Bulk viscosity of color-superconducting quark matter

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Reviews: M. Alford, K. Rajagopal, hep-ph/0606157
Color superconductivity: Cooper pairing of quarks

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

\[ \mu = E_F \]

But quarks have attractive QCD interactions.

\[ F = E - \mu N \]

Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

Color superconductivity in three flavor quark matter

Unpaired

\[ p_F \]

\[ M_{s/4\mu} \]

\[ \begin{array}{c}
  u \\
  u \\
  d \\
  s \\
\end{array} \]

red green blue

\[ \begin{array}{c}
  \text{red} \\
  \text{green} \\
  \text{blue} \\
\end{array} \]

2SC pairing

\[ p_F \]

\[ \begin{array}{c}
  u \\
  u \\
  d \\
  s \\
\end{array} \]

red green blue

\[ \begin{array}{c}
  \text{red} \\
  \text{green} \\
  \text{blue} \\
\end{array} \]

CFL pairing

\[ p_F \]

\[ \begin{array}{c}
  u \\
  u \\
  d \\
  s \\
\end{array} \]

red green blue

\[ \begin{array}{c}
  \text{red} \\
  \text{green} \\
  \text{blue} \\
\end{array} \]

2SC: Two-flavor pairing phase. May occur at intermediate densities.

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij} \text{ i.e., } (rg - gr)(ud - du) \]

CFL: Color-flavor-locked phase, favored at the highest densities.

\[ \langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN} \]

(color \( \alpha, \beta \), flavor \( i, j = u, d, s \)); (Alford, Rajagopal, Wilczek, hep-ph/9804403)
I. High density QCD

Conjectured phase diagram

Right panels: NJL model with coupled chiral and color-superconducting condensates.

(Rüster, Werth, Buballa, Shovkovy, Rischke, hep-ph/0503184)
Signatures of color superconductivity in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

<table>
<thead>
<tr>
<th>mass</th>
<th>radius</th>
<th>density</th>
<th>initial temp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 1.4 M_\odot$</td>
<td>$\mathcal{O}(10 \text{ km})$</td>
<td>$\geq \rho_{\text{nuclear}}$</td>
<td>$\sim 30$ MeV</td>
</tr>
</tbody>
</table>

The star cools by neutrino emission for the first million years.
How would color superconductivity affect the star?

Pairing energy \( \text{affects Equation of state. Hard to detect.} \)

(Alford, Braby, Paris, Reddy, nucl-th/0411016)

Gaps in quark spectra and Goldstone bosons \( \text{affect Transport properties:} \)

- emissivity, heat capacity, viscosity (shear, bulk),
- conductivity (electrical, thermal)

1. Cooling by neutrino emission, neutrino pulse at birth
   Reddy, Sadzikowski, Tachibana, nucl-th/0306015; Grigorian, Blaschke, Voskresensky

2. Glitches and crystalline ("LOFF") pairing
   (Alford, Bowers, Rajagopal, hep-ph/0008208)

3. Gravitational waves: r-mode instability, shear and bulk viscosity
   (Madsen, astro-ph/9912418; Manuel, Dobado, Llanes-Estrada, hep-ph/0406058,
   Alford, Schmitt nucl-th/0608019, Alford, Braby, Reddy, Schäfer nucl-th/0701067,
   Manuel, Llanes-Estrada arXiv:0705.3909)
r-modes: gravitational spin-down of compact stars

An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.

The Lindblom group at Caltech has made a movie of r-mode evolution.

http://www.cacr.caltech.edu/projects/hydrligo/rmode.html

r-modes are unstable if rotation rate $\Omega > \Omega_{\text{crit}}(T)$, and they can spin the star down within months (Andersson gr-qc/9706075; Friedman and Morsink gr-qc/9706073; Lindblom astro-ph/0101136).

Once we measure $T$ and $\Omega$ for a star, we can put an upper limit on $\Omega_{\text{crit}}(T)$. 
**Constraints from r-modes** (Madsen, astro-ph/9912418)

Predicted $\Omega_{\text{crit}}(T)$ for various phases. Shaded regions above curves are unstable: viscosity is too low to hold back the $r$-modes.

**Nuclear matter**

**Unpaired** ($m_s = 200$)

**2SC** ($m_s = 200$)

Dotted lines: $m_s = 100$ MeV; $\underline{m_s} = \text{millisecond pulsars}$

According to Madsen’s original calculation, pairing always lowers bulk viscosity, making **2SC** more vulnerable to $r$-modes.

We find that actually $\zeta_{2SC} > \zeta_{\text{unp}}$ at high $T$. We expect this will move the $\Omega_{\text{crit}}$ line outward (dark red arrows).
What is bulk viscosity?

(L. viscum = mistletoe; It. vischio, Jp. ickle, Gm. Mistelzweig, Sp. muérdago, Fr. gui, Ru. omela)

A sticky glue was made from mistletoe berries and coated onto small tree branches to catch birds.

Energy consumed in a compression cycle:

\[ V(t) = \bar{V} + \text{Re}[\delta V \exp(i\omega t)] \]

\[ p(t) = \bar{p} + \text{Re}[\delta p \exp(i\omega t)] \]

\[
\langle \frac{dE}{dt} \rangle = -\frac{\zeta}{\tau} \int_0^\tau (\text{div} \ \vec{v})^2 \, dt = \frac{\zeta}{2} \omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}} \int_0^\tau p(t) \frac{dV}{dt} \, dt
\]

\[
\Rightarrow \zeta(\omega, T) = -\frac{\bar{V}}{\delta V} \frac{\text{Im}(\delta p)}{\omega}
\]

Physically, bulk viscosity arises from re-equilibration processes. If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to \( \tau \), then pressure gets out of phase with volume and energy is consumed. (Just like \( V \) and \( Q \) in a \( R-C \) circuit.)
Flavor re-equilibration processes

phase: 2SC  CFL (not CFL-$K^0$)

lightest modes: unpaired (“blue”) quarks  $H, K^0$

flavor equilibration: $u + d \leftrightarrow s + u$  $K^0 \leftrightarrow H$  $H$  $K^0 H \leftrightarrow H$

\[ u \xrightarrow{W^\pm} s \]
\[ d \xrightarrow{W^\pm} u \]
CFL thermal kaon bulk viscosity

(Alford, Braby, Reddy, Schäfer nucl-th/0701067)

\[ K^0 \text{ dispersion relation:} \]

\[ E(p) = -\frac{M_s^2}{2\mu} + \sqrt{\frac{1}{3}p^2 + m_{K^0}^2} \]

\[ \approx m_{K^0} - \frac{M_s^2}{2\mu} + \frac{1}{3}p^2 \]

\[ \underbrace{\delta m}_{\text{phonon splitting}} \]

Thermal kaon density \( \sim \exp(-\delta m/T) \), drops rapidly for \( T \ll \delta m \).

Kaons dominate bulk viscosity for \( T \gtrsim \delta m/30 \).

Superfluid mode ("phonon") splitting dominates in some temp range if \( \delta m \gtrsim 2 \text{ MeV} \) (Manuel, Llanes-Estrada arXiv:0705.3909)
How bulk viscosity depends on equilibration rate

\[ \zeta(\omega, T) = C(T) \frac{\gamma_K(T)}{\gamma_K(T)^2 + \omega^2} \]

- \( \omega \) is angular frequency of applied compression cycle.
- \( C \) measures the sensitivity of \( n_K \) and \( n_q \) to changes in \( \mu_K \) and \( \mu \).
- \( \gamma_K \) is the average kaon width, from \( K^0 \leftrightarrow H \).

**Graphical representations:**
- **Left graph:** Shows the function \( C(\tau, \delta m) \) for different values of \( \delta m \) and \( \tau = 1 \text{ ms} \).
- **Right graph:** Displays \( \gamma_{\text{eff}}(\delta m) \) for various \( \delta m \) values with \( \tau = 1 \text{ ms} \). The graph includes lines for different \( \delta m \) values such as 0.1 MeV, 0.5 MeV, 5 MeV, and 10 MeV.
CFL kaonic bulk viscosity: dependence on $\omega$

\[ \zeta(\omega, T) = C(T) \frac{\gamma_K(T)}{\gamma_K(T)^2 + \omega^2} \]

At high temp, $\gamma_K(T)$ rises, and $\omega$ becomes negligible.

For unpaired quark matter, $C$ is indp of $T$, and the resonance peak at $\gamma_K(T) = \omega$ is clear.

As the frequency of compression drops,

- The peak in $\zeta_{\text{unp}}$, which occurs where $\gamma_K(T) = \omega$, drops to lower temp.
- The peak value rises: $\zeta_{\text{max}} = \frac{1}{2} C/\omega$. 
Quark matter bulk viscosity: Summary

- Unpaired and 2SC have the largest bulk viscosity, because they have unpaired modes at Fermi surface (large phase space).
- $K^0$ density $\sim \exp(-\delta m/T)$ drops rapidly for $T \lesssim \delta m/10$.
- $\delta m = m_{K^0} - M_s^2/(2\mu)$ could be anything from negative (kaon condensation) to $\sim 10$ MeV.
- Superfluid modes ("phonons") alone contribute some bulk viscosity.

Alford, Schmitt nucl-th/0608019; Alford, Braby, Reddy, Schäfer nucl-th/0701067;
Manuel, Llanes-Estrada arXiv:0705.3909
Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - shear and bulk viscosity of $\text{CFL}-K^0$, other phases...
  - detailed analysis of $r$-mode profiles in hybrid star
  - heat capacity, conductivity and emissivity (neutrino cooling)
  - structure: nuclear-quark interface (gravitational waves?)
  - crystalline phase (glitches)
  - $\text{CFL}$: vortices but no flux tubes

- More general questions:
  - magnetic instability of gapless phases
  - better weak-coupling calculations, include vertex corrections
  - go beyond mean-field, include fluctuations
  - solve the sign problem and do lattice QCD at high density.
Calculating bulk viscosity for a known equilibration rate

Suppose the equilibrating quantity is \( n_y \) (this will be \( n_d - n_s \)).

Corresponding chemical potential \( \mu_y = \delta\mu_y \exp(i\omega t) \).

We want \( \text{Im}(\delta p) = \text{Im}\left(\frac{dp}{d\mu_y} \delta\mu_y\right) = n_y \text{Im}(\delta\mu_y) \).

Write \( \dot{n}_y \) two ways:

\[
\frac{dn_y}{d\mu_y} \dot{\mu}_y = -\frac{n_y}{V} \dot{V} - (n_y - \bar{n}_y) \Gamma
\]

\[
\Rightarrow \frac{dn_y}{d\mu_y} (i\omega + \gamma) \delta\mu_y = -\frac{n_y}{V} i\omega \delta V
\]

writing \( \Gamma \equiv \gamma \frac{dn_y}{d\mu_y} \)

\[
\Rightarrow \delta\mu_y = \frac{-i\omega}{i\omega + \gamma} \bar{n}_y \left( \frac{dn_y}{d\mu_y} \right)^{-1} \frac{\delta V}{V}
\]

\[
\Rightarrow \text{Im}(\delta p) = \bar{n}_y \text{Im}(\delta\mu_y) = \frac{-\omega \gamma}{\omega^2 + \gamma^2} \frac{\delta V}{V} \bar{n}_y^2 \left( \frac{dn_y}{d\mu_y} \right)^{-1}
\]

\[
\Rightarrow \zeta = -\frac{\bar{V}}{\delta V} \frac{\text{Im}(\delta p)}{\omega} = \bar{n}_y^2 \left( \frac{dn_y}{d\mu_y} \right)^{-1} \frac{\gamma}{\gamma^2 + \omega^2}
\]