Large-N gauge theories

Mike Teper (Oxford) – SMFT 2006, Bari

- preamble : large N

- preamble : lattice

- Is SU($N \to \infty$) confining? Is it close to SU(3)?

- Physics of SU($N \to \infty$):
(glueball) mass spectrum ; k-strings ; deconfinement and high T ; topology ; θ-vacua ; running coupling ; chiral symmetry breaking ; strong-to-weak coupling transitions ; effective string theory ; ...

- D=3+1 $\longrightarrow$ D=2+1

some particular topics:

- deconfinement

properties of transition — Lucini, Wenger

Hagedorn transition ; high-T pressure — Bringoltz

't Hooft string tension — Bursa

- string tensions in D=2+1

Karabali-Nair ; effective string theory — Bringoltz

- (twisted) Eguchi-Kawai

very large N – reduced models — Vairinhos

- Wilson loop eigenvalue spectra

– remarkable universalities — Bursa

.....

- ‘Oxford’ group’

Bringoltz, Bursa, Liddle, Lucini, Meyer, Teper, Vairinhos, Wenger, ...

some other groups:

- ‘Pisa’

Del Debbio, Panagopoulos, Rossi, Vicari, ...
Campostrini, ...

- ‘Rutgers’

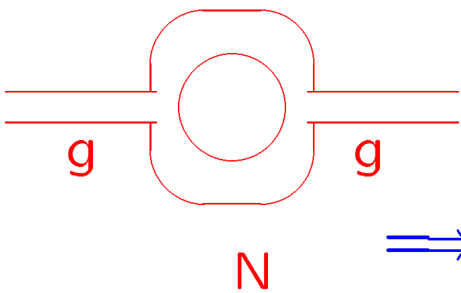
Narayanan, Neuberger, ...

- ‘Torino’

D’Adda, Caselle, Gliozzi, Hasenbusch, Panero, Rago, ...

Large N – Preamble

• QCD : no small expansion parameter
($g^2 \leftrightarrow$ scale)
$\stackrel{'tHooft}{\Longrightarrow}$ try SU(N) $\simeq$ SU(∞) + O($1/N^2$)



$\Longrightarrow$ g^2Nconstant at large N

• $N \rightarrow \infty$ colour singlet phenomenology

't Hooft, Witten, Manohar, ...

zero decay widths; no mixing; exact OZI, η'; SU(6) for baryons; ...

• no scattering of colour singlets – integrability?
but strongly interacting bound states

• factorisation colour singlet operators: e.g.
$\langle \Phi_1(x_1)\Phi_2(x_2)\rangle = \langle \Phi_1\rangle\langle \Phi_2\rangle \left\{1 + O\left(\frac{1}{N^2}\right)\right\}$
$\Longrightarrow$ Witten's Master Field $\rightarrow$ translation invariant $\rightarrow$
Eguchi-Kawai single point reduction

• Feynman diagrams on 2D surfaces – as $g^2N \rightarrow \infty$
continuous with vertices $\sim$ stringy sheets $\Longrightarrow$
$N = \infty$ gauge theory $\sim$ a string theory 't Hooft
$N = \infty$ gauge theory $\sim$ dual to a string theory
Maldacena

Lattice – Preamble

Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4

$x_\mu \bullet - \bullet\, x_\mu + \widehat{\mu}\delta x$: $A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

$x_\mu \bullet - - - \bullet\, x'_m u$: $P\left\{ e^{\int_x^{x'} A.dx} \right\} \in$ SU(N) group

$x_\mu \underset{\longrightarrow}{=} an_\mu$

$an_\mu \bullet - - - \bullet\, an_\mu + a\widehat{\mu}$: $U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

Lattice – Preamble

gauge transformation: $U_\mu(n) \rightarrow g(n) U_\mu(n) g^\dagger(n+\widehat{\mu})$

$\longrightarrow$

Tr $\prod_{l\in\partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$

$Z = \int \prod_l dU_l e^{-\beta S}$ where $S = \sum_p \left\{1 - \frac{1}{N}\text{ReTr}\, u_p\right\}$
where u_p is product links around the plaquette p.

$\longrightarrow$

$\int \prod_l dU_l e^{-\beta S} \quad \xrightarrow{a\to 0} \quad \int DA e^{-\frac{4}{g^2}\int d^4x Tr F_{\mu\nu} F_{\mu\nu}}$
with $\beta = \frac{2N}{g^2}$

and then numerical Monte carlo evaluation

Calculating masses from correlators:

$\Phi(t)$ a gauge invariant operator

$$\begin{aligned} C(t = an_t) &= \langle \Phi^\dagger(t = an_t)\Phi(0)\rangle \\ &= \langle \Phi^\dagger e^{-Han_t}\Phi\rangle \\ &= \sum_i |c_i|^2 e^{-aE_i n_t} \\ &\stackrel{t\to\infty}{\simeq} |c|^2 e^{-man_t} \end{aligned}$$

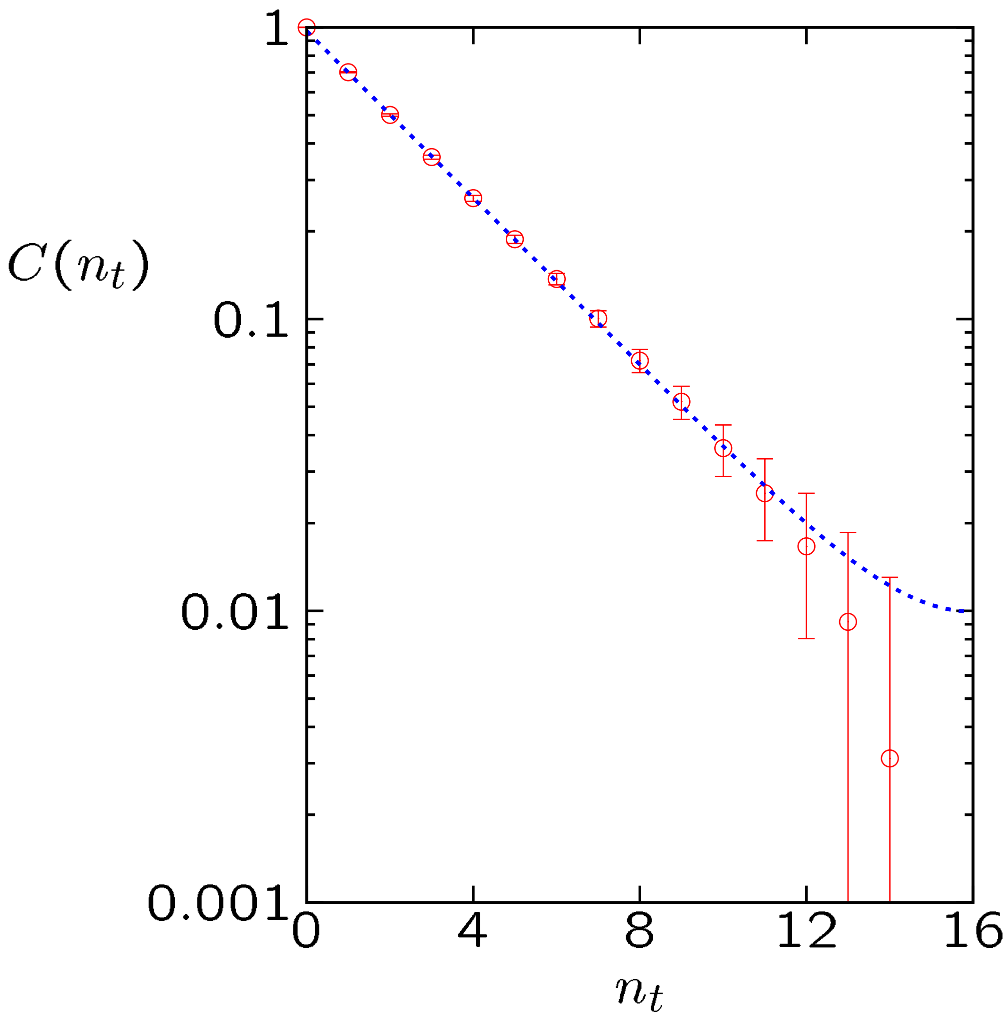
where m is lightest mass with quantum numbers of Φ

Continuum limit :

$$\frac{am_i(a)}{am_j(a)} = \frac{m_i(a)}{m_j(a)} = \frac{m_i(0)}{m_j(0)} + c_{ijk}a^2 m_k^2 + O(a^4)$$

Calculating masses from correlators

$$C(t = an_t) = \langle \Phi^\dagger(t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-E_i t} \overset{t\uparrow}{\simeq} |c|^2 e^{-man_t}$$



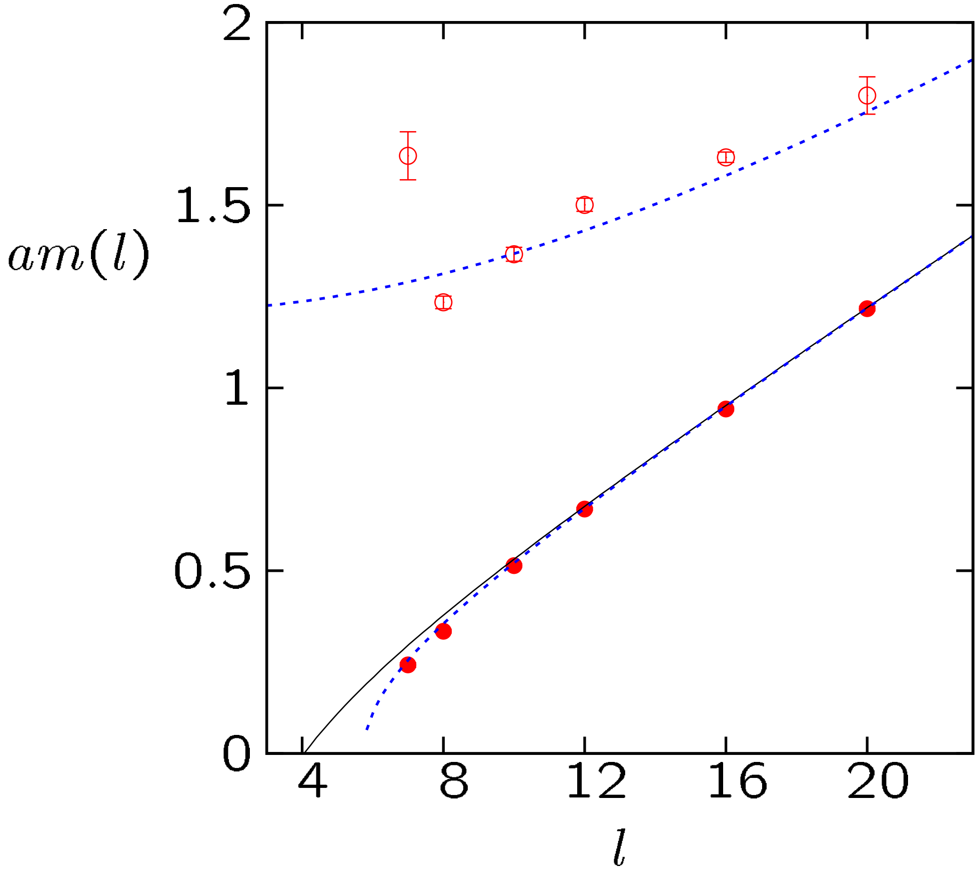
SU(3), 32^4, $a \simeq 0.046$ 'fm'

$\Longrightarrow$

fit : $am_{0^{++}} = 0.330(7)$

Linear confinement in SU(6)?

H. Meyer, M. Teper: hep-lat/0411039



linear + bosonic string correction (—):

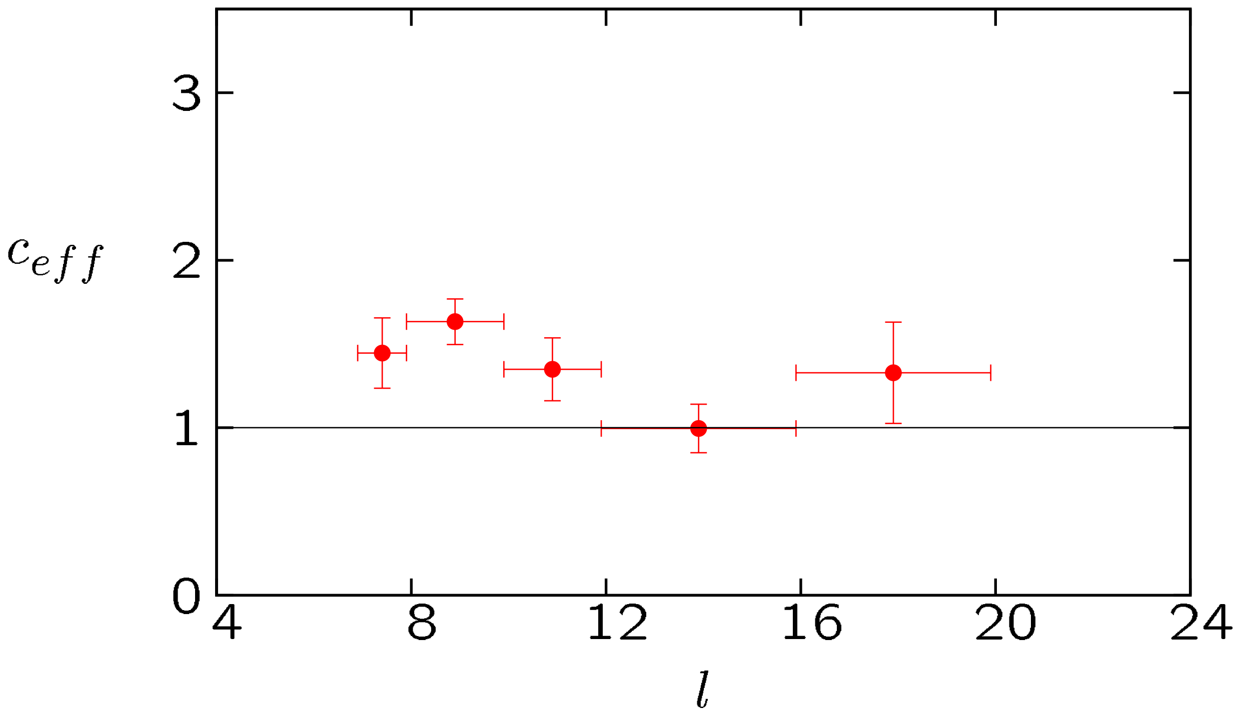
$$m(l) = \sigma l - \frac{\pi(D-2)}{6l}$$

Nambu-Goto string action ($\cdots$):

$$E_n(l) = \sigma l[1 + \frac{8\pi}{\sigma l^2}(n - \frac{D-2}{24})]^{\frac{1}{2}}$$

... bosonic string?

try : $m(l) = \sigma l - \frac{c_{eff}\pi}{3l}$

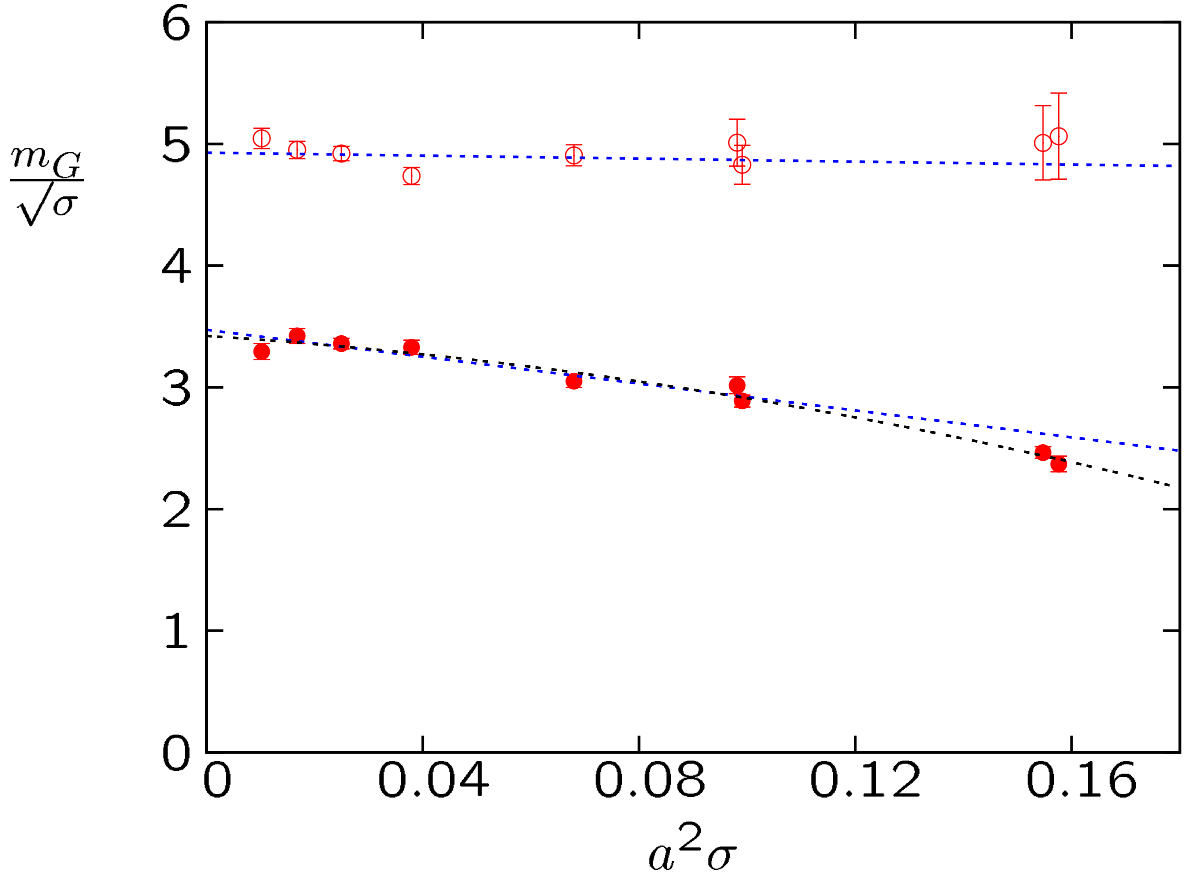


$$\Longrightarrow c_{eff} \overset{l\to\infty}{\longrightarrow} 1$$

a very good fit:

$$m(l) = \sigma l - \frac{\pi}{3l} - \frac{1.23(21)}{(\sigma l^2)l}$$

Continuum limit mass spectrum: SU(3)



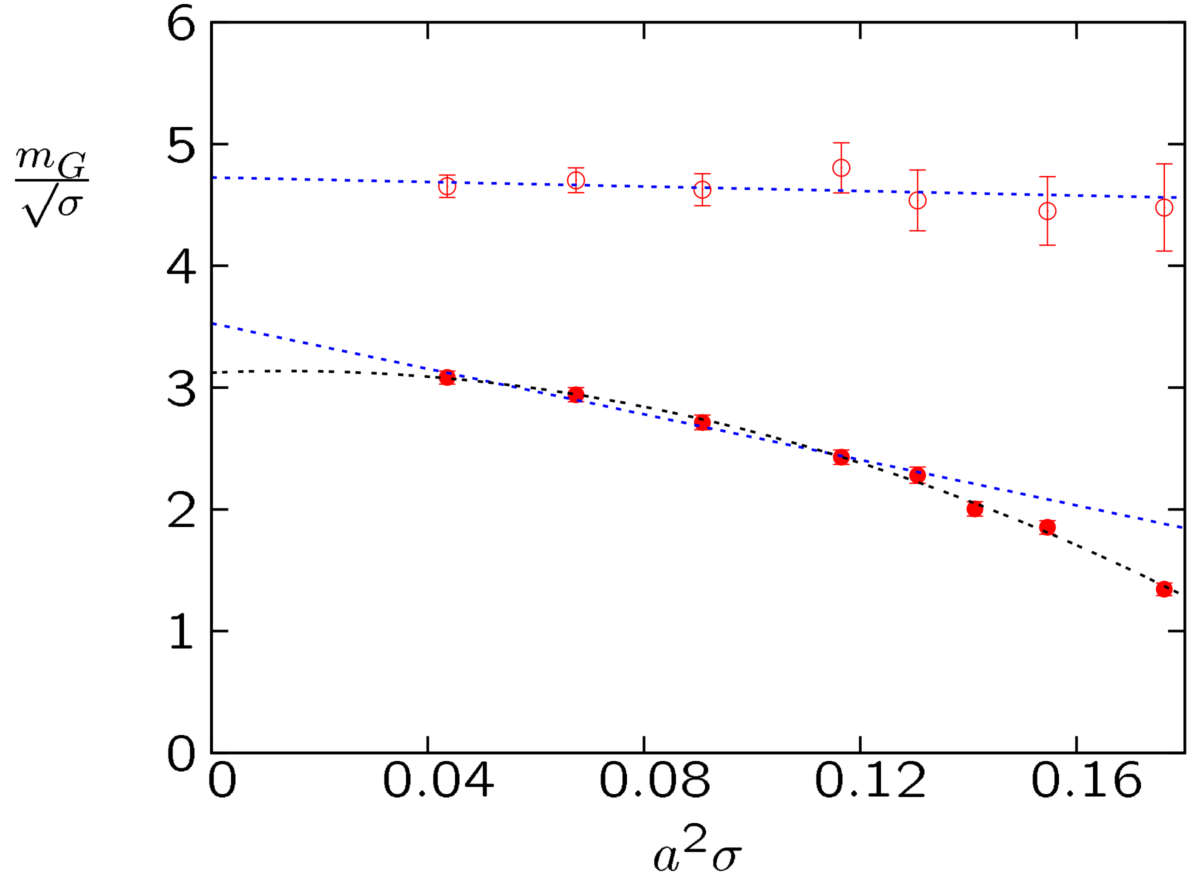
$O(a^2)$ continuum extrapolations:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Continuum limit mass spectrum: SU(8)



$O(a^2)$ continuum extrapolations:

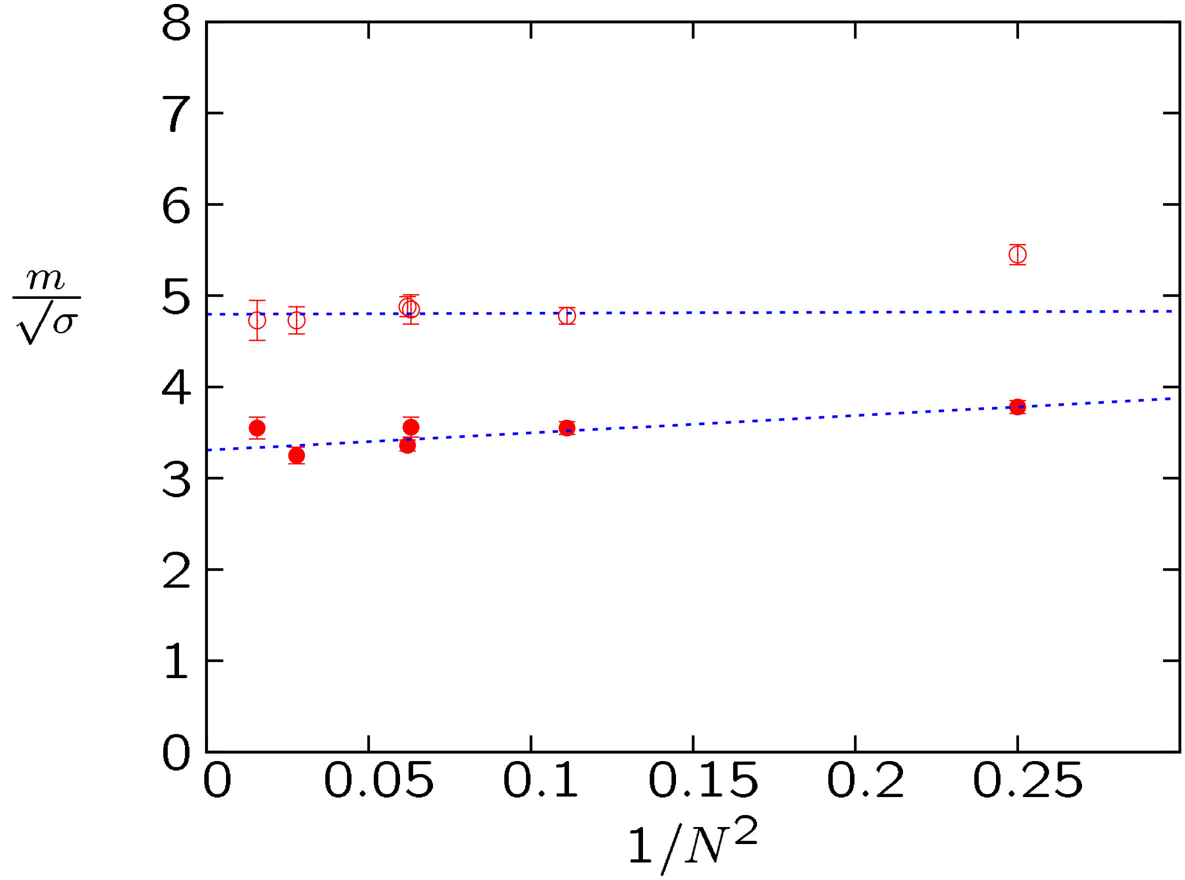
$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.53(8) - 9.3(1.0)a^2\sigma$$
$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.73(13) - 0.9(1.8)a^2\sigma$$

BUT $O(a^4)$ extrapolation

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.13(25) + 1.66a^2\sigma - 66.0(a^2\sigma)^2$$

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



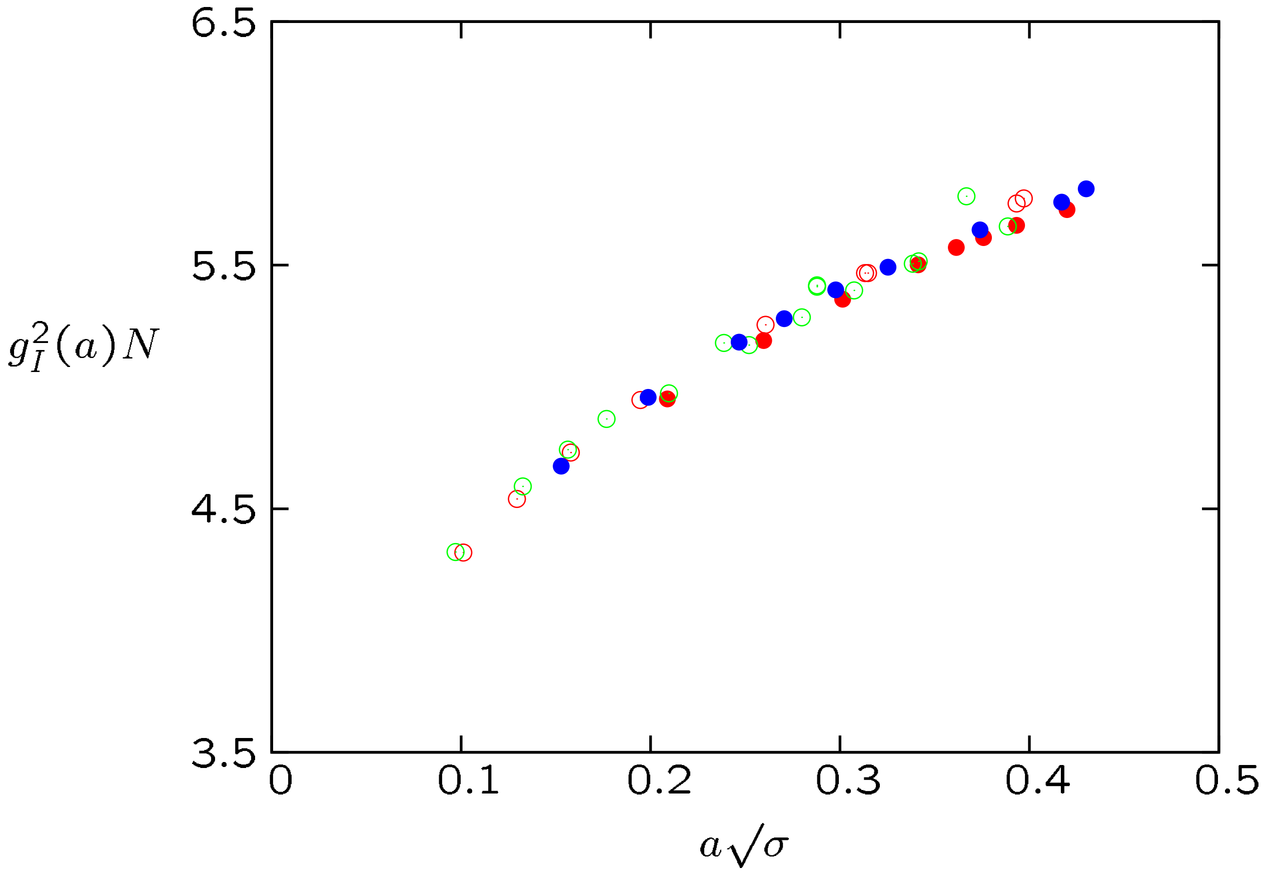
$O(1/N^2)$ extrapolations to $N = \infty$:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}}|_N = 3.31 + \frac{1.90}{N^2}$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}}|_N = 4.80 + \frac{0.11}{N^2}$$

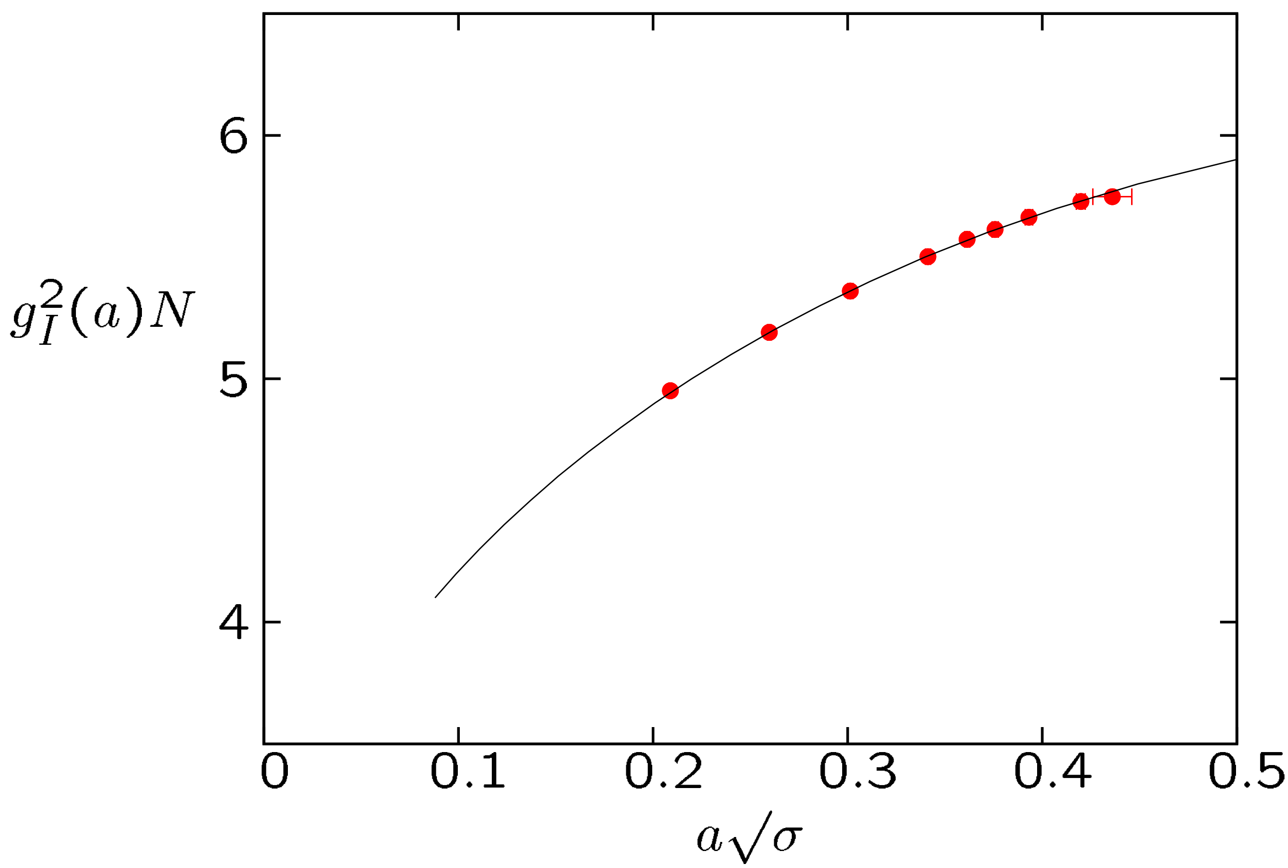
g^2N fixed as $N \to \infty$?

- $g^2(l)$ versus $\frac{l}{\xi}$ with $\xi = \frac{1}{\sqrt{\sigma}}$, $l = a$
- $\frac{1}{g_I^2(a)} = \frac{\beta}{2N^2} \times \langle u_p \rangle$



SU(2) ○ ; SU(3) ○; SU(4) ● ;
SU(6) ○ ; SU(8) ●

β–function : SU(8)

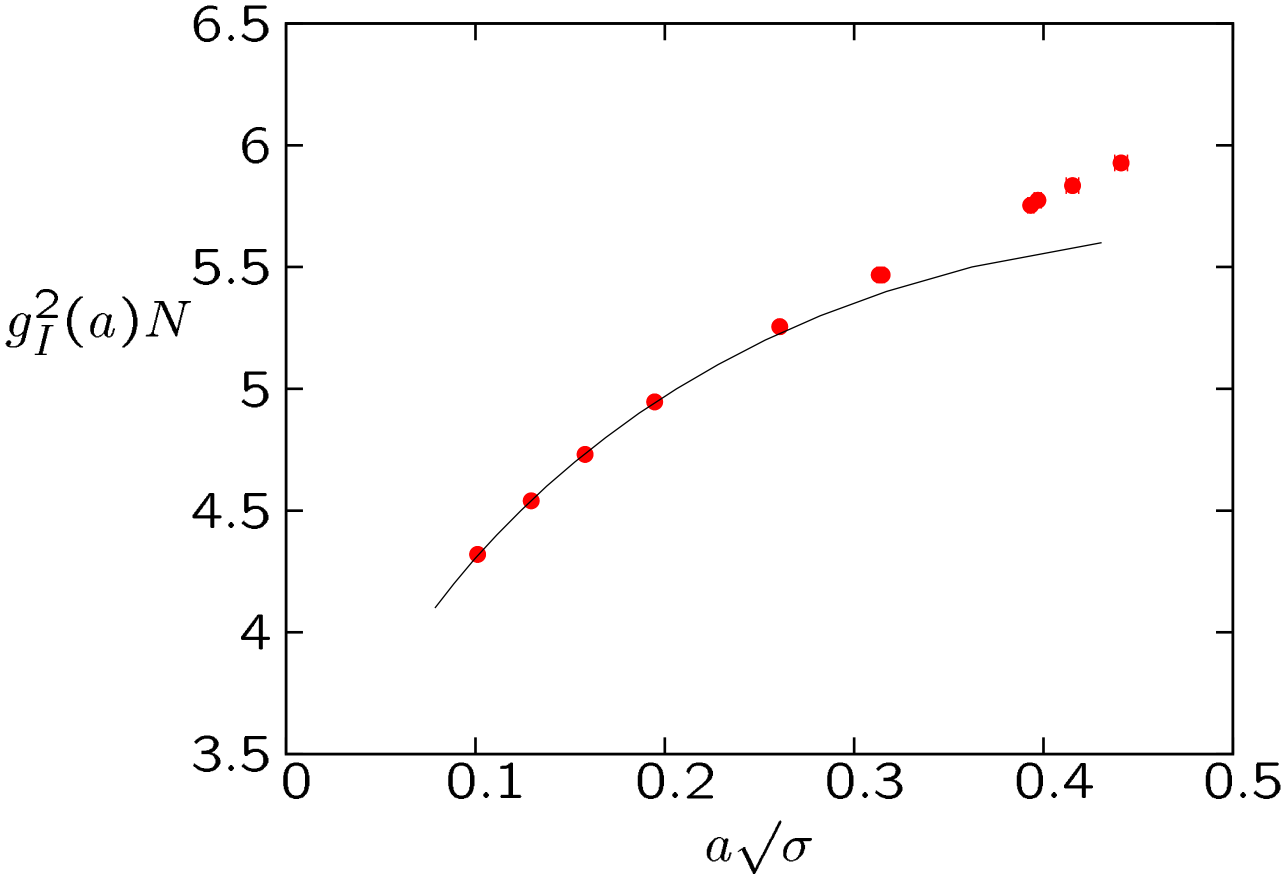


try 2-loop × leading lattice correction:

$$a\sqrt{\sigma} = \frac{\sqrt{\sigma}}{\Lambda_I}(1 + ca^2\sigma)\left(\frac{11}{48\pi^2}g_I^2 N\right)^{-\frac{51}{121}}\left(1 + \frac{17}{88\pi^2}g_I^2 N\right)^{-\frac{51}{121}} e^{-\frac{24\pi^2}{11}\frac{1}{g_I^2 N}}$$

now, $\Lambda_{\overline{MS}} = 2.633\Lambda_I \rightsquigarrow \frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.427(3)$

β–function : SU(3)

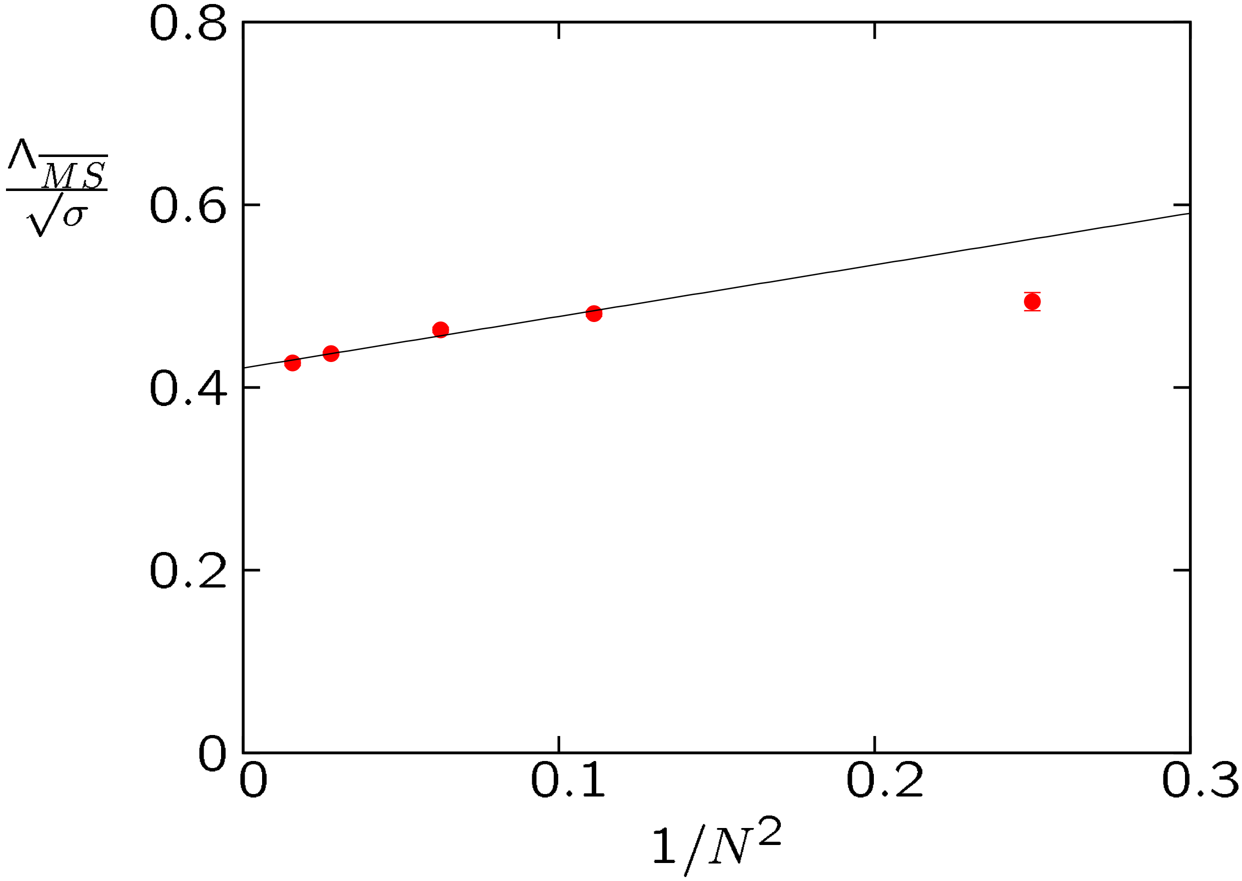


$\Rightarrow$

$$\Lambda_{\overline{MS}} = 0.481(4)\sqrt{\sigma} \simeq 215 \text{ 'MeV'}$$

warning : errors statistical – systematic errors much larger!!

$\Lambda_{\overline{MS}}$



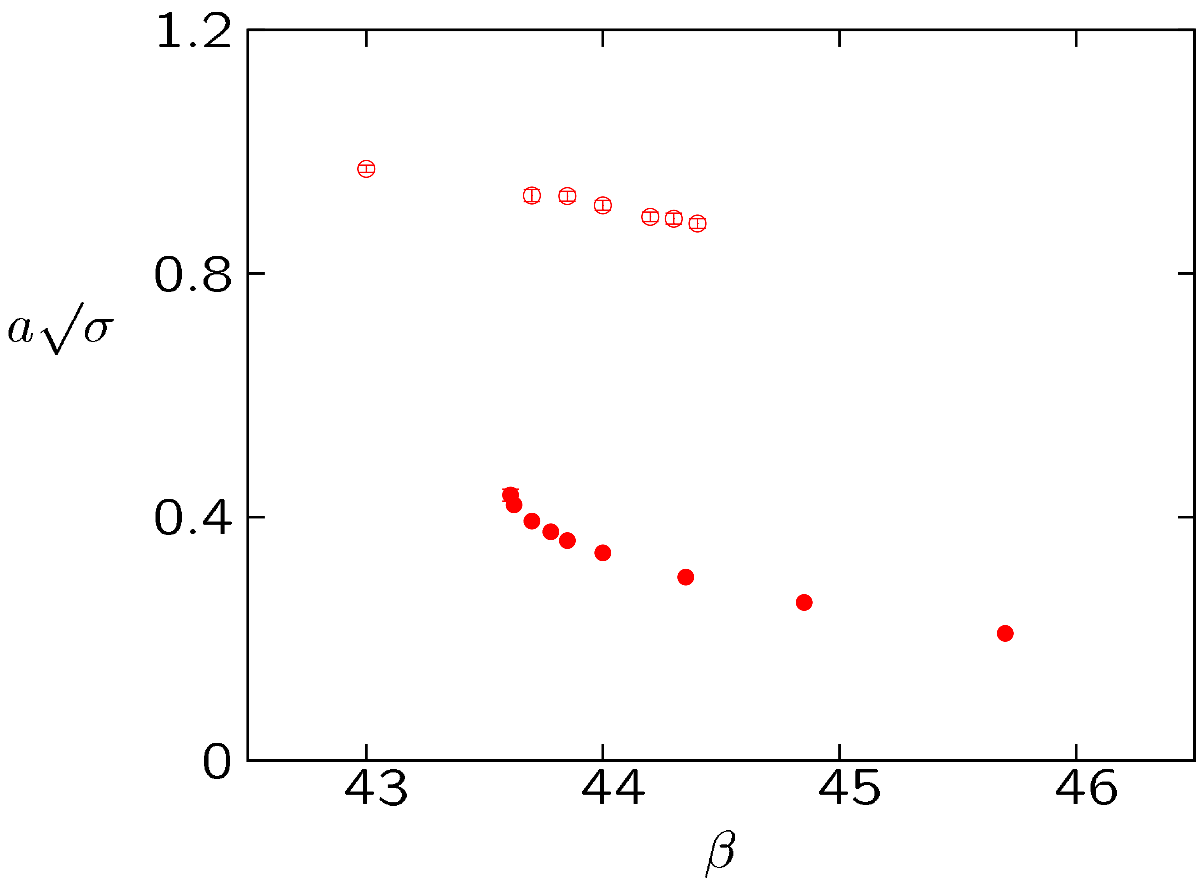
$\Rightarrow$

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.421 + \frac{0.565}{N^2}$$

$\leadsto$

$$\lim_{N\to\infty} \Lambda_{\overline{MS}} \simeq 190 \text{ `MeV'}$$

Strong–to–weak coupling phase transition
SU(8) in 3+1 dimensions



$\Longrightarrow$

1st order bulk transition ensures clean weak-coupling physics beyond a certain β

So :

- SU(3) $\sim$ SU(∞) for many quantities

- linear confinement persists at large N

- g^2N fixed gives smooth large-N limit – leading lattice correction + 2 loop continuum β-function describes whole of large-N weak-coupling phase very well

Motivated by the apparent phenomenological relevance of large-N, we delve deeper into the properties of this theory ...

k-strings

Lucini, Teper, Wenger: hep-lat/0404008

also Pisa group

Casimir scaling:

$$\frac{\sigma_k}{\sigma} = \frac{k(N-k)}{N-1}$$

'MQCD':

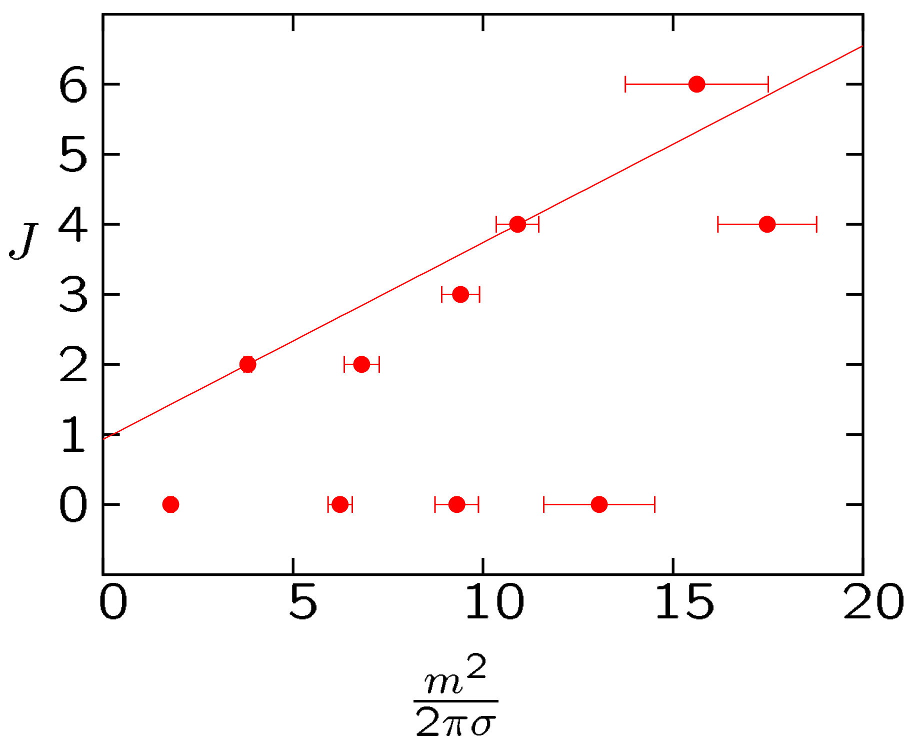
$$\frac{\sigma_k}{\sigma} = \frac{\sin \frac{k\pi}{N}}{\sin \frac{\pi}{N}}$$

σ_k/σ			
(N,k)	Casimir scaling	this paper	'MQCD'
(4,2)	1.333	1.370(20)	1.414
(4,2)	1.333	1.358(33)	1.414
(6,2)	1.600	1.675(31)	1.732
(6,3)	1.800	1.886(61)	2.000
(8,2)	1.714	1.779(51)	1.848
(8,3)	2.143	2.38(10)	2.414
(8,4)	2.286	2.69(17)	2.613

Pomeron: the leading glueball Regge trajectory?

H. Meyer, M. Teper: hep-th/0409183

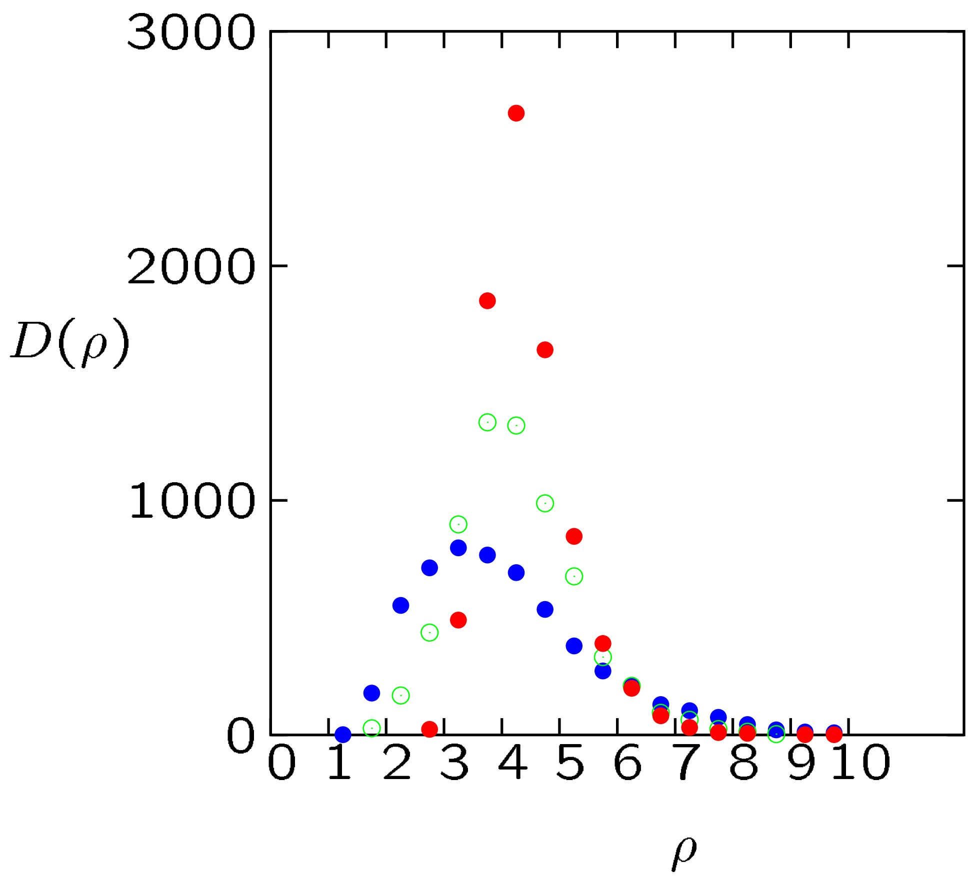
Chew-Frautschi plot: $PC = ++$ states in SU(3) gauge theory



$$\alpha(t) = 0.93(24) + 0.28\alpha'_R t$$

Instanton size density

$$D(\rho) \stackrel{N\to\infty}{\longrightarrow} \delta(\rho - \rho_c) \ ?$$



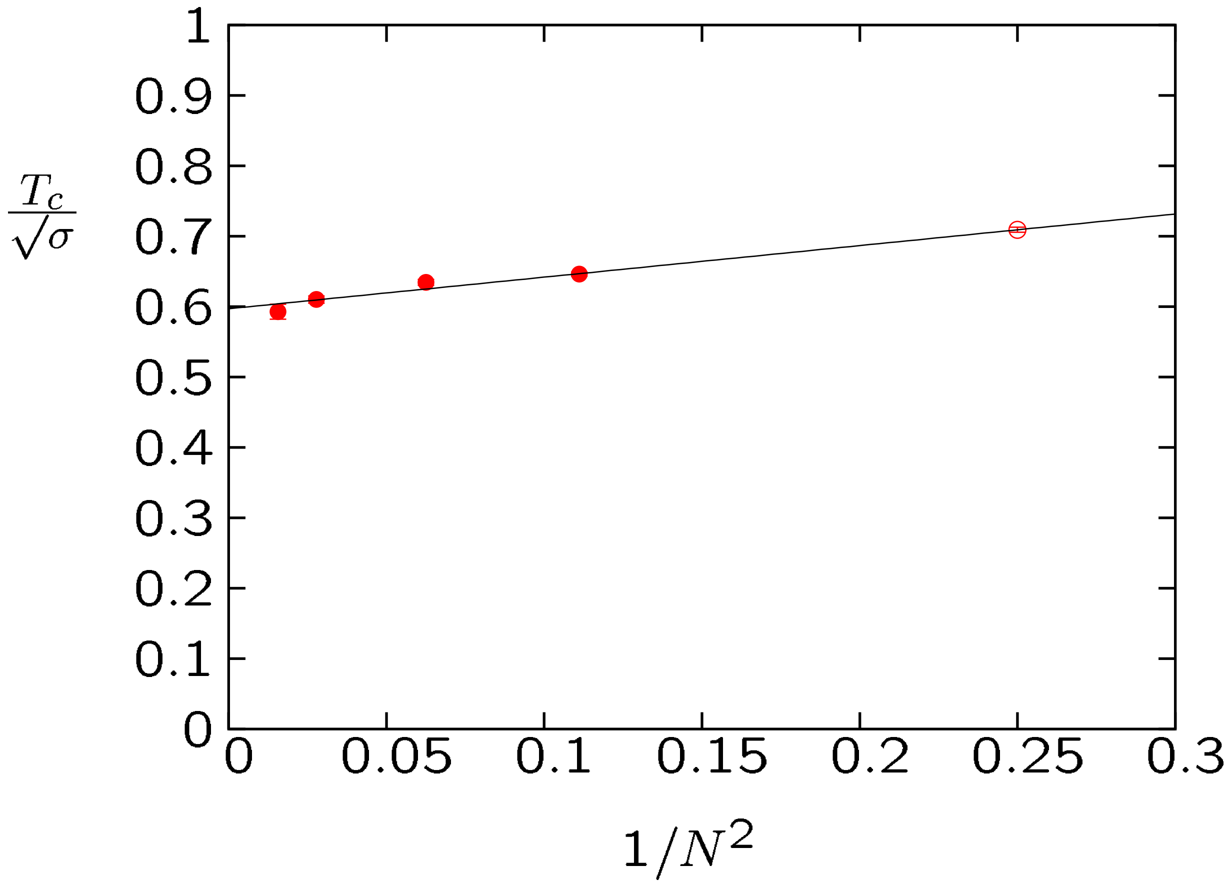
SU(3) ● ; SU(6) ○ ; SU(12) ●

$$\rho_c \simeq \frac{1}{T_c}$$

Deconfining temperature in D=3+1

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003

$L_s^3 L_t \Rightarrow T = \frac{1}{a(\beta)L_t}$ if L_s is large enough



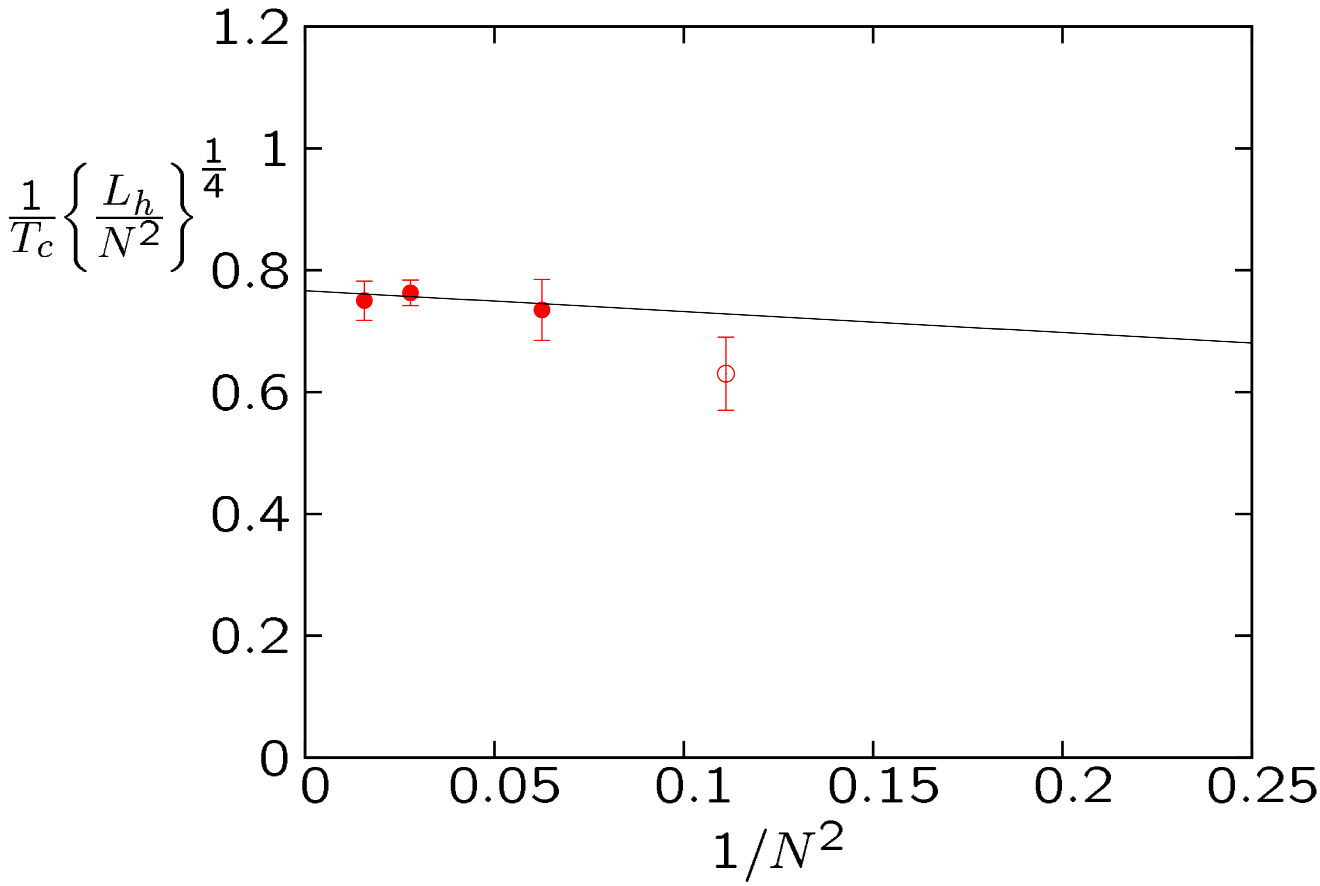
2nd order ∘ ; 1st order •

⇒

fit : $\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$

Confinement-deconfinement latent heat

B.Lucini, M.Teper, U.Wenger: hep-lat/0307017,0502003



$\Rightarrow$

large-N deconfinement is 'normal' first order

$N = 3$ 'weakly' first order

N counting of free energies (heuristic)

$$\begin{aligned} Z &= e^{-\frac{F}{T}} = \sum_n e^{-\frac{E_n}{T}} &(1)\\ &= \sum_{c=singlet} e^{-\frac{E_c}{T}} + \sum_{g=gluons} e^{-\frac{E_g}{T}} &(2)\\ &= e^{-\frac{F_c}{T}} + e^{-\frac{F_g}{T}} &(3)\end{aligned}$$

and at $T = T_c$ we have

$$F_c = F_g$$

but

$$F_g \sim N^2 \qquad \text{colour singlet entropy} \sim N^0$$

so reason that $T_c \not\to 0$ as $N \to \infty$ is that

$$E_c = hadron\, masses + E_{vac}$$

and

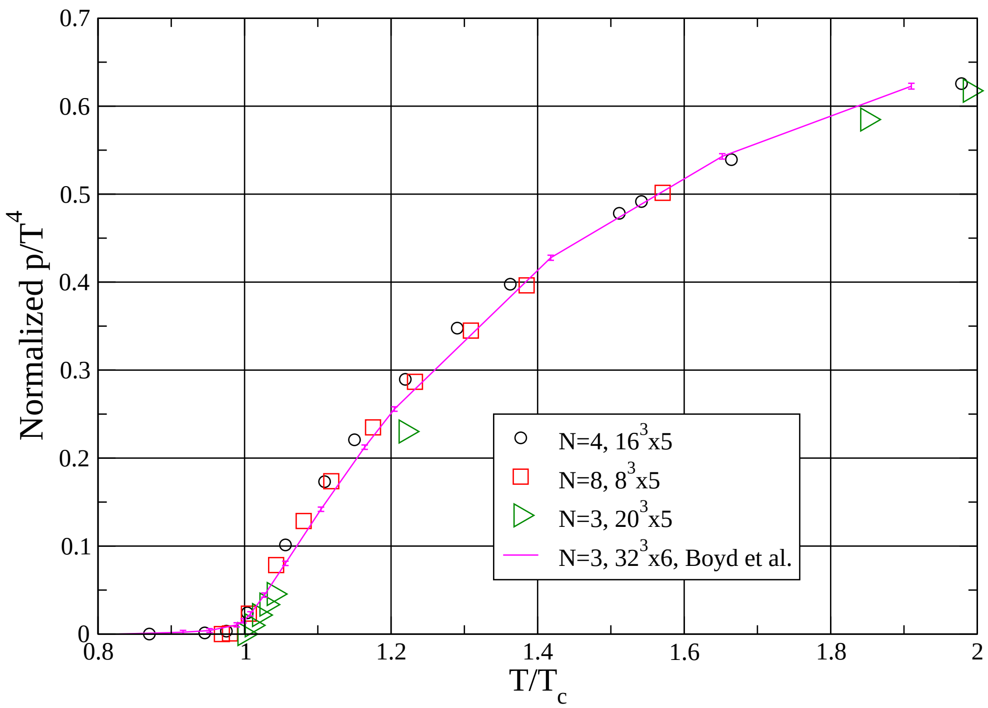
$$E_{vac} \sim -N^2 \sim gluon\, condensate$$

so

$$F_g = -E_{vac} \text{ at } \; T = T_c$$

Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

$\Rightarrow$

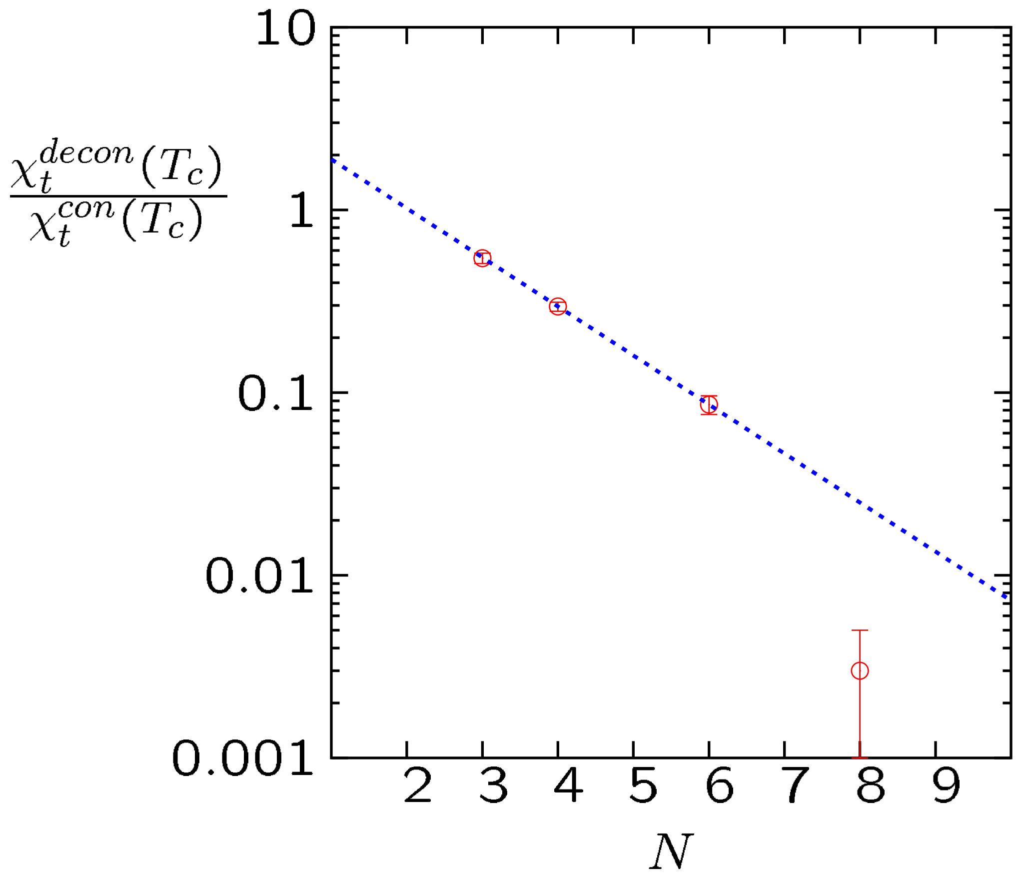
- not (colour singlet) hadrons above T_c
- not topology (instantons)

no topological fluctuations in deconfined phase ...

Lucini,Teper,Wenger: hep-lat/0401028

(Del Debbio,Panagopoulos,Vicari: hep-lat/0407068)

$\chi_t \equiv \langle Q^2 \rangle / V$ in confining/deconfining phases at $T = T_c$



$\Rightarrow$

deconfined topological fluctuations
vanish with N exponentially fast

not so surprising ...

Lucini,Teper,Wenger: hep-lat/0401028

- for small instantons, $\rho \ll 1/\sqrt{\sigma}$,

$$D(\rho) \sim \exp\{-8\pi^2/g^2(\rho)\} \sim \exp\{-c(\rho)N\}$$

with $c(\rho) = 8\pi^2/g^2(\rho)N$

- for T high enough for perturbation theory in $g^2(T)$ to be good, Gross,Pisarski,Yaffe RMP 53(19981)43

$$D(\rho, T) \sim D(\rho, T = 0) \exp(-\tfrac{2N}{3}\{\pi\rho T\}^2 - \gamma(\rho T))$$

and instantons with $\rho T > 1$ again vanish exponentially in N

So

the lattice results suggest that these two regions of exponential suppression overlap for any T in the deconfined phase, so that all instantons vanish $\propto \exp\{-cN\}$

!all values provisional!

string tensions in D=2+1

Barak Bringoltz

Karabali, Nair analytic treatment:

e.g. hep-th/0309061

(also: Freidel, Leigh, Minic hep-th/0604184)

$$\frac{\sqrt{\sigma}}{g^2 N} = \sqrt{\frac{1-\frac{1}{N^2}}{8\pi}}$$

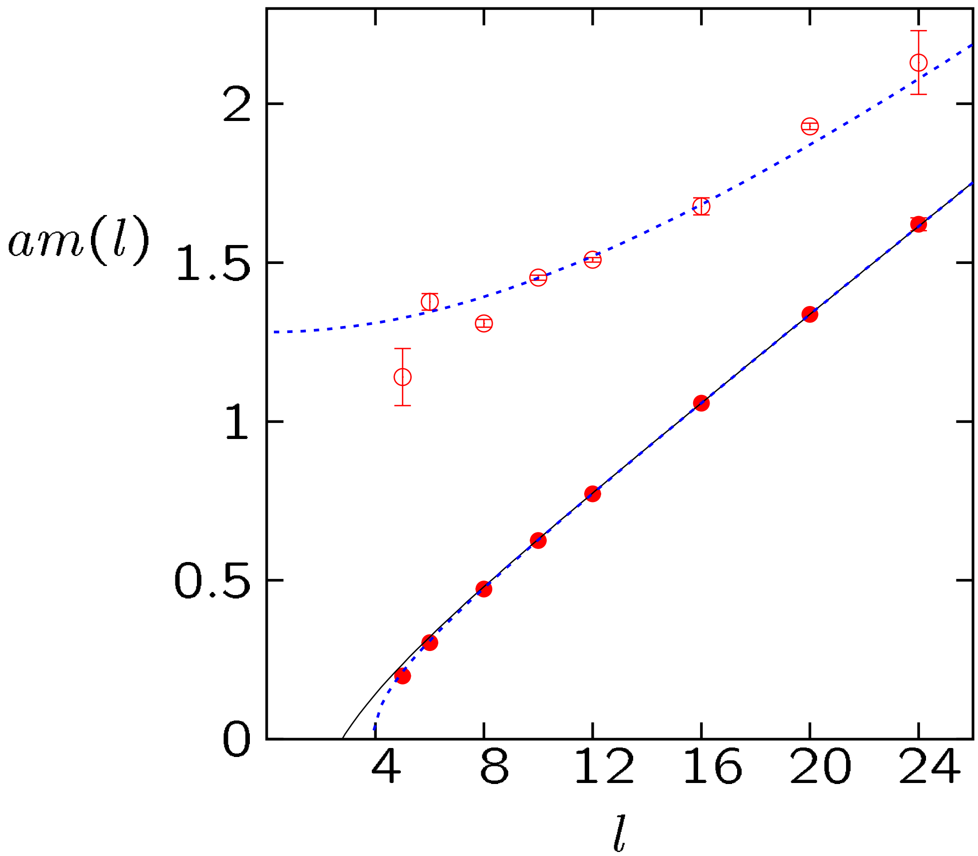
and

Casimir scaling

$$\frac{\sigma_k}{\sigma} = \frac{\mathcal{C}_k}{\mathcal{C}_f} = \frac{k(N-k)}{N-1}$$

D=2+1 : linear confinement in SU(3)

B. Bringoltz, M. Teper: in progress



linear + bosonic string correction (—):

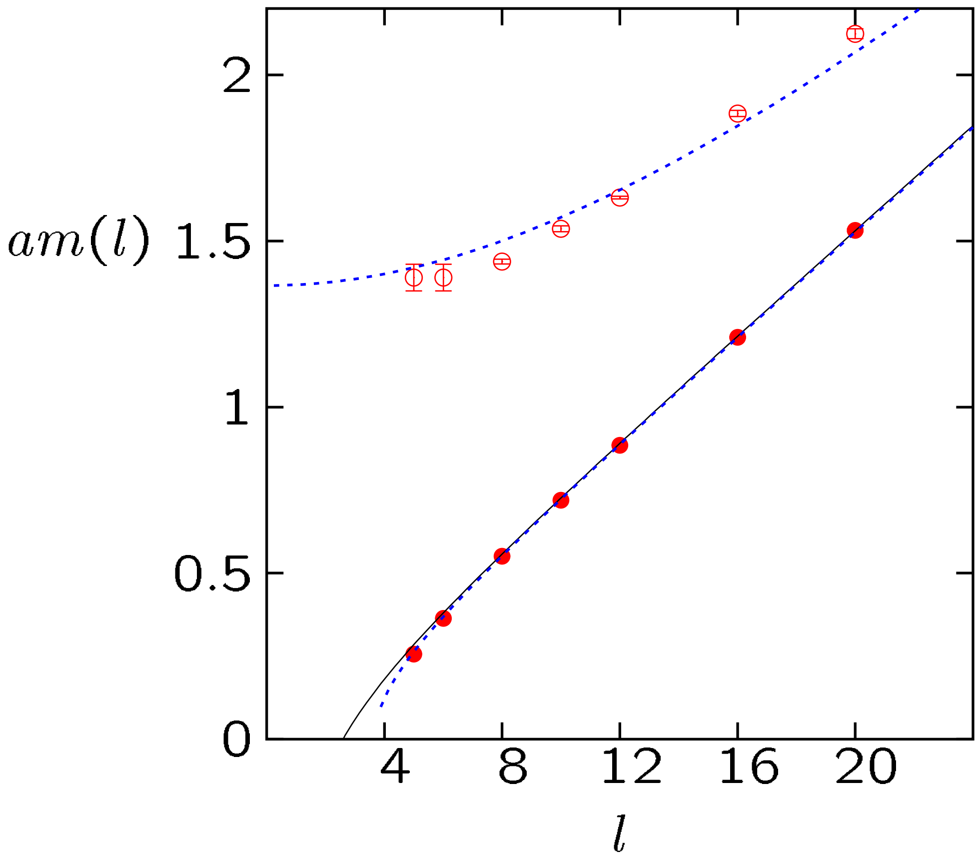
$$m(l) = \sigma l - \frac{\pi(D-2)}{6l}$$

Nambu-Goto string action ($\cdots$):

$$E_n(l) = \sigma l\left[1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right]^{\frac{1}{2}}$$

D=2+1 : linear confinement in SU(6)?

B. Bringoltz, M. Teper: in progress



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Nambu-Goto string action ($\cdots$):

$$E_n(l) = \sigma l[1 + \frac{8\pi}{\sigma l^2}(n - \frac{D-2}{24})]^{\frac{1}{2}}$$

expansion parameter:

ground state: $\sim \frac{\pi}{6}\frac{1}{\sigma l^2}$

first excited state: $\sim \frac{4\pi}{\sigma l^2}$

We typically use $\sigma l^2 \sim 10$ and extrapolate (ground state) to $l = \infty$ using fitted corrections, e.g.

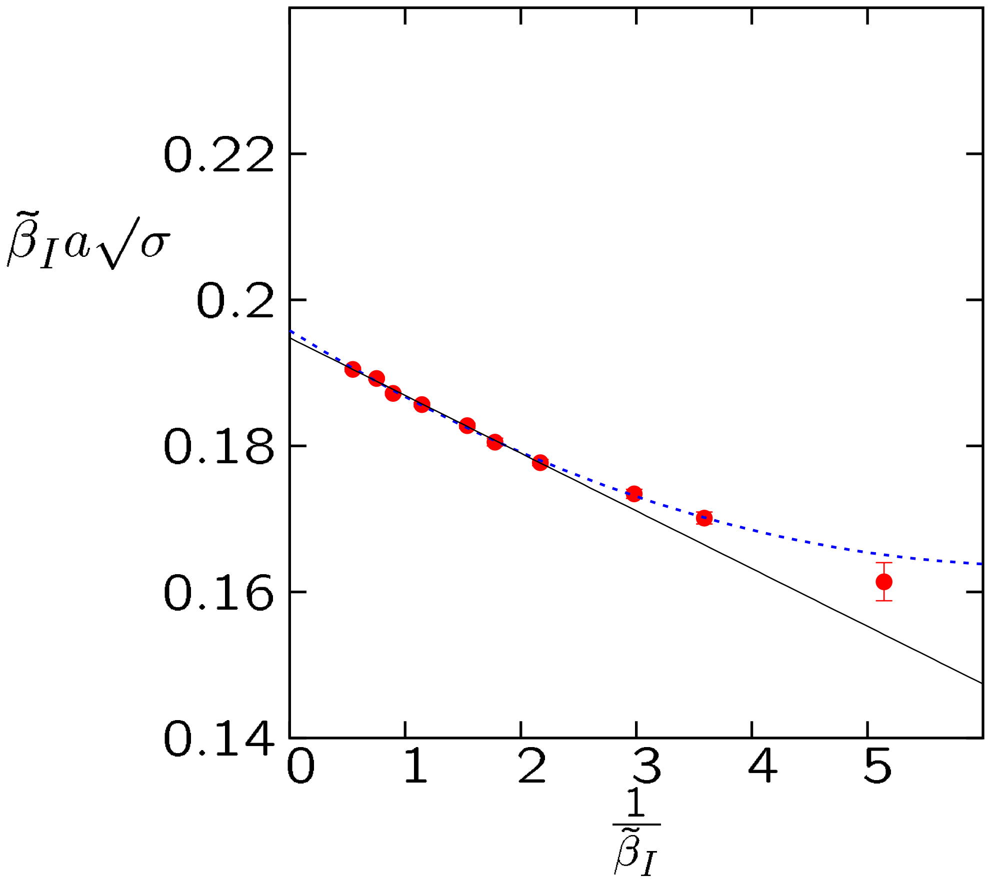
$$am(l) = a^2\sigma l - \frac{\pi}{6l} - \frac{0.26(5)}{(\sigma l^2)l}$$

or, better,

$$am(l) = a^2\sigma l - \frac{\pi}{6l} - \frac{1}{2}\left\{\frac{\pi}{6}\right\}^2 \frac{1}{(\sigma l^2)l} - \frac{0.15(10)}{(\sigma l^2)^2 l}$$

where the errors cover all N and a, and we extrapolate to the continuum e.g.

SU(8) continuum limit $\sqrt{\sigma}/g^2N$:



where $\tilde{\beta}_I = \beta\langle u_p\rangle/2N^2 \stackrel{a\to 0}{=} 1/ag^2N$, and we fit using $(\cdots)$

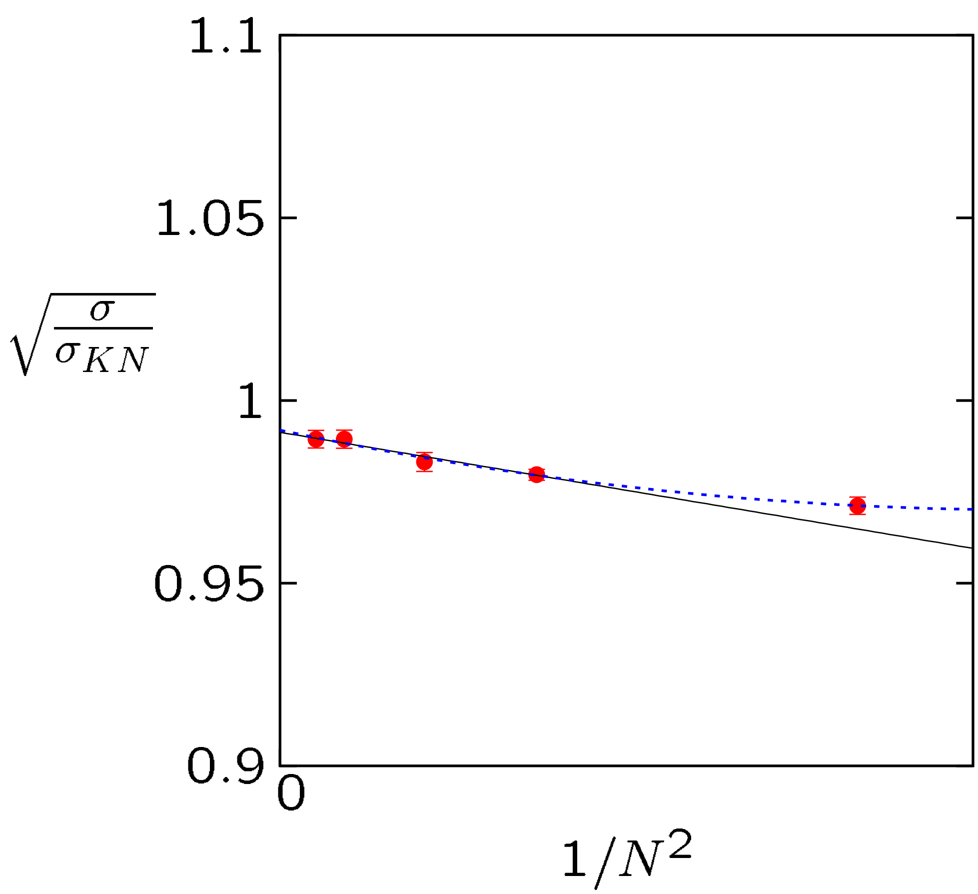
$$\tilde{\beta}_I a\sqrt{\sigma} = c_0 + \frac{c_1}{\tilde{\beta}_I} + \frac{c_2}{\tilde{\beta}_I^2}$$

giving a continuum value

$$\frac{\sqrt{\sigma}}{g^2N} = c_0 = 0.19580(47)$$

$N \to \infty$ and Karabali-Nair comparison:

dividing by $\sqrt{\sigma_{KN}} = g^2\sqrt{(N^2-1)/8\pi}$



we obtain

$$\lim_{N\to\infty}\sqrt{\frac{\sigma}{\sigma_{KN}}} = 0.9913 \pm 0.0021$$

very close, but not exact!

k-strings and Casimir scaling

32^3 lattice ; SU(8) ; $a \sim 0.1/\sqrt{\sigma}$

usual mass plateau analysis $\Rightarrow$

$\frac{\sigma_{k=2}}{\sigma_{k=1}} = 1.751(6)$

versus

1.714 ... Casimir 1.848... 'MQCD' sine formula

But

(approx) subtraction excited masses $\Rightarrow$

$\frac{\sigma_{k=2}}{\sigma_{k=1}} = 1.706(14)$

almost right on Casimir scaling!

So

usual claim that CS misses the k-string tensions by $\sim 1-2\%$ is not robust – CS may be even better than that!

.....

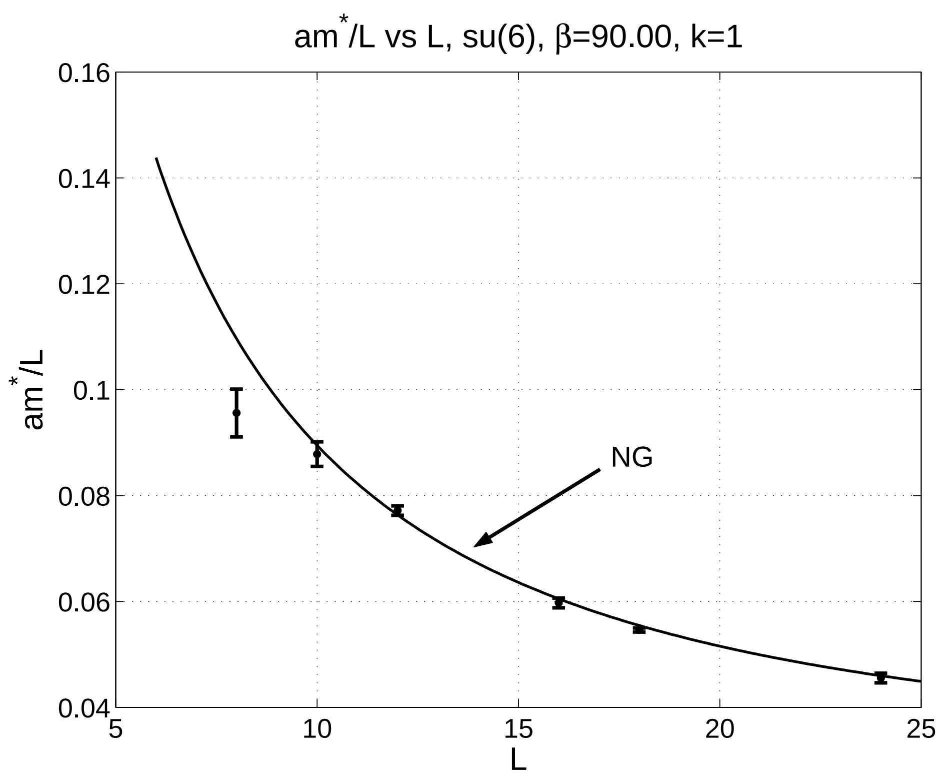
Nambu-Goto works amazingly well
as $a \to 0$ for larger N!

in SU(6) for $a \sim 0.17/\sqrt{\sigma} \sim 0.08$'fm',

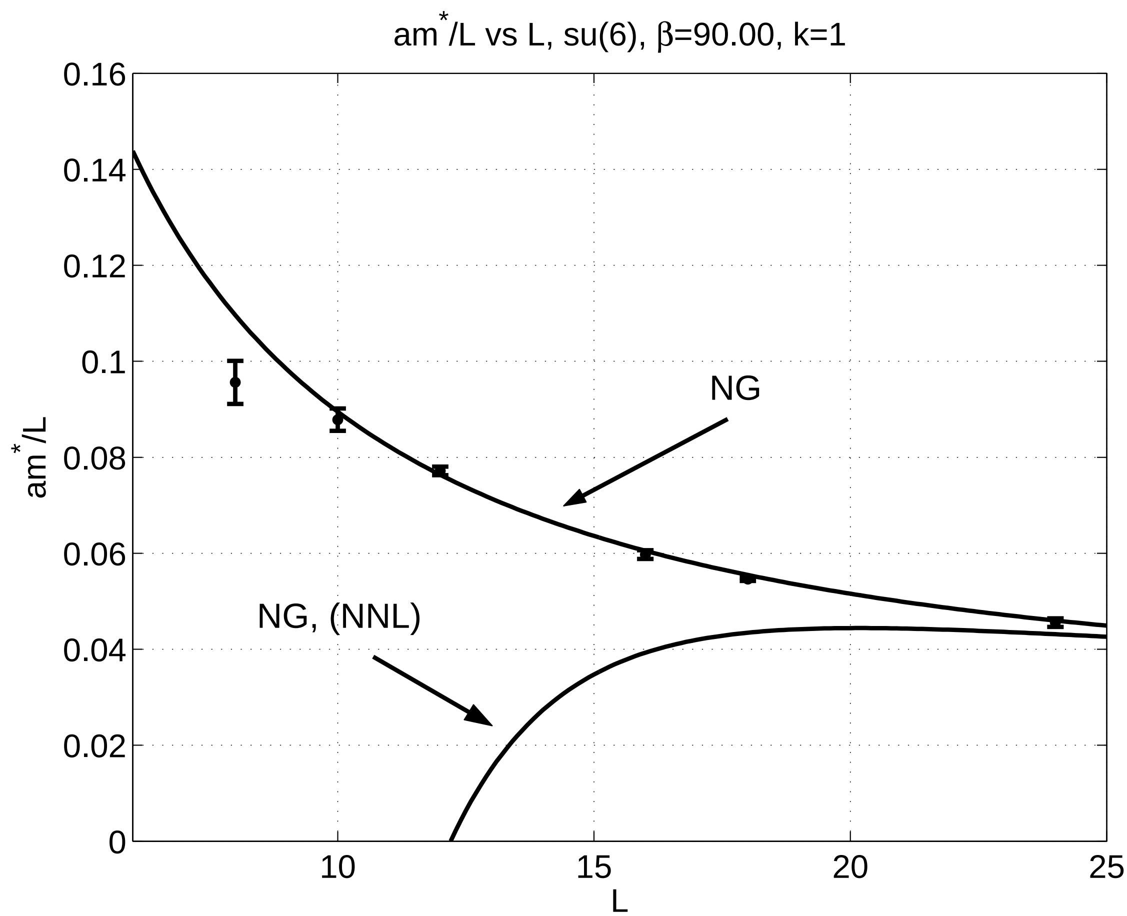
first excited string state compared to NG prediction:

$$E^{\star} = \sigma l[1 + \frac{23\pi}{3\sigma l^2}]^{\frac{1}{2}}$$

where σ fixed by ground state fit - so no parameters!



compare first 2 terms of expansion ...



Recall:

Nambu-Goto is a *free* string theory, and :

$$E_n(l) = \sigma l [1 + \frac{8\pi}{\sigma l^2}(n - \frac{D-2}{24})]^{\frac{1}{2}}$$

is the relativistic expression for the string energy

but

it is *not* a free field theory of the transverse displacements, and can be expanded in a perturbative expansion of these about the free (Gaussian) theory:

$$m(l) = \sigma l - \frac{\pi}{6l} - \frac{1}{2}\left\{\frac{\pi}{6}\right\}^2 \frac{1}{(\sigma l^2) l} + \frac{c}{l^{\gamma \geq 4}}$$

where the first 2 terms are as for Nambu-Goto

Luscher, Weisz hep-th/0406205;

Hari Dass, Matlock hep-th/0606265

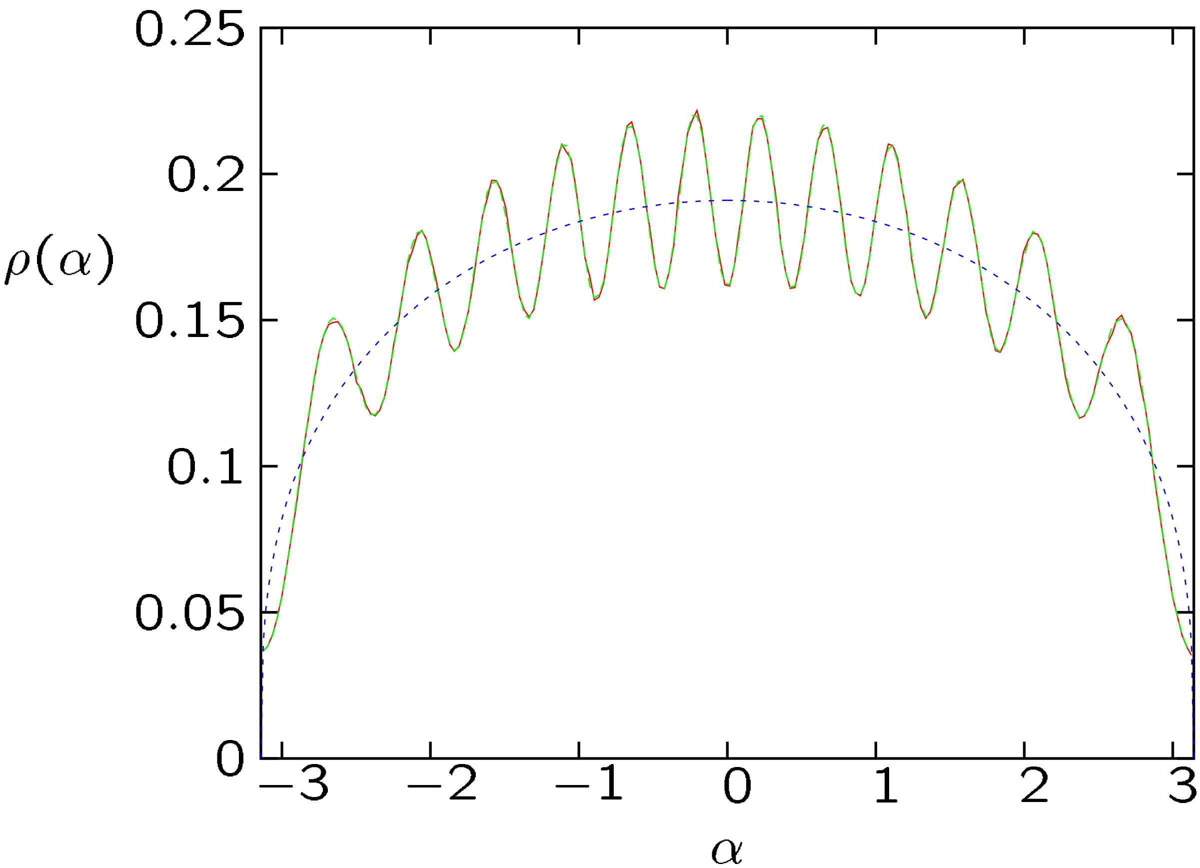
see also

Caselle, Hasenbusch, Panero hep-lat/0501027

and the next term is unknown

Matching Wilson loop spectra in D=2+1 and D=1+1

3×3 Wilson loop eigenvalue spectra $\lambda = e^{i\alpha}$ in SU(12),
matched average traces,
D=1+1 —— ; D=2+1 —— and
D=1+1, continuum, $N = \infty$ ——



Consider a Wilson (Polyakov) loop $l_{\mathcal{C}}$
around a contour $\mathcal{C}$ in SU(N) ...

in dimensions $D = d + 1$ and $D' = d' + 1$, if we choose couplings such that the average traces are equal

$$\langle \mathrm{Tr} l_{\mathcal{C}} \rangle_{D',\beta'} = \langle \mathrm{Tr} l_{\mathcal{C}} \rangle_{D,\beta}$$

then we find that the eigenvalue spectra are ‘identical’

. . .

for 1+1, 2+1, 3+1 dimensions,
for large as well as small loops,
for all N

Unexpected !

the trace of a $L \times L$ Wilson loop comes from

$$\langle \mathrm{Tr} l_{L\times L} \rangle \sim e^{-\{self-energy+potential\}}$$

and the (divergent) self energy depends on D

$$\begin{array}{ll} \sim 0 & ;\ \mathrm{D=1+1} \\ \sim \log a & ;\ \mathrm{D=2+1} \\ \sim \frac{1}{a} & ;\ \mathrm{D=3+1} \end{array}$$

$\Longrightarrow$

very different mix of physics leads to same trace for different D

e.g. $A\sigma \stackrel{a\to 0}{\longrightarrow} 0$

in D=2+1 and D=3+1 for any non-zero value of the trace, but not in D=1+1

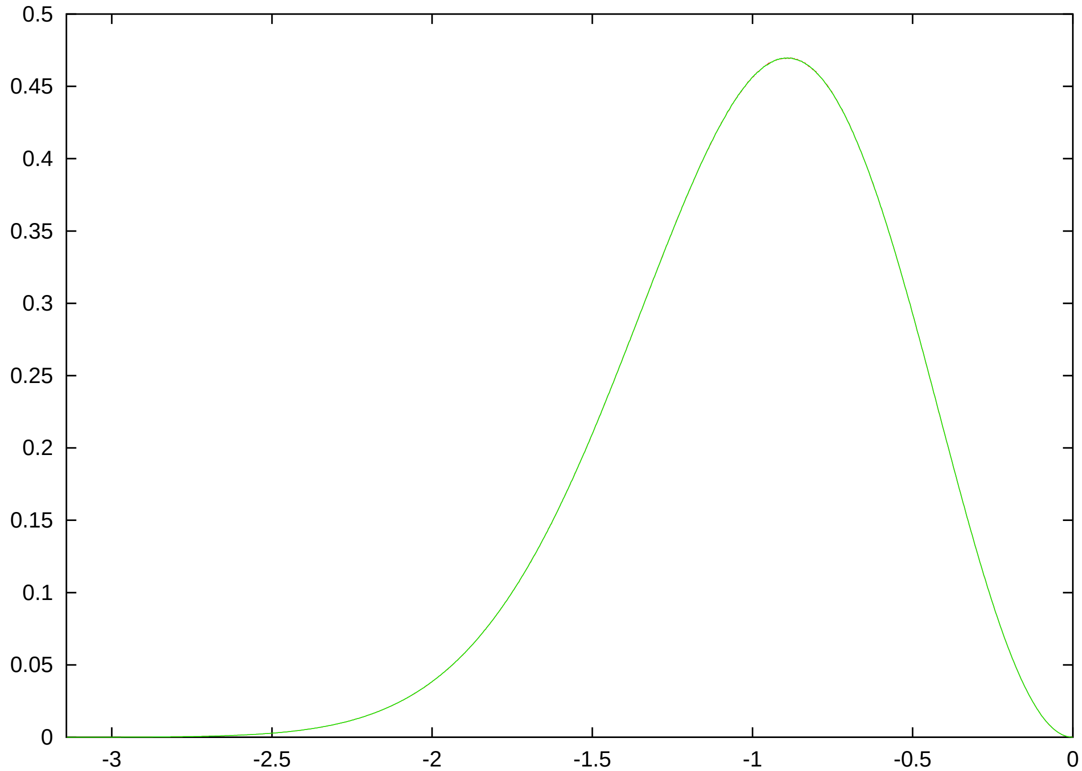
...

how precise is this matching?

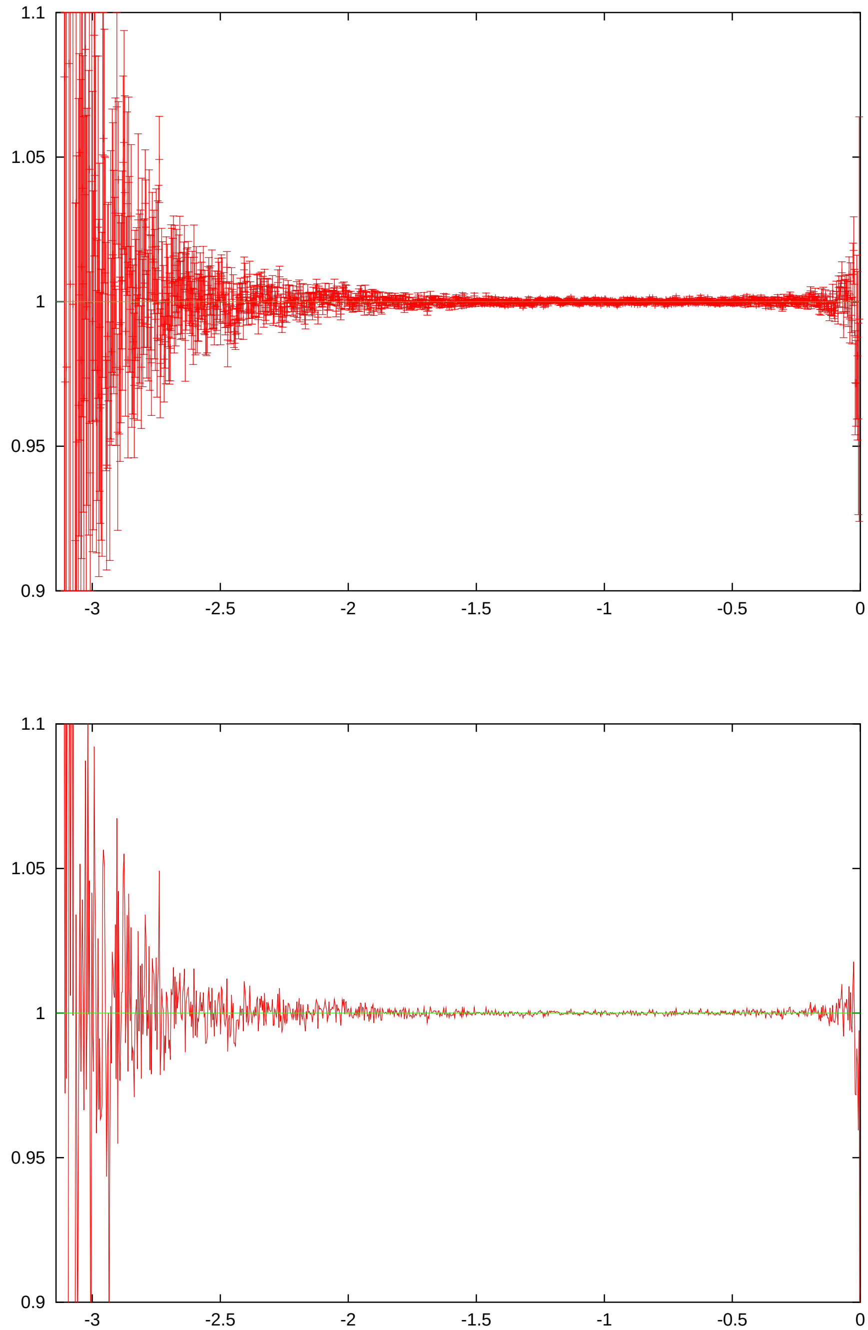
Precision comparison of Wilson loop eigenvalue spectra in $D = 1 + 1$ and $D = 2 + 1$

e.g.

4×4 loops in SU(2) for $\langle TrW \rangle = 0.5$



Ratio 4×4 eigenvalue spectra in D=1+1 and D=2+1



So ...

Wilson loop eigenvalue spectra match between D=d+1 and D=1+1 for same average trace – unexpected and intriguing

also

There is a strong-to-weak coupling lattice transition at larger N

D=3+1 : first order at finite N, with simple weak coupling physics beyond that.

D=2+1 : third order at $N = \infty$, identical to Gross-Witten in D=1+1 with some sign of a simultaneous $N = \infty$ second order transition reflecting Z_N monopole-instanton condensation

As $a \to 0$ $L \times L$ Wilson loop eigenvalue spectrum goes from $\sim$ flat to $\sim$ Wigner semicircle, with a gap forming at some coupling at $N = \infty$

what physical implications (re: Narayanan,Neuberger)?

Cascade of finite volume transitions related to finite T deconfinement on ever more dimensionally reduced gauge theories – some at finite N, some at $N = \infty$

Some Conclusions

• $SU(N \to \infty)$ is confining and $SU(3) \sim SU(\infty)$ for the simplest observables

• the Strongly-coupled Gluon Plasma is a large-N phenomenon and so demands a large-N explanation : *not* instantons, or (colour singlet) hadrons ...

• vanishing of the string tension at $T_H = T_c + \epsilon$ and of the 't Hooft tension at $T_{\tilde{H}} = T_c - \tilde{\epsilon}$ hints at possible dualities ...

• In $D = 2 + 1$ the spectrum of (excited) strings is remarkably like Nambu-Goto ... particularly as $a \to 0$

• In $D = 2 + 1$, the fundamental string tension is within 1% of Karabali-Nair prediction at $N = \infty$, but there is a discrepancy

• Again in $D = 2+1$, the k-string tensions are very close to Casimir Scaling (as in Karabali-Nair - also CKA) and any difference is likely to be less (possibly much less) than $\sim 1\%$.

• Anisotropic TEK has spontaneous centre symmetry breaking at very large (but finite?) N, at intermediate β, and for the isotropic case needs $N \gg 100$ ... so not very practical!

• Partially reduced TEK looks like a viable compromise that may prove useful...

• there is a strong-to-weak coupling lattice transition at larger N

In D=3+1 : first order at finite N, with simple weak coupling physics beyond that.

In D=2+1 : third order at $N = \infty$, identical to Gross-Witten in D=1+1 but a suggestion of a simultaneous $N = \infty$ second order transition reflecting Z_N monopole-instanton condensation

• cascade of finite volume transitions

• Wilson loop eigenvalue spectra match with amazing accuracy between D=3+1, D=2+1 and D=1+1 for same average trace