Lattice QCD in Heavy Flavour Physics:
Recent results for $m_b$, $f_B$ and $f_{Bs}$ by ETMC

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The SM turns out to be very successful in describing essentially all processes. However, it is expected to be an effective theory valid up to a cutoff scale as it has some important limits.

The SM is a quantum theory for strong and electroweak interactions but NOT for gravitation. Quantum effects in gravitation are expected to become important at very high energies ($M_{Pl} \sim 10^{23}$ GeV).

There is cosmological evidence of Dark Matter (not made up of SM particles) in the Universe.

In the SM the Higgs mass receives large radiative corrections, quadratic in the cutoff $\Lambda \sim M_{Pl} \sim 10^{23}$ GeV (energy scale where the SM fails). To have a Higgs mass of $O(100$ GeV$)$, indicated by electroweak precision tests, an innatural fine-tuning is required (hierarchy problem).
The solution doesn’t seem to be trivial: the FLAVOUR PROBLEM

“NP is expected at the TeV scale (in order to solve the hierarchy problem $\delta M_{ii}^2 \approx \Lambda^2$) but in flavour processes NP effects are not observed (hinting for NP at higher scales)”

The flavour structure of the NP model cannot be generic
In order to reveal NP and understand its nature, Flavour Physics has a fundamental role besides the direct production at LHC.

The next decades will see a great experimental activity, not only in the direct NP search at LHC, but also in the Flavour Sector.

LHCb, SuperB-Factory, NA62, MEG, J-PARC, ...

It is crucial to have theoretical uncertainties well under control, in particular those of the hadronic parameters computed on the Lattice.
The Unitarity Triangle Analysis (UTA)

The experimental constraints:

\[
\begin{pmatrix}
\Delta m_d \\
\Delta m_s \\
V_{ub} \\
V_{cb}
\end{pmatrix}
\]

\[
\sin 2\beta, \cos 2\beta, \alpha, \gamma (2\beta + \gamma)
\]

overconstrain the CKM parameters consistently

Within the OPE and the QCD series, the small expansion parameter \(\Lambda_{QCD}/m_b\) and \(\alpha_s(m_b)\) helps a good convergence

The peculiar role of B-physics

Within the OPE and the QCD series, the small expansion parameter \(\Lambda_{QCD}/m_b\) and \(\alpha_s(m_b)\) helps a good convergence

Mass eigenstates

Weak eigenstates

\[
\begin{pmatrix}
d' \\
s' \\
b'
\end{pmatrix}
= V_{CKM}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\]

SM analysis

\[
\rho = 0.129 \pm 0.022
\]

\[
\eta = 0.346 \pm 0.015
\]

\[
\sim 17\%
\]

\[
\sim 4\%
\]
Theorist's Golden Modes

Suppression within the SM → Sensitivity to NP

- FCNCs forbidden at tree-level in the SM (radiative and rare decays: $b \to (s,d) \gamma$, $b \to (s,d) l^+ l^-$, $b \to s \nu \bar{\nu}$, $B_{d,s} \to l^+ l^-$, ...)
- CKM-, helicity-suppression (semileptonic CP-asymmetry: $A_{s,SL}^s$, t-dep. CP-asymmetries: $A_{CP}(B \to K^* \gamma)$)

Small hadronic uncertainties → Theoretically clean

- At most one hadron in the final state (leptonic and semileptonic decays: $B_{d,s} \to l^+ l^-$, $b \to (s,d) l^+ l^-$, $b \to s \nu \bar{\nu}$, ...)
- Smearing of bound-effects in the final state (Inclusive quantities: lifetimes, $\Delta M_q$, $\Delta \Gamma_q/\Gamma_q$, $A_q^{s,SL}$, $\phi_s$, ...)
- Suppression/cancellation of some hadronic uncertainties (clean dominant contributions, peculiar ratios/correlations: $A_{CP}(B \to J_\psi K_S)$, $\Delta M_s/\Delta M_d$, ...)
The LHC will study also the flavour structure of NP

LHCb stands for LHCbeauty

It is dedicated to the study of b-physics (all kinds of b-hadrons are produced)

@LHC (p-p collider with 7 TeV for each beam) a huge amount of b-\bar{b} couples (10^{12}/year) is produced but with a high background

First, very promising, results have been obtained
**Summer 2011 Hot Topics**
(EPS@Grenoble, LeptonPhoton@Mumbai)

\[ B_q \rightarrow \ell^+ \ell^- \]

- Highly sensitive to NP (loop FCNC: Z-penguin dominated)
- Theoretically clean (purely leptonic)
- \( F_{B_s} \) from Lattice QCD

\[ Br(B_s \rightarrow \ell^+ \ell^-) = \tau(B_s) \frac{G_F^2}{\pi} \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 \frac{F_{B_s}^2 m_{\ell}^2 m_{B_s}}{m_{B_s}^2} \left[ 1 - 4 \frac{m_{\ell}^2}{m_{B_s}^2} |V_{tb}^* V_{ts}|^2 Y^2(x_t) \right] \]

**New Summer 2011 results**

CDF (Tevatron) measures an excess

\[ Br( B_s \rightarrow \mu^+ \mu^-) = \left(1.8^{+1.1}_{-0.9}\right) \cdot 10^{-8} \]  
[CDF, 7 fb\(^{-1}\)]

But the LHC does not!

\( Br( B_s \rightarrow \mu^+ \mu^-) < 1.9 \cdot 10^{-8} \)  
[CMS, 1.14 fb\(^{-1}\)]

\( Br( B_s \rightarrow \mu^+ \mu^-) < 1.5 \cdot 10^{-8} \)  
[LHCb, 300 pb\(^{-1}\)]

CMS+LHCb

\[ Br( B_s \rightarrow \mu^+ \mu^-) < 1.1 \cdot 10^{-8} \]
\[ \text{BR}(B \to \tau \nu)_{\text{SM}} = (0.79 \pm 0.08) \times 10^{-4} \]
[UTfit, update of 0908.3470]
turns out to be smaller by \( \sim 2.4 \sigma \)
than the experimental value
\[ \text{BR}(B \to \tau \nu)_{\text{exp}} = (1.64 \pm 0.34) \times 10^{-4} \]
[Heavy Flavor Averaging Group]

\[
\text{BR}(B \to \tau \nu) = \frac{G_F^2 m_B m_{\tau}^2}{8 \pi} \left(1 - \frac{m_{\tau}^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B
\]

• \( \text{BR}(B \to \tau \nu)_{\text{exp}} \) prefers a large value for \( |V_{ub}| \) (close to the incl. determination)
• But a shift in the central value of \( |V_{ub}| \) would enhance the \( \beta \) tension

Important role of Lattice QCD
Results for the $B_s$ mixing amplitude:

Neutral mesons are not eigenstates of the Weak Interactions:

⇒ “particle-antiparticle oscillations”:

highly sensitive to NP

In 2009, CDF and D0 results for $\phi_{B_s}$

More than 2σ deviation from the SM!

Update before summer 2011

$$\Delta m_{q/K} = C_{B_q/\Delta m_{q/K}}^{SM}$$

$$A_{CP}^{B_s \rightarrow J/\psi \phi} \sim \sin 2(-\beta_s + \phi_{B_s})$$

=1 in SM

=0 in SM

$$C_{B_s} = 0.87 \pm 0.12$$

$$\phi_{B_s} = (-23 \pm 10)^\circ$$

SM at ~2.2σ
Still, the dimuon charge asymmetry (measured by D0) \( a_{\mu\mu} \) points to a large value of \( \phi_{Bs} \).

CDF (5.2 fb\(^{-1}\)) and D0 (8 fb\(^{-1}\)) measurements reduce the significance of the deviation to 0.8 \( \sigma \) and 1.0 \( \sigma \).

A combined measurement is in plan.

Further confirmations from experiments are looked forward!

LHCb has performed the world most precise measurement of \( \phi_{Bs} \).

It is found to be compatible to the SM.

LHCb Preliminary

\[ \phi_s = 0.03 \pm 0.16 \pm 0.07 \text{ rad} \]
The SuperB project has been approved!
(the Super KEKB project has been approved as well in Japan)

- The SuperB is an international project (~80 Institutions)
- It will be realized in Italy, (TorVergata area)
- The Technical-Design-Report will be ready in 2012
**SuperB**

- **$e^+e^-$ collider with the appropriate energy to produce couples of B and anti-B mesons, in a clean environment** (like BaBar and Belle, but with $\sim 100$ times higher luminosity)
- **it aims at improving the accuracy of the B-factories by a factor 5-10**
- **It will test the CKM matrix at 1% level**
- **It will increase the sensitivity for several channels sensitive to NP by one order of magnitude**

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**Role of B-factories in constraining the UT**

**Before B-factories**

**After B-factories**

**After SuperB-factories?**
It is crucial to have theoretical accuracy at the same level, in particular those of the hadronic parameters computed on the Lattice.

I am going to present our recent results for $m_b, f_B, f_{B_s}$.
Our Collaboration

Members from all over Europe:
Cyprus, France, Germany, Great Britain, Italy, Netherlands, Poland, Spain, Switzerland

Our choice for the Lattice action

Gauge Action: tree-level improved Symanzik ($N_f = 2$ dynamical fermions, degenerate in mass)

Fermionic Action: twisted mass (tm) at maximal twist

[R.Frezzotti, G.Rossi, hep-lat/0306014]

Important advantage of tm at maximal twist

($\omega = \pi/2 \leftrightarrow m_{\text{PCAC}} = 0$)

• Physical observables are automatically $O(a)$ improved
## Simulation details

- **Four values of the lattice spacing**
  
  \[
  \beta = \{3.80, 3.90, 4.05, 4.20\}
  \]

  \[
  a = \{0.098(3), 0.085(2), 0.067(2), 0.054(1)\} \text{ fm}
  \]

- **Large volumes**
  
  \[24^3 \times 48, \ 32^3 \times 64, \ 48^3 \times 96\]

- **Simulated sea and valence quark masses:**

  \[
  0.15 \cdot m_s^{\text{phys}} \leq \mu_l \leq 0.5 \cdot m_s^{\text{phys}}
  \]

  \[
  0.9 \cdot m_s^{\text{phys}} \leq \mu_s \leq 1.2 \cdot m_s^{\text{phys}}
  \]

  \[
  0.9 \cdot m_c^{\text{phys}} \leq \mu_h \leq 2.4 \cdot m_c^{\text{phys}}
  \]

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(a\mu_l)</th>
<th>(a\mu_s)</th>
<th>(a\mu_h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.80</td>
<td>0.0080, 0.0110</td>
<td>0.0165, 0.0200, 0.0250</td>
<td>0.2143, 0.2406, 0.2701, 0.3032</td>
</tr>
<tr>
<td>3.90</td>
<td>0.0030, 0.0040, 0.0064, 0.0085, 0.0100</td>
<td>0.0150, 0.0180, 0.0220</td>
<td>0.2049, 0.2300, 0.2582, 0.2898</td>
</tr>
<tr>
<td>4.05</td>
<td>0.0030, 0.0060, 0.0080</td>
<td>0.0135, 0.0150, 0.0180</td>
<td>0.1663, 0.1867, 0.2096, 0.2352</td>
</tr>
<tr>
<td>4.20</td>
<td>0.0020, 0.0065</td>
<td>0.0130, 0.0148, 0.0180</td>
<td>0.1477, 0.1699, 0.1954, 0.2247</td>
</tr>
</tbody>
</table>
Two methods for studying the heavy mass dependence

- B-physics on the lattice has the difficulty of large discretization effects of $O(a^* m_b)$

- The physical $b$-quark mass ($\approx 4$ GeV) cannot be directly simulated on present ($a^{-1} \leq 4$ GeV) lattices

- Several approaches have been investigated and used so far (HQET, NRQCD, step-scaling, ...)

We have followed two methods

**Ratio Method**: based on suitable ratios having an exactly known static limit

**Interpolation Method**: interpolating relativistic data in the charm region and HQET data
The Ratio Method

• Basic idea: known static limit

\[ \lim_{\mu_{h}^{\text{pole}} \to \infty} \left( \frac{M_{h\ell}}{\mu_{h}^{\text{pole}}} \right) = \text{constant} \]

• Consider several heavy quark masses: \( \bar{\mu}_{h}^{(1)}, \bar{\mu}_{h}^{(2)}, \ldots, \bar{\mu}_{h}^{(N)} \), with

\[ \frac{\bar{\mu}_{h}^{(n)}}{\bar{\mu}_{h}^{(m-1)}} = \lambda \]

• Build the ratios

\[
y(\bar{\mu}_{h}^{(n)}; \lambda, \bar{\mu}_{\ell}, a) \equiv \frac{M_{h\ell}(\bar{\mu}_{h}^{(n)}; \bar{\mu}_{\ell}, a)}{M_{h\ell}(\bar{\mu}_{h}^{(n-1)}; \bar{\mu}_{\ell}, a)} \cdot \frac{\rho(\bar{\mu}_{h}^{(n)}; \mu^{*})}{\rho(\bar{\mu}_{h}^{(n)}; \mu^{*})} = \lambda^{-1} \frac{M_{h\ell}(\bar{\mu}_{h}^{(n)}; \bar{\mu}_{\ell}, a)}{M_{h\ell}(\bar{\mu}_{h}^{(n-1)}; \bar{\mu}_{\ell}, a)} \cdot \frac{\rho(\bar{\mu}_{h}^{(n)}; \lambda, \mu^{*})}{\rho(\bar{\mu}_{h}^{(n)}; \mu^{*})}, \quad n = 2, \ldots, N
\]

where

\[ \lim_{\bar{\mu}_{h} \to \infty} y(\bar{\mu}_{h}, \lambda; \bar{\mu}_{\ell}, a = 0) = 1 \]

• Perform chiral and continuum extrapolation (smoother in ratios)

• Study the dependence on \( \mu_{h} \)

\[ y(\bar{\mu}_{h}) = 1 + \frac{\eta_{1}}{\bar{\mu}_{h}} + \frac{\eta_{2}}{\bar{\mu}_{h}^{2}} \]

and build the product of ratios:

\[
y(\bar{\mu}_{h}^{(2)}) y(\bar{\mu}_{h}^{(3)}) \ldots y(\bar{\mu}_{h}^{(K+1)}) = \lambda^{-K} \frac{M_{h\ell}(\bar{\mu}_{h}^{(K+1)})}{M_{h\ell}(\bar{\mu}_{h}^{(1)})} \cdot \left[ \frac{\rho(\bar{\mu}_{h}^{(1)}, \mu^{*})}{\rho(\bar{\mu}_{h}^{(K+1)}, \mu^{*})} \right]
\]

• Determine the physical b-quark mass, using the experimental B-meson mass in input:

\[ \bar{\mu}_{b} = \lambda^{K_{b}} \bar{\mu}_{h}^{(1)} = 4.91(15) \text{ GeV} \]
Smooth chiral and continuum limit

Dependence on the heavy quark mass (static limit=1)

Figure 1: Light quark mass dependence of the meson mass $M_{ht}(\mu_h^{(1)})$ (left) and of the ratio $y(\mu_h^{(2)})$ (right) at the four values of the lattice spacing.

Figure 2: Heavy quark mass dependence of the ratio $y(\mu_h)$ extrapolated to the physical value of the light quark mass and to the continuum limit. The vertical line represents the value of the physical $b$ quark mass.
Similarly for the decay constants

- Known static limit

\[
\lim_{\mu_h \to \infty} \frac{f_{h\ell}}{\sqrt{\mu_h}} = \text{constant}
\]

- Build the ratios

\[
\begin{align*}
  z(\mu_h, \lambda; \bar{\mu}, a) & = \lambda^{1/2} \frac{f_{h\ell}(\mu_h, \bar{\mu}, a)}{f_{h\ell}(\mu_h/\lambda, \bar{\mu}, a)} \cdot \frac{C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h/\lambda)}{C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h)} \cdot \frac{[\rho(\mu_h, \mu^*)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*)]^{1/2}} \\
  z_s(\mu_h, \lambda; \bar{\mu}_s, a) & = \lambda^{1/2} \frac{f_{hs}(\mu_h, \bar{\mu}_{s}, \bar{\mu}_s, a)}{f_{hs}(\mu_h/\lambda, \bar{\mu}_{s}, \bar{\mu}_s, a)} \cdot \frac{C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h/\lambda)}{C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h)} \cdot \frac{[\rho(\mu_h, \mu^*)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*)]^{1/2}}
\end{align*}
\]

where

\[
\Phi_{hs}(\mu_b^*) = [C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h)]^{-1} \cdot \Phi_{QCD}^{\text{QCD}}(\bar{\mu}_h)
\]

- Perform **chiral** (HMChPT or polynomial) and **continuum extrapolation** (of \( z_s \) and \( z_s/z \))

- Study the **dependence on** \( \mu_h \) (exploiting the known static limit)

- Determine \( f_B \) and \( f_{Bs} \) by inserting the previously determined \( m_b \) value
Smooth chiral and continuum limit

Figure 3: Light quark mass dependence of the decay constant $f_{hs}(\bar{\mu}_h^{(1)})$ (left) and of the ratio $z_s(\bar{\mu}_h^{(2)})$ (right) at the four values of the lattice spacing.

Figure 4: Light quark mass dependence of the ratio of decay constants $f_{hs}(\bar{\mu}_h^{(1)}) / f_{hl}(\bar{\mu}_h^{(1)})$ (left) and of the double ratio $z_s(\bar{\mu}_h^{(2)}) / z(\bar{\mu}_h^{(2)})$ (right) at the four values of the lattice spacing.
Figure 5: Heavy quark mass dependence of the ratio $z_s(\bar{\mu}_h)$ (left) and of the double ratio $z_s(\bar{\mu}_h)/z(\bar{\mu}_h)$ (right) extrapolated to the physical value of the light and strange quark masses and to the continuum limit. The vertical line represents the value of the physical $b$ quark mass.
The Interpolation Method:
Interpolation between relativistic data in the charm region and HQET data

Basic quantities for determining $f_{B_s}$ and $f_{B_s}/f_B$:

\[
\Phi_{hs} = f_{hs} \sqrt{M_{hs}} \quad \text{and} \quad \frac{\Phi_{hs}}{\Phi_{h\ell}} = \frac{f_{hs}}{f_{h\ell}} \sqrt{\frac{M_{hs}}{M_{h\ell}}}
\]

Static data have been obtained with:

- HYP2 static action
- A subset of the configuration ensembles ($\beta=3.9, 4.05$)
- Perturbative estimate of the RCs ($Z_P^{\text{stat}}$ and $Z_S^{\text{stat}}$)

Chiral and continuum limit of static data (small discretization effects)

Chiral and continuum limit of relativistic data is based on the same lattice data and uses the same predictions as in the ratio method.
Dependence on the heavy quark mass:
Interpolation between the charm region and the static point, with relativistic data matched to HQET according to

$$\Phi_{hs}(\mu_b^*) = \left[ C_A \sigma_{(\mu_b^*, \mu_h)} \right]^{-1} \cdot \Phi_{hs}^{QCD}(\mu_h)$$

Figure 8: Dependence of $\Phi_{hs}$ (left) and $\Phi_{hs}/\Phi_{h\ell}$ (right), in the chiral and continuum limit, on the inverse of the heavy quark mass.
The results of two chiral fits are averaged. The systematic uncertainty includes the errors due to:

- Chiral extrapolation (comparing two different fits)
- Continuum limit (excluding data at $\beta=3.8$)
- Heavy mass dependence (including two larger masses, varying the fit ansatz)
- Pole mass definition (using the LO definition instead of NLO)

<table>
<thead>
<tr>
<th>Ratio Method</th>
<th>Interpol. Method</th>
<th>$f_{Bs}/f_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lin.</td>
<td>Lin.</td>
<td>HMChPT</td>
</tr>
<tr>
<td>Quad.</td>
<td>Quad.</td>
<td>Polyn.</td>
</tr>
</tbody>
</table>

$m_{\overline{b}}(m_{\overline{b}}) = 4.29(14) \text{ GeV}$,

$f_B = 195(12) \text{ MeV}$, $f_{Bs} = 232(10) \text{ MeV}$, $\frac{f_{Bs}}{f_B} = 1.19(5)$
Comparison with recent \((N_f=2, 2+1)\) results (Lattice 2011)

Our \(N_f=2\) results

\[
\begin{align*}
\langle m_b \rangle &= 4.29(14) \text{ GeV}, \\
 f_B &= 195(12) \text{ MeV}, \\
 f_{B_s} &= 232(10) \text{ MeV}, \\
 \frac{f_{B_s}}{f_B} &= 1.19(5)
\end{align*}
\]

are well compatible with recent \(N_f=2, 2+1\) results

<table>
<thead>
<tr>
<th>(N_f=2+1)</th>
<th>(m_b) (GeV)</th>
<th>(f_B) (MeV)</th>
<th>(f_{B_s}) (MeV)</th>
<th>(f_{B_s}/f_B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPQCD10</td>
<td>4.16(2)</td>
<td>191(9)</td>
<td>226(10)</td>
<td>1.18(2)</td>
</tr>
<tr>
<td>FNAL/MILC11</td>
<td>197(8)</td>
<td>242(9)</td>
<td>1.23(3)</td>
<td></td>
</tr>
<tr>
<td>ALPHA11</td>
<td>4.23(10)</td>
<td>172(12)</td>
<td></td>
<td></td>
</tr>
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Simulations with $N_f=2+1+1$ dynamical flavors are being performed by ETMC.

In the next future we will perform $N_f=2+1+1$ analyses, in order to improve the determination of flavor observables, in particular:

• $m_b$, $f_B$, $f_{Bs}$

• B-parameters entering $B$-$\bar{B}$ mixing

• Semileptonic form factors required for $V_{ub}$ and $V_{cb}$

• ...
BACKUP
Tests/comparison of $m_b$

av. inc. contnm results
CD+M.Steinhauser 2011

$4.16(2) \text{ GeV}$

- $u, d, s$ sea
  - $4.164(23)$
  - $a^2$ dominates
  - HPQCD HISQ
  - 1004.4285

- $u, d$ sea
  - $4.234(95)$
  - stats dominates
  - ALPHEA LAT11
  - Fritzsch(Fri)

- $m_h$ fit dominates
  - ETMC 1107.1441

C. Davies@Lattice2011
$f_{B_s}, f_B$ comparison

$f_B$ average: $194(7)$ MeV (inc. corrs)
down from LAT10

PDG av BR($B \rightarrow \tau \nu$) + PDG av $V_{ub}$

$2.4\sigma$
apart for $f_{B_s}$

HPQCD NRQCD
LAT11 Shigemitsu (Mon)

HPQCD HISQ
prelim.

FNAL/MILC LAT11
Neil (poster)

ETMC 1107.1441

ALPHA LAT11
Fritzsch (Fri)
static +1/M
cont. + chiral extrap
$a:0.075,0.065,0.048$ fm

NOTE:
$f_{B_s} < f_{D_s}$ now quite clear
$f_{B_s}/f_B$ comparison

expect 2% higher than $f_{D_s}/f_D$ for matching ratios

HPQCD NRQCD
LAT11
Shigemitsu (Mon)

FNAL/MILC LAT11
Neil (poster)

RBC/UKQCD
static h; domain-wall l;
1001.2023 a=0.11fm

ETMC 1107.1441

improved results soon from 2+1+1 configs at physical $m_l$?
A look at the future

by Vittorio Lubicz

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<thead>
<tr>
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<tbody>
<tr>
<td>$f^+_K(0)$</td>
<td>0.9%</td>
<td>0.5%</td>
<td>0.7%</td>
<td>0.4%</td>
<td>&lt; 0.1%</td>
</tr>
<tr>
<td>$\hat{B}_K$</td>
<td>11%</td>
<td>5%</td>
<td>5%</td>
<td>3%</td>
<td>1%</td>
</tr>
<tr>
<td>$f_B$</td>
<td>14%</td>
<td>5%</td>
<td>3.5 - 4.5%</td>
<td>2.5 - 4.0%</td>
<td>1 - 1.5%</td>
</tr>
<tr>
<td>$f_{B_s}B_{B_s}^{1/2}$</td>
<td>13%</td>
<td>5%</td>
<td>4 - 5%</td>
<td>3 - 4%</td>
<td>1 - 1.5%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>5%</td>
<td>2%</td>
<td>3%</td>
<td>1.5 - 2%</td>
<td>0.5 - 0.8%</td>
</tr>
<tr>
<td>$\mathcal{F}_{B \to D/D^*}$</td>
<td>4%</td>
<td>2%</td>
<td>2%</td>
<td>1.2%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$f_{B_s}^{*}$</td>
<td>11%</td>
<td>11%</td>
<td>5.5 - 6.5%</td>
<td>4 - 5%</td>
<td>2 - 3%</td>
</tr>
<tr>
<td>$T_1^{B \to K^*/\rho}$</td>
<td>13%</td>
<td>13%</td>
<td>----</td>
<td>----</td>
<td>3 - 4%</td>
</tr>
</tbody>
</table>
B-mesons decay constants $f_B, f_{Bs}$ and $B\bar{B}$ mixing, $B_{Bd/s}$

$N_f=2$ $\xi = 1.243 \pm 0.028$ $2\%$

$N_f=2+1$ $f_{Bs}/f_B = 1.231 \pm 0.027$ $2\%$

Combining with the only modern calculation HPQCD [0902.1815]:

$\hat{B}_{Bd} = 1.26 \pm 0.11, \hat{B}_{Bs} = 1.33 \pm 0.06$

$f_{Bs} = 238.8 \pm 9.5$ MeV

$f_B = 192.8 \pm 9.9$ MeV

$\sqrt{\hat{B}_{Bs}} = 275 \pm 13$ MeV $5\%$

$\xi = 1.243 \pm 0.028$ $2\%$
**Exclusive vs Inclusive $V_{ub}$**

**THEORETICALLY CLEAN**
but more lattice calculations are certainly desired

\[ |V_{ub}|_{\text{excl.}} = (35.0 \pm 4.0) \times 10^{-4} \]

\[ |V_{ub}|_{\text{incl.}} = (42.0 \pm 1.5 \pm 5.0) \times 10^{-4} \]

**IMPORTANT LONG DISTANCE CONTRIBUTIONS.**
The results have some model dependence

\[ |V_{ub}|_{\text{SM-Fit}} = (35.5 \pm 1.4) \times 10^{-4} \]
$$S_F = \int d^4x \bar{\psi} \left[ D + m_\varrho + i\mu \gamma_5 \tau_3 \right] \psi$$

- Dirac operator
- Twisted mass parameter
- Third Pauli matrix in flavour space (2 flavours)
\[\rho(\bar{\mu}_h, \mu^*) = \left[ 1 + \frac{16}{3} \cdot \frac{\alpha \overline{\text{MS}}(\bar{\mu}_h)}{4\pi} \right] \cdot \left( \frac{\alpha \overline{\text{MS}}(\bar{\mu}_h)}{\alpha \overline{\text{MS}}(\mu^*)} \right)^{12/(33 - 2N_f)} \cdot \left[ 1 + \left( \frac{2(4491 - 252N_f + 20N_f^2)}{3(33 - 2N_f)^2} \right) \frac{\alpha \overline{\text{MS}}(\bar{\mu}_h) - \alpha \overline{\text{MS}}(\mu^*)}{4\pi} \right],\]

\[C_A^{\text{stat}}(\mu_b^*, \bar{\mu}_h) = \left( \frac{\alpha \overline{\text{MS}}(\bar{\mu}_h)}{\alpha \overline{\text{MS}}(\mu_b^*)} \right)^{-\frac{6}{33 - 2N_f}} \cdot \left[ 1 - \left( \frac{-3951 + 300N_f + 60N_f^2 + (924 - 56N_f)\pi^2}{9(33 - 2N_f)^2} \right) \left( \frac{\alpha \overline{\text{MS}}(\bar{\mu}_h) - \alpha \overline{\text{MS}}(\mu_b^*)}{4\pi} \right) \right] \cdot \left[ 1 - \frac{8\alpha \overline{\text{MS}}(\bar{\mu}_h)}{3} \right],\] (17)