(N)$ deconfinement transition.

The model successfully reproduces the main non-perturbative spectral and thermodynamical features of the $SU(3)$ YM theory.

Can also be used to derive predictions for observables such as the plasma bulk viscosity, drag force and jet quenching parameter [Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009].
Outline

1 Physical motivation

2 The large-$N$ limit

3 Lattice QCD

4 Equation of state in $D = 3 + 1$ dimensions

5 Equation of state in $D = 2 + 1$ dimensions

6 Conclusions
Lattice QCD: The basics

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing $a$
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Discretization of the continuum gauge action with the Wilson lattice action \([\text{Wilson, 1974}]\):
  \[
  S = \beta \sum_\square \left( 1 - \frac{1}{N} \, \text{Re} \, \text{Tr} \, U_\square \right), \quad \text{with: } \beta = \frac{2N}{g_0^2}
  \]
- A gauge-invariant, non-perturbative regularization
- Amenable to numerical simulation: Sample configuration space according to a statistical weight proportional to $\exp(-S)$
- Physical results recovered by extrapolation to the continuum limit $a \to 0$
Thermodynamics on the lattice

- Thermal averages from simulations on a lattice with compactified Euclidean time direction, with $T = 1/(aN_T)$
- Pressure $p(T)$ via the ‘integral method’ [Engels et al., 1990]:

\[
p = T \frac{\partial}{\partial V} \log Z \simeq \frac{T}{V} \log Z = \frac{1}{a^4 N_s^3 N_T} \int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'} = \frac{6}{a^4} \int_{\beta_0}^{\beta} d\beta' \left( \langle U^2 \rangle_T - \langle U^2 \rangle_0 \right)
\]
Other equilibrium thermodynamic observables obtained from indirect measurements

- Trace of the stress tensor $\Delta = \epsilon - 3p$:
  $$\Delta = T^5 \frac{\partial}{\partial T} \frac{p}{T^4} = \frac{6}{a^4} \frac{\partial \beta}{\partial \log a} \left( \langle U^2 \rangle_0 - \langle U^2 \rangle_T \right)$$

- Energy density:
  $$\epsilon = \frac{T^2}{V} \frac{\partial}{\partial T} \log Z = \Delta + 3p$$

- Entropy density:
  $$s = \frac{S}{V} = \frac{\epsilon - f}{T} = \frac{\Delta + 4p}{T}$$
Simulation details

- Lattice sizes $N_s^{D-1} \times N_\tau$, with $N_s$ from 16 to 64, and $N_\tau$ from 5 to 12
- Cross-check with $T = 0$ simulations run using the Chroma suite [Edwards and Joó, 2004]
- Physical scale for SU(3) in 4D determined from $r_0$ [Necco and Sommer, 2001]
- Physical scale for SU($N > 3$) in 4D determined from the string tension $\sigma$ [Lucini, Teper and Wenger, 2004; Lucini and Teper, 2001; Del Debbio, Panagopoulos, Rossi and Vicari, 2001] in combination with the 3-loop lattice $\beta$-function [Allés, Feo and Panagopoulos, 1997; Allton, Teper and Trivini, 2008] in the mean-field improved lattice scheme [Parisi, 1980; Lepage and Mackenzie, 1993]
- Physical scale for SU($N$) in 3D determined from lattice computations of $\sigma$ [Liddle and Teper, 2008]
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Equation of state in $D = 3 + 1$ dimensions

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Improved holographic QCD model vs. lattice data
QCD thermodynamics and the large-$N$ limit – A review

Equation of state in $D = 3 + 1$ dimensions

Improved holographic QCD model vs. lattice data

Pressure

![Graph showing pressure vs. $T/T_c$]
QCD thermodynamics and the large-$N$ limit – A review

Equation of state in $D = 3 + 1$ dimensions

**Improved holographic QCD model vs. lattice data**

Trace of the energy-momentum tensor

![Graph showing the trace of the energy-momentum tensor normalized to $(N^2 - 1) \pi^2 / 45$ for SU(3), SU(4), SU(5), SU(6), SU(8) improved holographic QCD model vs. lattice data.](image-url)
Improved holographic QCD model vs. lattice data

Energy density

Energy density graph with data points for SU(3), SU(4), SU(5), SU(6), SU(8) and improved holographic QCD model, normalized to the SB limit.
Improved holographic QCD model vs. lattice data

Entrophy density

- SU(3)
- SU(4)
- SU(5)
- SU(6)
- SU(8)

improved holographic QCD model
For $T \simeq 3T_c$, the lattice results reveal that the deconfined plasma, while still strongly interacting and far from the Stefan-Boltzmann limit, approaches a scale-invariant regime . . .

$p(\varepsilon)$ equation of state and approach to conformality
AdS/CFT vs. lattice data in a ‘quasi-conformal’ regime

... in which the entropy density is comparable with the supergravity prediction for \( \mathcal{N} = 4 \) SYM [Gubser, Klebanov and Tseytlin, 1998]

\[
\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32} \zeta(3)(2\lambda)^{-3/2} + \ldots
\]

Entropy density vs. ’t Hooft coupling
AdS/CFT vs. lattice data in a ‘quasi-conformal’ regime

...in which the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM [Gubser, Klebanov and Tseytlin, 1998]

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32} \zeta(3)(2\lambda)^{-3/2} + \ldots$$

Note that a comparison of $\mathcal{N} = 4$ SYM and full-QCD lattice results for the drag force on heavy quarks also yields $\lambda \simeq 5.5$ [Gubser, 2006]
The trace anomaly reveals a characteristic $T^2$-behavior, possibly of non-perturbative origin [Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007]
Extrapolation to $N \to \infty$

Based on the parametrization [Bazavov et al., 2009]:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45} (N^2 - 1) \cdot \left(1 - \left\{1 + \exp \left[\frac{(T/T_c) - f_1}{f_2}\right]\right\}^{-2}\right) \left(f_3 \frac{T_c^2}{T^2} + f_4 \frac{T_c^4}{T^4}\right)$$

Extrapolation to the large-$N$ limit
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The confining phase of Yang-Mills theories in $D = 2 + 1$ dimensions

For $T < T_c$, the equation of state is essentially independent of $N$ for all $SU(N \geq 3)$, and can be described by a gas of massive, non-interacting glueballs, with spectral density modelled by a closed bosonic string [Isgur and Paton, 1985]

$$\tilde{\rho}_D(m) = 2\frac{(D - 2)^{D-1}}{m} \left( \frac{\pi T_H}{3m} \right)^{D-1} e^{m/T_H}$$

Trace of the energy-momentum tensor and string model
The deconfined phase of Yang-Mills theories in $D = 2 + 1$ dimensions

Similarly to the $D = 3 + 1$ case, in the deconfined phase the equation of state scales proportionally to $N^2 - 1$ . . .
The deconfined phase of Yang-Mills theories in $D = 2 + 1$ dimensions

... and the trace of the energy-momentum tensor appears to be dominated by contributions proportional to $T^2$
Conclusions

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In the deconfined phase, the equation of state of non-supersymmetric Yang-Mills theories appears to be nearly exactly proportional to $N^2 - 1$; this holds both in both $D = 3 + 1$ and $D = 2 + 1$ dimensions.

The IHQCD model provides a quantitative description of the results for the $D = 3 + 1$ case.

For the $D = 3 + 1$ case, the bulk thermodynamic quantities in a nearly conformal, yet strongly coupled regime near $T \sim 3 T_c$ can be compared with holographic predictions for $\mathcal{N} = 4$ SYM.

Both in $D = 3 + 1$ and $D = 2 + 1$ dimensions, in the deconfined phase $\Delta$ exhibits a characteristic $T^2$-dependence.

In the confining phase, the equation of state of YM theories in $D = 2 + 1$ is described by a gas of massive, non-interacting glueballs (with multiplicities independent of $N$—except for the $N = 2$ case), whose spectral density can be modelled by a bosonic string model.