Effective gluon mass and infrared fixed point in QCD

Arlene C. Aguilar[1] J. Papavassiliou[1]

[1]University of Valencia, Spain

QCD @ Work,
Martina Franca, 16-20th June 2007

General Considerations for the Effective Gluon Mass

- IR finite gluon propagator $(\Delta^{-1}(0) \neq 0)$ allows for a definition of a Effective Gluon Mass.

Features:

- Generated dynamically.
- It is not a hard mass! - Local gauge invariance is preserved
- A momentum dependent mass $m(p^2)$.
- Drops off at UV - recover the pertubative behavior.
- No fundamental scalars appear in the spectrum.
- $\boxed{\texttt{Purely non-perturbative effect.}}$

General Considerations for the Effective Gluon Mass

- IR finite gluon propagator ($\Delta^{-1}(0) \neq 0$) allows for a definition of a Effective Gluon Mass.

Features:

- Generated dynamically.
- It is not a hard mass! - Local gauge invariance is preserved
- A momentum dependent mass $m(p^2)$.
- Drops off at UV - recover the pertubative behavior.
- No fundamental scalars appear in the spectrum.
- Purely non-perturbative effect.

General Considerations for the Effective Gluon Mass

- IR finite gluon propagator ($\Delta^{-1}(0) \neq 0$) allows for a definition of a Effective Gluon Mass.

Features:

- Generated dynamically.
- It is not a hard mass! - Local gauge invariance is preserved
- A momentum dependent mass $m(p^2)$.
- Drops off at UV - recover the pertubative behavior.
- No fundamental scalars appear in the spectrum.
- `Purely non-perturbative effect.`

Non-perturbative scheme - SDE

Non-perturbative tools:

- Lattice QCD (discrete approach)
- Schwinger-Dyson Equation (continuum approach)

What is Schwinger-Dyson Equation?

- The equations of motion for Green's functions.
- They are coupled integral equations. Forming an infinite tower of coupled equations.
- Obviously, the complete infinite tower is insoluble.
- Truncation scheme is necessary to make any problem tractable.

Non-perturbative scheme - SDE

Non-perturbative tools:

- Lattice QCD (discrete approach)
- Schwinger-Dyson Equation (continuum approach)

What is Schwinger-Dyson Equation?

- The equations of motion for Green's functions.
- They are coupled integral equations. Forming an infinite tower of coupled equations.
- Obviously, the complete infinite tower is insoluble.
- Truncation scheme is necessary to make any problem tractable.

Non-perturbative tools:

- Lattice QCD (discrete approach)
- Schwinger-Dyson Equation (continuum approach)

What is Schwinger-Dyson Equation?

- The equations of motion for Green's functions.
- They are coupled integral equations. Forming an infinite tower of coupled equations.
- Obviously, the complete infinite tower is insoluble.
- Truncation scheme is necessary to make any problem tractable.

Truncation scheme must respect:

- Gauge and renormalization-group invariances
- The truncation is guided through the Pinch Technique.

$\downarrow$

Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with desirable properties.

- To all orders,

Pinch-Technique $\longleftrightarrow$ Feynman gauge of Background Field Method

PT Properties

- New Feynman rules
- Green's functions respect QED-like Ward-Identities instead of Slavnov-Taylor Identities

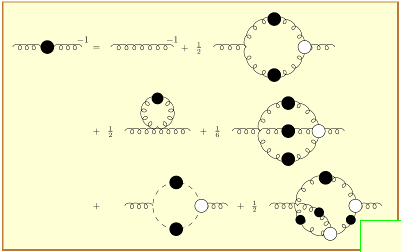
$$
\begin{aligned}
q_1^{\mu}\, \underbrace{\widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1, q_2, q_3)}_{\text{3-gluon vertex}} &= g f^{abc} \underbrace{[\Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3)]}_{\text{gluon propagators}} \\
q_1^{\mu}\, \underbrace{\widetilde{\Gamma}^{acb}_{\mu}(q_2, q_1, q_3)}_{\text{gluon-ghost vertex}} &= g f^{abc} \underbrace{[D^{-1}(q_2) - D^{-1}(q_3)]}_{\text{ghost propagators}}
\end{aligned}
$$

- Consequently the fundamental QED-like relation,

$$
\widehat{Z}_g = \widehat{Z}_A^{-1/2}, \quad \text{where} \quad \begin{aligned} g_o &= \widehat{Z}_g\, g\,, \\ \widehat{\Delta}_o &= \widehat{Z}_A\, \widehat{\Delta} \end{aligned}
$$

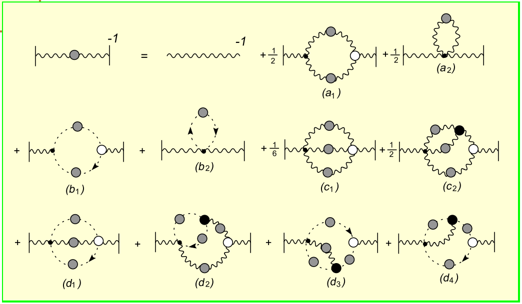
is held in the PT-BFM framework.

Conventional X BFM

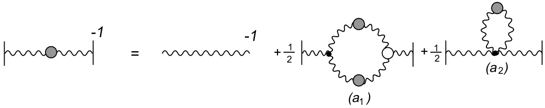


⟵ Conventional SDE

PT-BFM SDE ⟶



Approximation



$$\widehat{\Delta}^{-1}(q^2)\mathrm{P}_{\mu\nu}(q) = q^2\mathrm{P}_{\mu\nu}(q) + i\left[\widehat{\Pi}^{(\mathbf{a_1})}_{\mu\nu}(q) + \widehat{\Pi}^{(\mathbf{a_2})}_{\mu\nu}\right]$$

$$\begin{aligned}
\widehat{\Pi}^{(\mathbf{a_1})}_{\mu\nu}(q) &= \frac{1}{2}\,C_A\,g^2\int[dk]\widetilde{\Gamma}_{\mu\alpha\beta}\widehat{\Delta}^{\alpha\alpha'}(k)\widetilde{\mathrm{I}\Gamma}_{\nu\alpha'\beta'}\widehat{\Delta}^{\beta\beta'}(k+q) \\
\widehat{\Pi}^{(\mathbf{a_2})}_{\mu\nu} &= -C_A\,g^2\,g_{\mu\nu}\int[dk]\widehat{\Delta}(k)
\end{aligned}$$

The expression for the vertex that we will use is given by

$$\widetilde{\rm I\Gamma}^{\mu\alpha\beta} = L^{\mu\alpha\beta} + T_1^{\mu\alpha\beta} + T_2^{\mu\alpha\beta}$$

with

$$\begin{aligned}
L^{\mu\alpha\beta}(q,p_1,p_2) &= \widetilde{\Gamma}^{\mu\alpha\beta}(q,p_1,p_2) + ig^{\alpha\beta}\,\frac{q^\mu}{q^2}\left[\widehat{\Pi}(p_2) - \widehat{\Pi}(p_1)\right] \\
T_1^{\mu\alpha\beta}(q,p_1,p_2) &= -i\frac{c_1}{q^2}\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)\left[\widehat{\Pi}(p_1) + \widehat{\Pi}(p_2)\right] \\
T_2^{\mu\alpha\beta}(q,p_1,p_2) &= -ic_2\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)\left[\frac{\widehat{\Pi}(p_1)}{p_1^2} + \frac{\widehat{\Pi}(p_2)}{p_2^2}\right]
\end{aligned}$$

SD equation

$$\Delta^{-1}(x) = Kx + \tilde{b}g^2 \sum_{i=1}^{8} a_i A_i(x) + \Delta^{-1}(0)$$

$$\begin{aligned}
A_1(x) &= a_1\, x \int_x^\infty dy y \Delta^2(y) \\
A_2(x) &= a_2\, x \int_x^\infty dy \Delta(y) \\
A_3(x) &= a_3\, x \Delta(x) \int_0^x dy y \Delta(y) \\
A_4(x) &= a_4 \int_0^x dy y^2 \Delta^2(y)
\end{aligned}$$

$$\begin{aligned}
A_5(x) &= a_5\, \Delta(x) \int_0^x dy y^2 \Delta(y) \\
A_6(x) &= a_6 \int_0^x dy y \Delta(y) \\
A_7(x) &= a_7\, \frac{\Delta(x)}{x} \int_0^x dy y^3 \Delta(y) \\
A_8(x) &= a_8\, \frac{1}{x} \int_0^x dy y^3 \Delta^2(y)
\end{aligned}$$

The renormalization condition K is fixed by $\widehat{\Delta}^{-1}(\mu^2) = \mu^2$

The UV behavior of effective gluon mass

$$m^2(x)\ln x \quad = \quad d^{-1}(0) + \gamma_1 \int_0^x dy\, m^2(y)\tilde{\Delta}(y) + \frac{\gamma_2}{x}\int_0^x dy\, y m^2(y)\tilde{\Delta}(y)$$

with

$$\tilde{\Delta}(q^2) = \frac{1}{q^2 + m^2(q^2)},$$

$$\gamma_1 = \frac{6}{5}(1 + c_2 - c_1) \qquad \gamma_2 = \frac{4}{5} + \frac{6c_1}{5},$$

There are two possible asymptotic solutions:

$$\begin{aligned} m_1^2(x) &= \lambda_1^2(\ln x)^{-1+\gamma_1} && \Longrightarrow \langle A_\mu^a A_a^\mu \rangle \\ m_2^2(x) &= \frac{\lambda_2^4}{x}(\ln x)^{\gamma_2 - 1} && \Longrightarrow \langle G_{\mu\nu}^a G_a^{\mu\nu} \rangle \end{aligned}$$

The UV behavior of effective gluon mass

$$m^2(x)\ln x = d^{-1}(0) + \gamma_1 \int_0^x dy\, m^2(y)\tilde{\Delta}(y) + \frac{\gamma_2}{x}\int_0^x dy\, y m^2(y)\tilde{\Delta}(y)$$

with

$$\tilde{\Delta}(q^2) = \frac{1}{q^2 + m^2(q^2)},$$

$$\gamma_1 = \frac{6}{5}(1 + c_2 - c_1) \quad \gamma_2 = \frac{4}{5} + \frac{6c_1}{5},$$

There are two possible asymptotic solutions:

$$m_1^2(x) = \lambda_1^2(\ln x)^{-1+\gamma_1} \Longrightarrow \langle A_\mu^a A_a^\mu \rangle$$

$$m_2^2(x) = \frac{\lambda_2^4}{x}(\ln x)^{\gamma_2 - 1} \Longrightarrow \langle G_{\mu\nu}^a G_a^{\mu\nu} \rangle$$

Propagator and Running Masses

The RG quantity, $d(q^2) = g^2\Delta(q^2)$, has the general form:

$$d(q^2) = \frac{\overline{g}^2(q^2)}{q^2 + m^2(q^2)}\,,$$

where the dynamical mass is

$$m_1^2(q^2) = m_0^2\left[\ln\left(\frac{q^2+\rho\,m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\,m_0^2}{\Lambda^2}\right)\right]^{-1+\gamma_1}$$

or

$$m_2^2(q^2) = \frac{m_0^4}{q^2+m_0^2}\left[\ln\left(\frac{q^2+\rho\,m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\,m_0^2}{\Lambda^2}\right)\right]^{\gamma_2-1}$$

Propagator and Running Masses

The RG quantity, $d(q^2) = g^2\Delta(q^2)$, has the general form:

$$d(q^2) = \frac{\overline{g}^2(q^2)}{q^2 + m^2(q^2)}\,,$$

where the dynamical mass is

$$m_1^2(q^2) = m_0^2\left[\ln\left(\frac{q^2+\rho\, m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{-1+\gamma_1}$$

or

$$m_2^2(q^2) = \frac{m_0^4}{q^2+m_0^2}\left[\ln\left(\frac{q^2+\rho\, m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{\gamma_2-1}$$

Propagator and Running Masses

The RG quantity, $d(q^2) = g^2\Delta(q^2)$, has the general form:

$$d(q^2) = \frac{\overline{g}^2(q^2)}{q^2 + m^2(q^2)}\,,$$

where the dynamical mass is

$$m_1^2(q^2) = m_0^2\left[\ln\left(\frac{q^2+\rho\, m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{-1+\gamma_1}$$

or

$$m_2^2(q^2) = \frac{m_0^4}{q^2+m_0^2}\left[\ln\left(\frac{q^2+\rho\, m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{\gamma_2-1}$$

Infrared Fixed Point

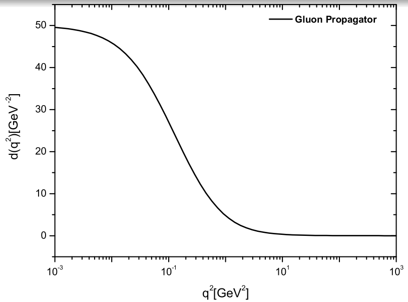
and $\overline{g}^2(q^2)$

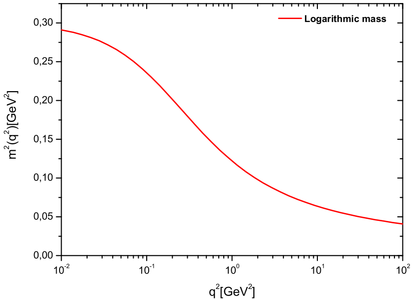
$$\boxed{\overline{g}^2(q^2) = \left[\tilde{b}\ln\left(\frac{q^2 + f(q^2, m^2(q^2))}{\Lambda^2}\right)\right]^{-1}}$$

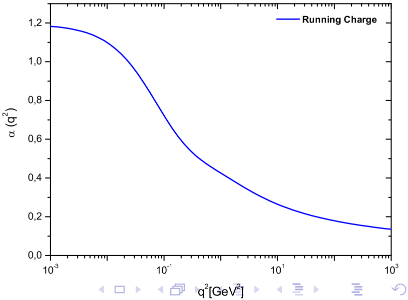
- It displays asymptotic freedom in the UV.
- Freezes at a finite value in the low energy regime

Infrared Fixed Point for QCD !.

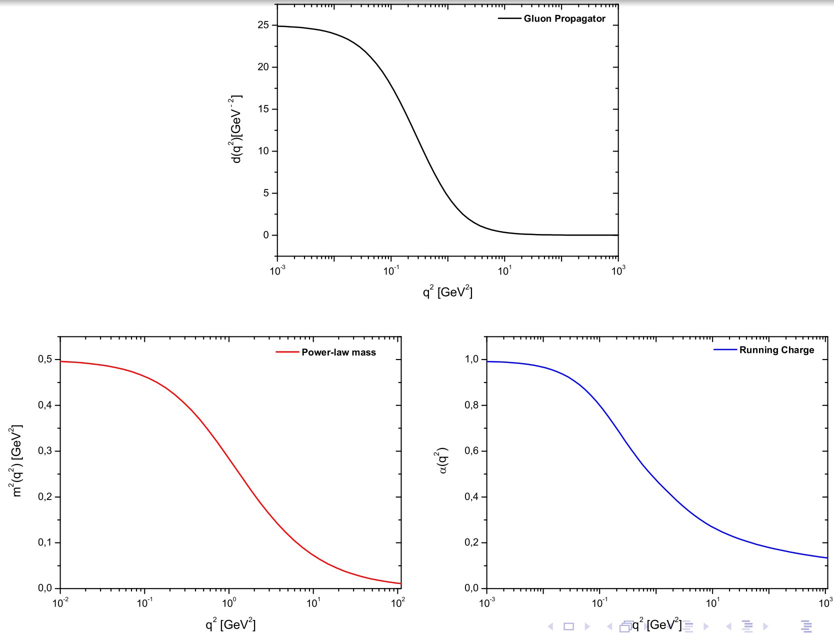
Numerical Analysis



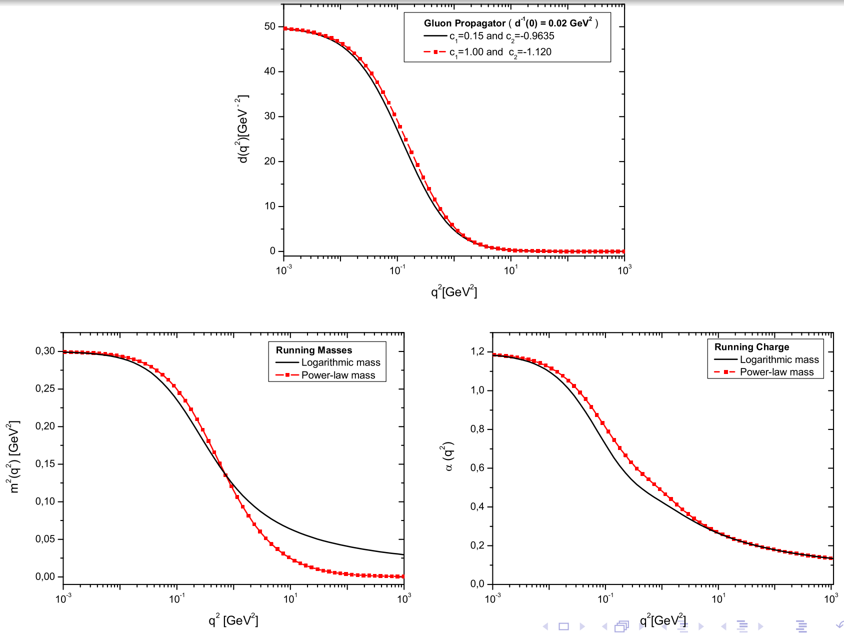




Numerical Analysis



Numerical Analysis



Summary

- We study the SD for the RG quantity $d(q^2)$
- Gluon propagator finite and fitted by a massive propagator
- The effective gluon mass has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

Summary

- We study the SD for the RG quantity $d(q^2)$
- **Gluon propagator finite** and fitted by a massive propagator
- The **effective gluon mass** has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

Summary

- We study the SD for the RG quantity $d(q^2)$
- Gluon propagator finite and fitted by a massive propagator
- The effective gluon mass has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

Summary

- We study the SD for the RG quantity $d(q^2)$
- Gluon propagator finite and fitted by a massive propagator
- The effective gluon mass has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

Summary

- We study the SD for the RG quantity $d(q^2)$
- Gluon propagator finite and fitted by a massive propagator
- The effective gluon mass has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

Summary

- We study the SD for the RG quantity $d(q^2)$
- Gluon propagator finite and fitted by a massive propagator
- The effective gluon mass has special properties:
  - It is a momentum dependent mass.
  - Vanishes in the deep ultraviolet region
  - Two asymptotic behaviors were found : Logarithmic a Power-law running
  - It is the 3-gluon vertex which decides what is behavior that the mass will develop!
- A effective charge (process-idependent) which displays:
  - Asympotic freedom in the UV
  - Infrared fixed point at IR! (Freezes at a finite value!)

- Outlook
  - Sctruture of the three-gluon vertex;
  - Ghost effects;
  - Lattice studies and phenomelogical applications;

References I

- A. C. Aguilar and J. Papavassiliou,
  In preparation
- A. C. Aguilar and J. Papavassiliou,
  "Gluon mass generation in the PT-BFM scheme,"
  JHEP **0612**, 012 (2006) [arXiv:hep-ph/0610040].
- A. C. Aguilar and J. Papavassiliou,
  "On dynamical gluon mass generation,"
  Eur. Phys. J. A **31**, 742 (2007)[arXiv:0704.2308 [hep-ph]].
- J. M. Cornwall,
  "Dynamical Mass Generation In Continuum QCD,"
  Phys. Rev. D **26** (1982) 1453.