PHASE-ORDERING IN ONE DIMENSION

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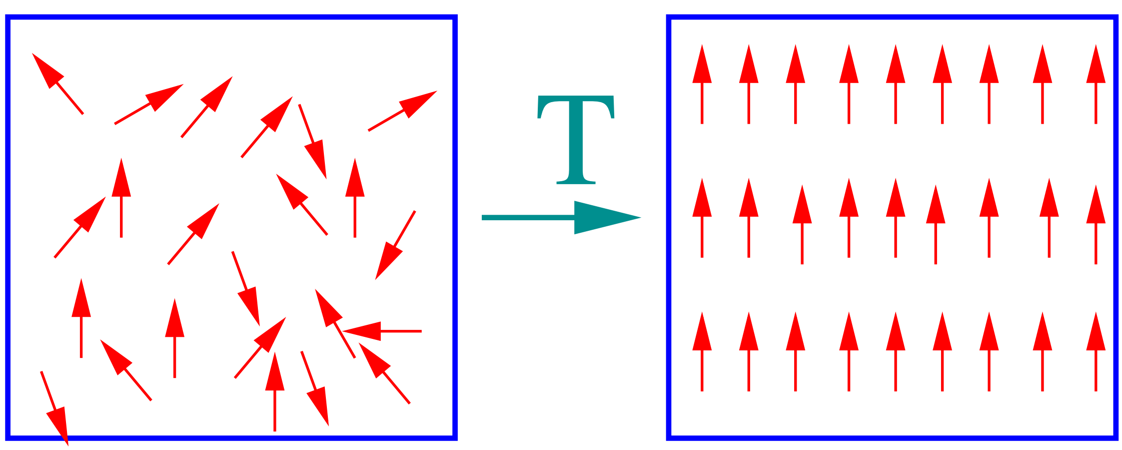
Phase-ordering in general

Statics

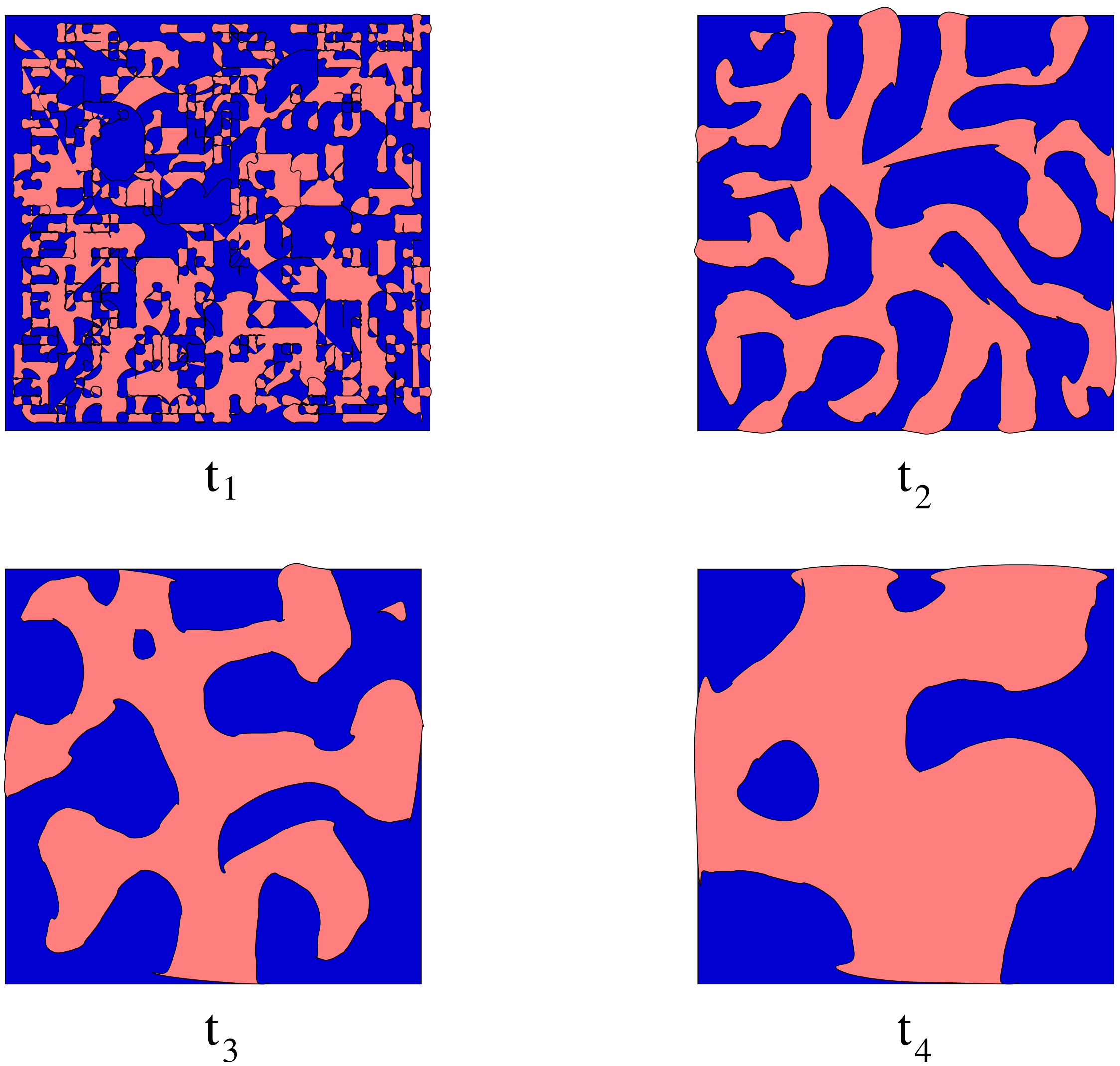
Classical system of N-component spins $\vec{\sigma}_i$ in a d-dimensional space, short range interactions (Ising, Heisenberg).



$T_c > 0$ if $d > d_l$

Dynamics

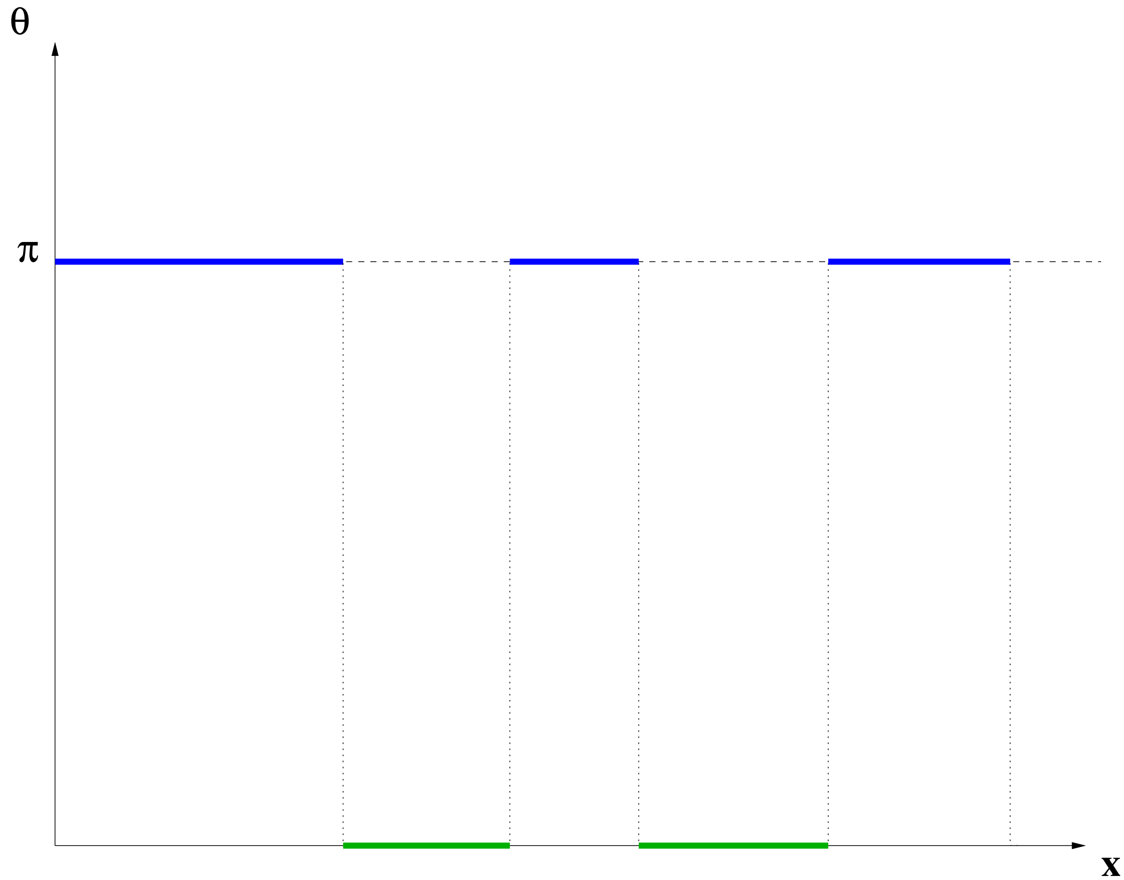
Formation and competition of ordered region (which grow and compete), and topological defects (which annihilate or shrink).



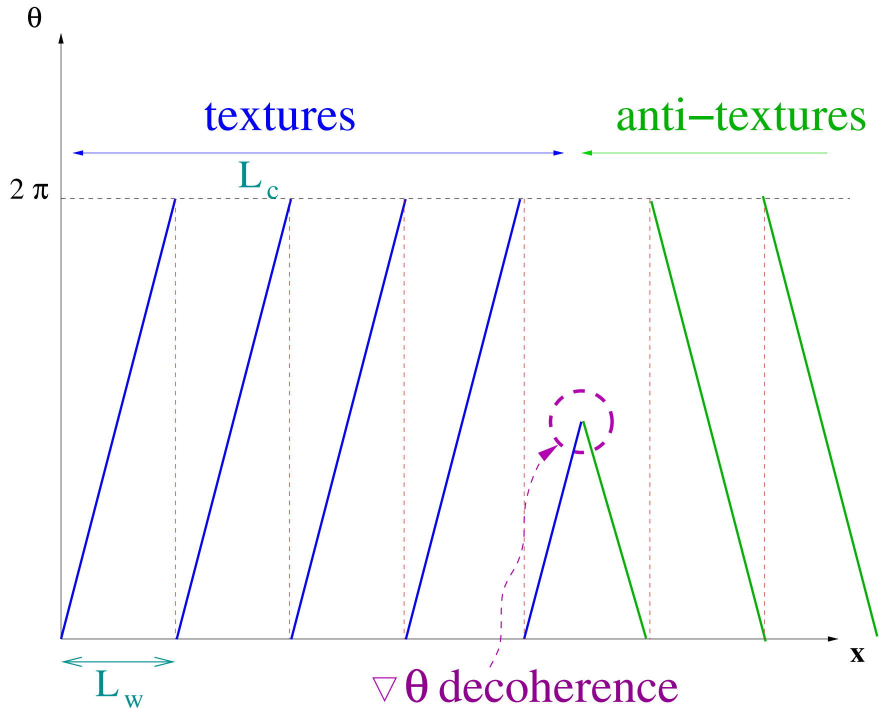
Typical size of ordered regions is $L(t)$. If Dynamical scaling holds ($G(r,t) = g(r/L(t))$ $L(t)$ is the only *relevant* scale, and usually $L(t) \sim t^{1/z}$. For NCOP usually $z = 2$ (short range interactions).

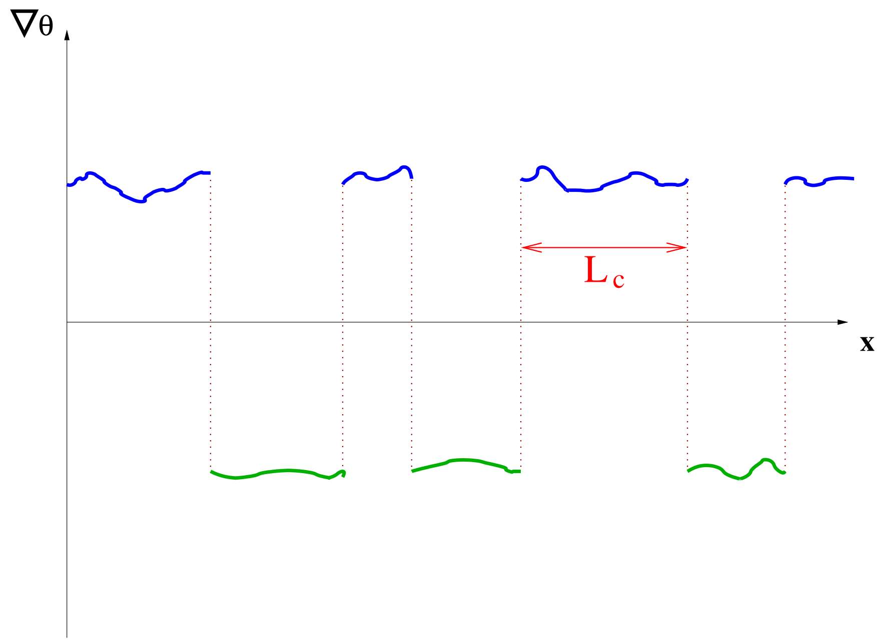
Typical example of this phenomenology is the Ising model in one dimension ($N = 1$, $d = 1$).

R. J. Glauber, J. Math. Phys. 4, 294 (1963).



Counterexample is the XY model, particularly in $d = 1$.



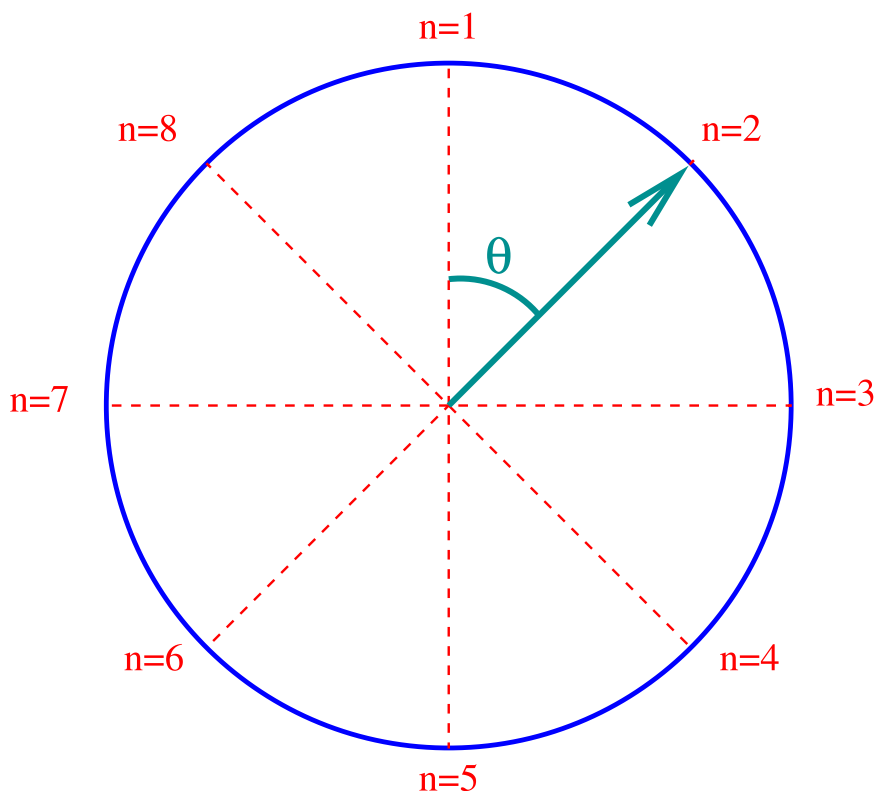


Two lengths, $L_w(t) \sim t^{1/4}$ and $L_c(t) \sim t^{1/2}$.

No dynamical scaling.

A. D. Rutenberg and A. J. Bray, PRL 74, 3836 (1995).

What happens between Ising and XY ?
(The p-state clock model (1d))



$$H[\sigma] = -J \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$$

For $p = 2$ is Ising, for $p = \infty$ is XY.

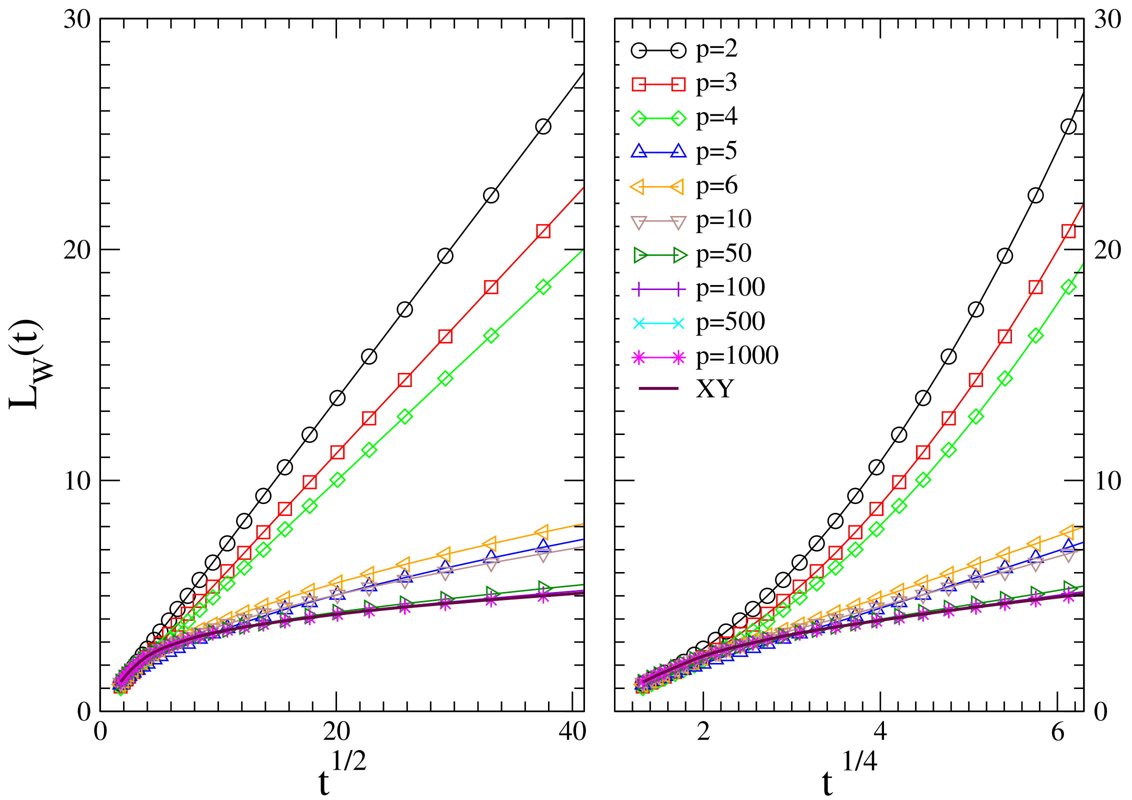
Generally, for arbitrary d, one expects the same dynamical exponents of the Ising model, and different (p-dependent) scaling functions. We will see that in $d = 1$ things are different.

F. Liu and G. F. Mazenko, Phys. Rev. B 47, 2866 (1993).

$T = 0$

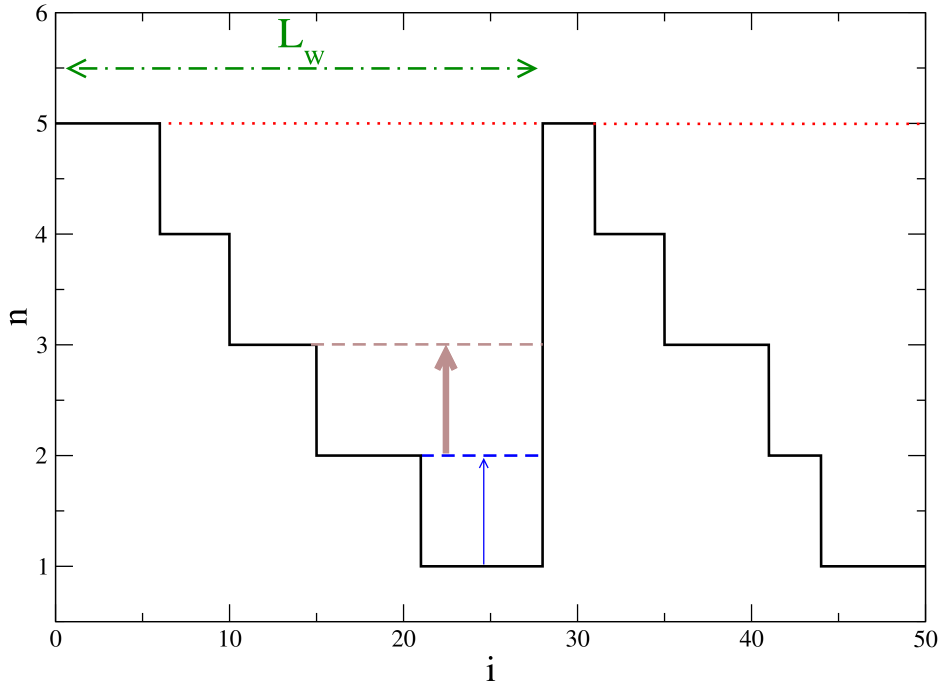
Simulations

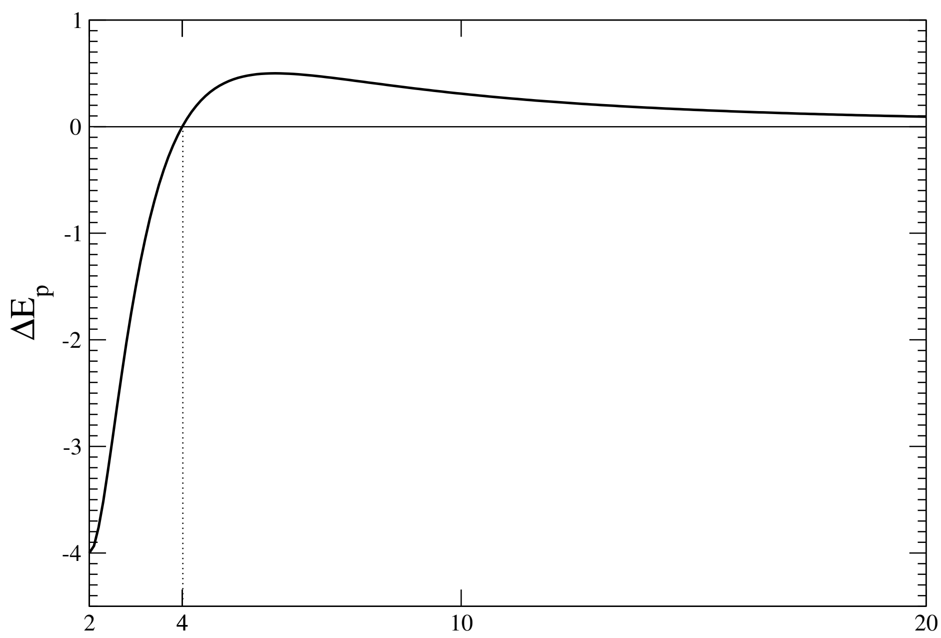
However, numerical simulations in $d = 1$, $T = 0$, give:



For $p > 4$ there is a length growing with $z = 4$, as for $p = \infty$.

Why ?





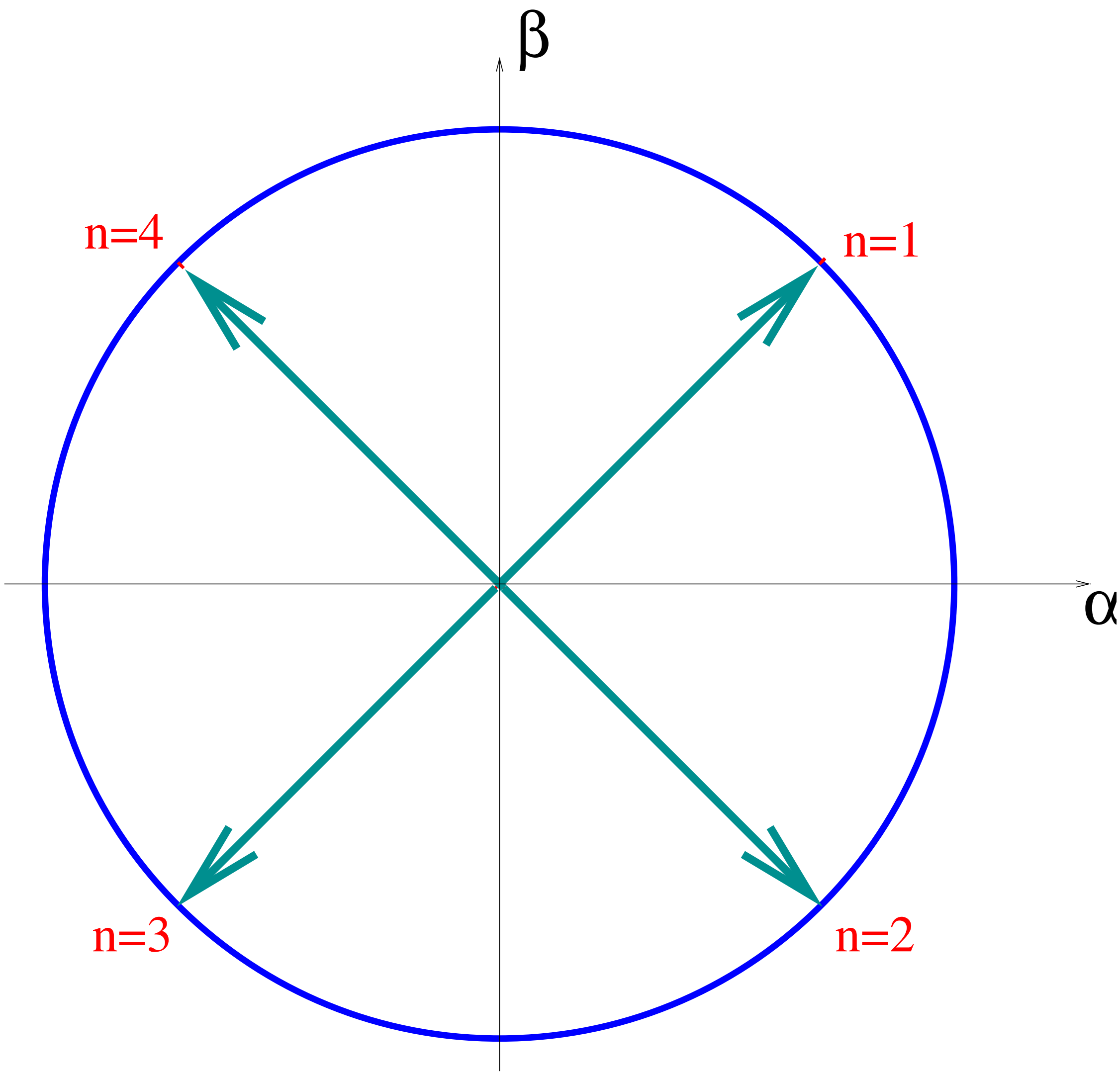
So there is $p_c = 4$ such that:

- for $p \leq p_c$ textures are destroyed and an Ising-like behavior sets in, with $z = 2$ and dynamical scaling
- for $p > p_c$ textures are stable and an XY-like behavior sets in, with violation of dynamical scaling

Can we say something more (for $p \leq p_c$) ?

For $p \leq p_c$ the model is equivalent to the Ising model.

- For $p = 4$ it is trivial



$$H[\sigma] = -J\sum_i \sigma_i^\alpha \sigma_{i+1}^\alpha - J\sum_i \sigma_i^\beta \sigma_{i+1}^\beta$$

Two non-interacting Ising models.

- For $p = 3$ it is more subtle.

Consider the correlation function $G(r,t) = \langle \vec{\sigma}_i(t) \cdot \vec{\sigma}_{i+r}(t) \rangle$ for instance. We can show that

$$G(r,t) = \frac{9}{2} G_P(r,t) - \frac{1}{2}$$

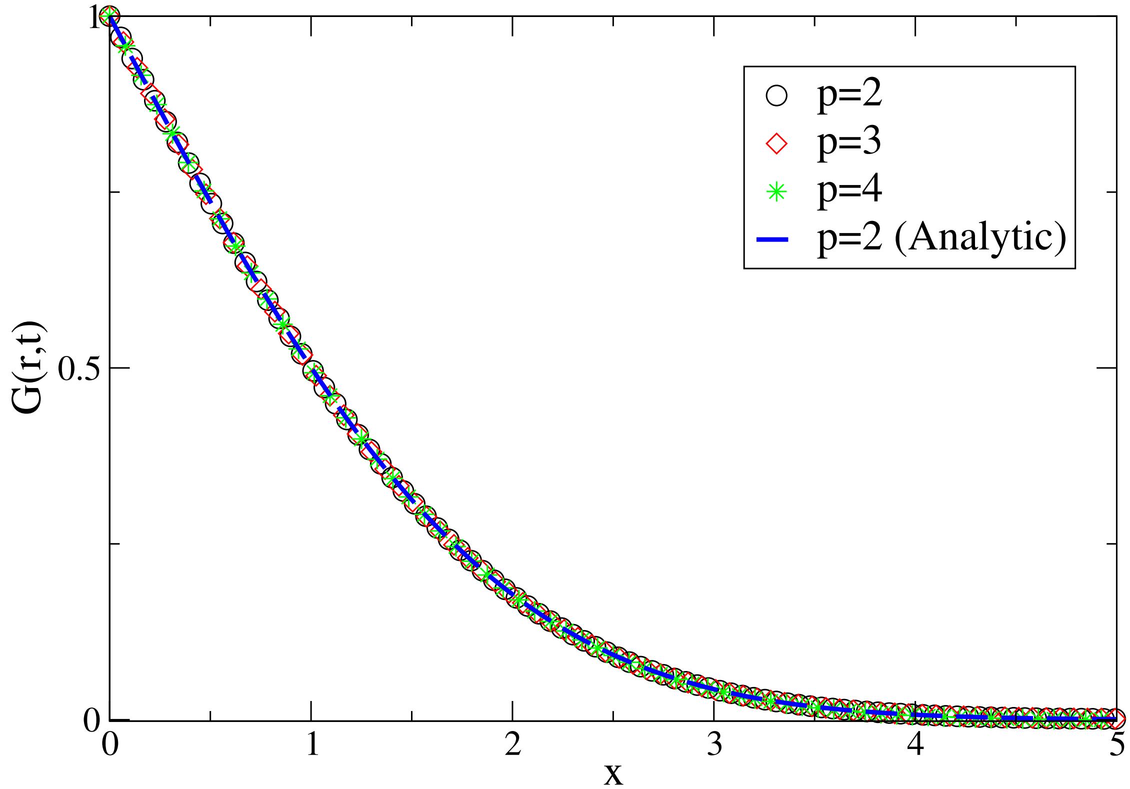
exactly, where $G_P(r,t)$ is the *single phase* correlation function of the 3-states 1d Potts model. This quantity was computed exacly by Sire and Majumdar yielding

C. Sire and S. N. Majumdar, PRL 74, 4321 (1995).

$$G_P(r,t) = \frac{2}{9} G_I(r,t) + \frac{1}{9}$$

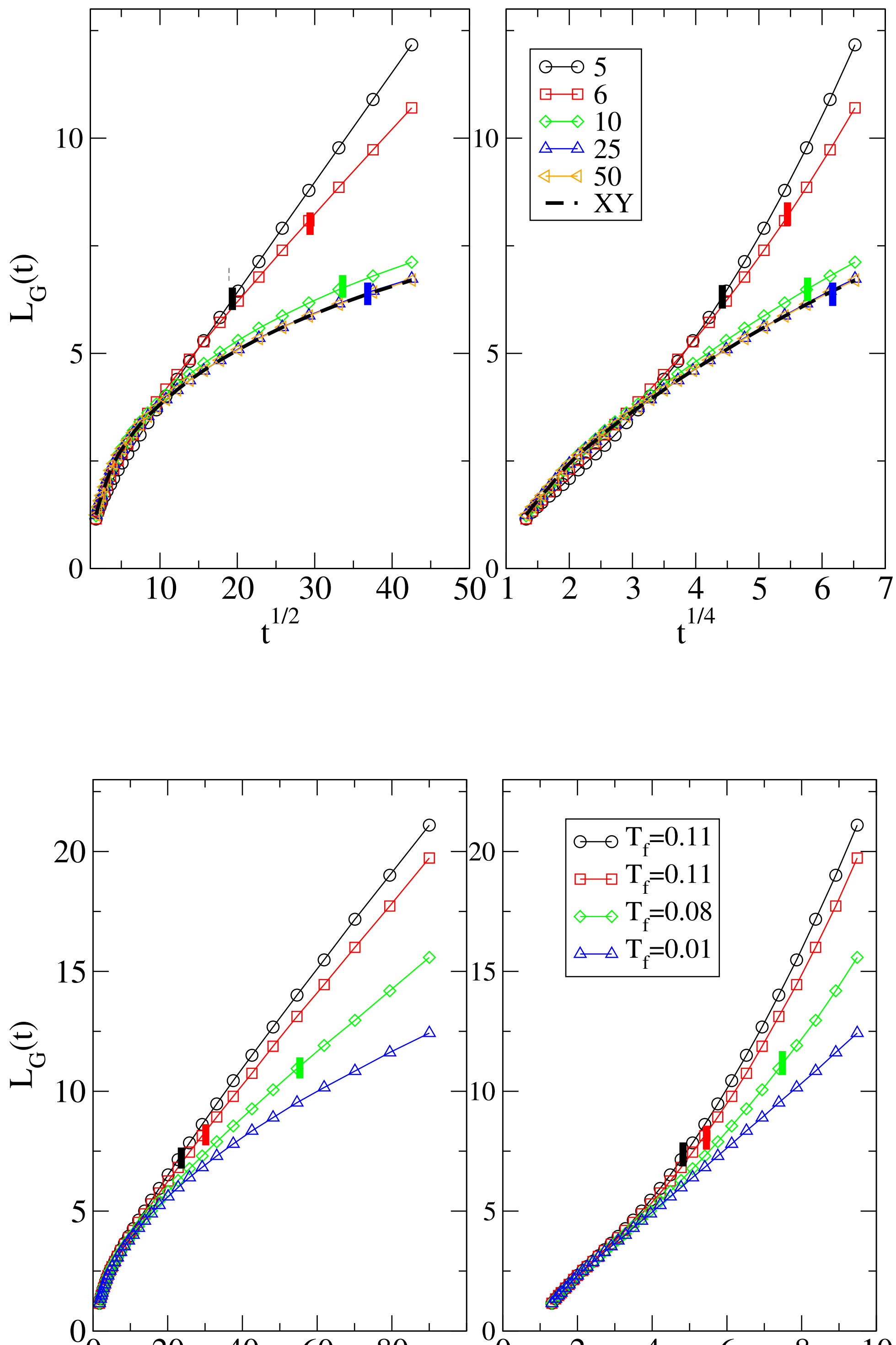
where $G_I(r,t)$ is the correlation function of the Ising model (with a global conservation law, which is irrelevant). Then

$$G(r,t) \equiv G_I(r,t).$$



p=2
p=3
p=4
p=2 (Analytic)
G(r,t)
x
0
0.5
1
0
1
2
3
4
5

$T > 0$ (before equilibration)



Conclusions

- Dynamics of the clock model in $d = 1$ is rich and different from the case $d > 1$ where same exponents are expected for all p with p-dependent scaling functions.

- Two radically different behaviors crossing $p_c = 4$.

- For $p \leq p_c$ the model is equivalent to the Ising model: Domains dynamics and scaling with the same exponents and same scaling functions.

- For $p > p_c$ a behavior similar to the XY model: Texture dynamics, two characteristic lengths growing with different exponents, violations of dynamical scaling.

- Crossover structure for $T > 0$.

- Possibly, something similar can happen in other systems with $N = d+1$ where extended defects without core analogous to textures are expected.