Fedora GNU/Linux; LaTeX 2ϵ; xfig

Bulk viscosity of color-superconducting quark matter

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Reviews:

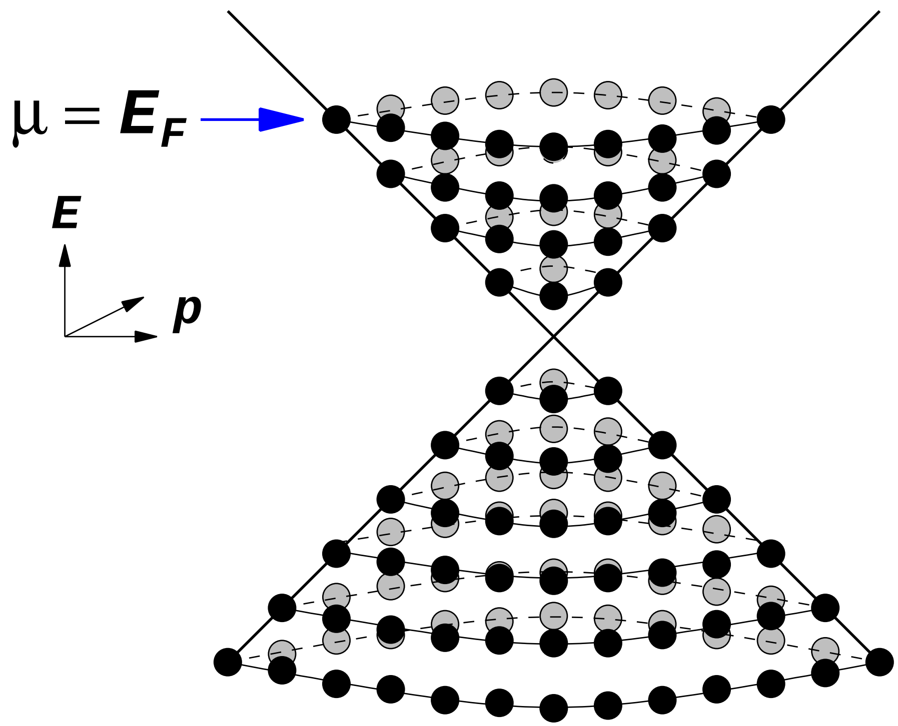
M. Alford, K. Rajagopal, hep-ph/0606157

T. Schäfer, hep-ph/0304281

K. Rajagopal, F. Wilczek, hep-ph/0011333

Color superconductivity: Cooper pairing of quarks

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



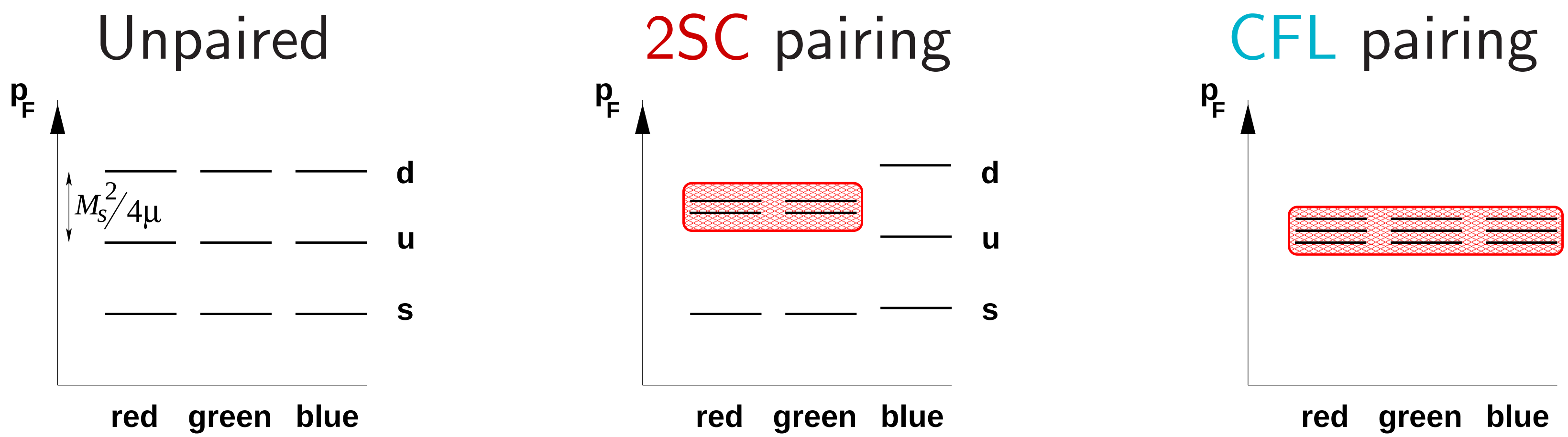
$$F = E - \mu N$$

But quarks have attractive QCD interactions.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

Color superconductivity in three flavor quark matter



2SC: Two-flavor pairing phase. May occur at intermediate densities.

$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3}\epsilon_{ij} \quad \text{i.e.,} \quad (rg - gr)(ud - du)$$

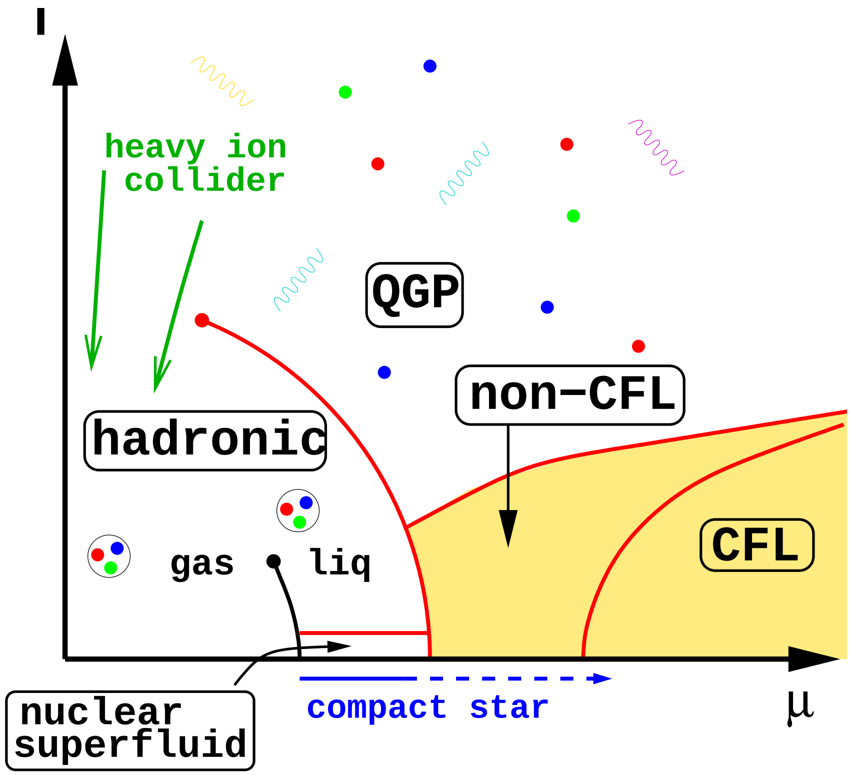
CFL: Color-flavor-locked phase, favored at the highest densities.

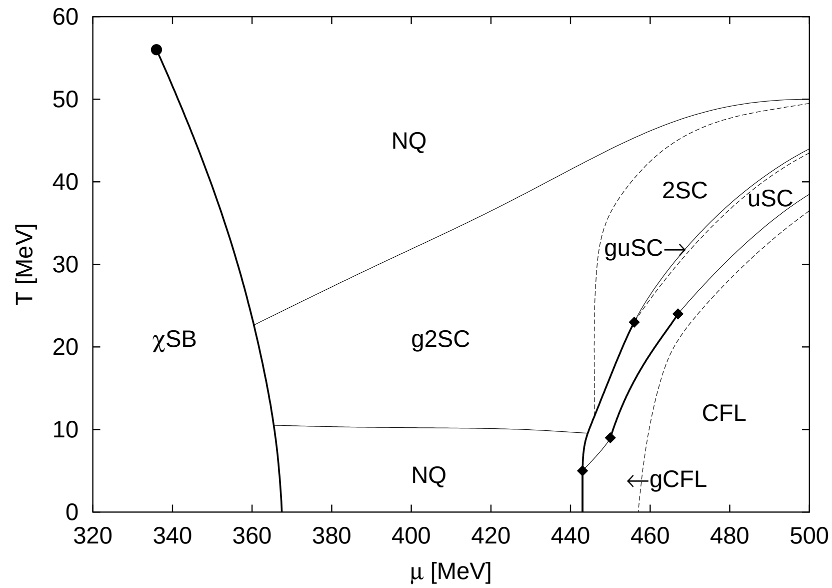
$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N}\epsilon_{ijN}$$

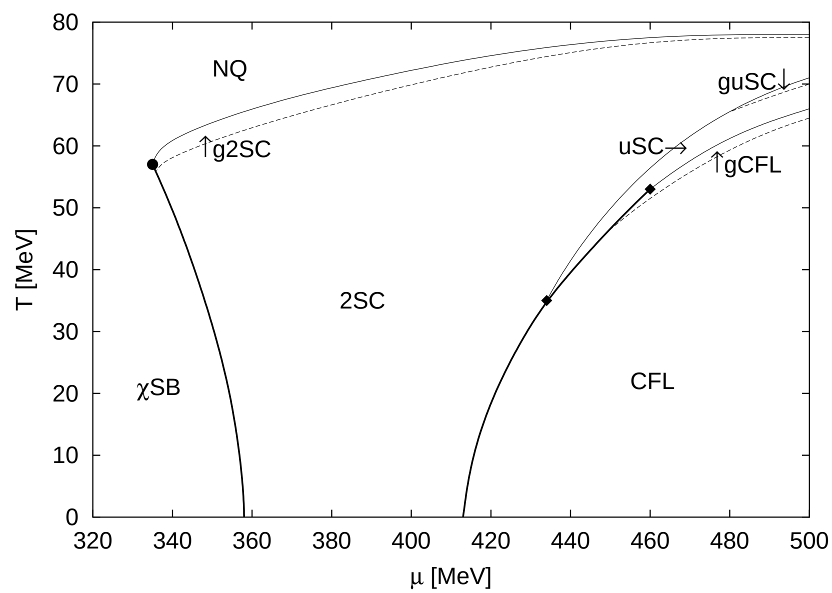
(color α, β, flavor $i, j = u, d, s$); (Alford, Rajagopal, Wilczek, `hep-ph/9804403`)

I. High density QCD

Conjectured phase diagram







Right panels: NJL model with coupled chiral and color-superconducting condensates.

(Rüster, Werth, Buballa, Shovkovy, Rischke, hep-ph/0503184)

Signatures of color superconductivity in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\ \text{km})$	$\geqslant \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

How would color superconductivity affect the star?

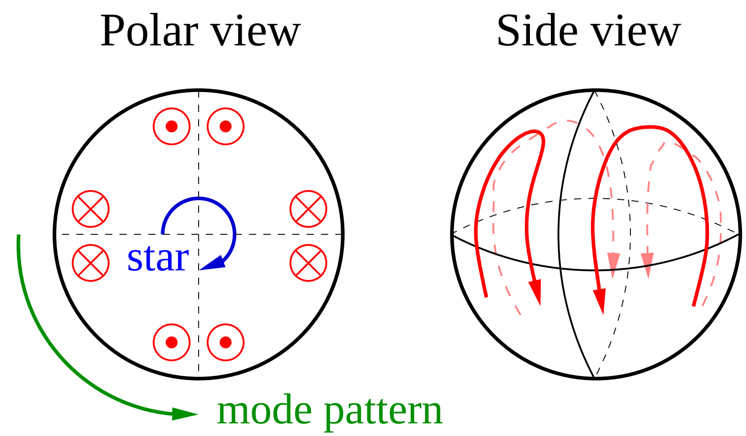
Pairing energy — affects Equation of state. Hard to detect.
(Alford, Braby, Paris, Reddy, `nucl-th/0411016`)

Gaps in quark spectra and Goldstone bosons — affect Transport properties:
emissivity, heat capacity, viscosity (shear, bulk),
conductivity (electrical, thermal)...

1. Cooling by neutrino emission, neutrino pulse at birth
   (Page, Prakash, Lattimer, Steiner, `hep-ph/0005094`; Carter and Reddy, `hep-ph/0005228`; Reddy, Sadzikowski, Tachibana, `nucl-th/0306015`; Grigorian, Blaschke, Voskresensky `astro-ph/0411619`).
2. Glitches and crystalline ("LOFF") pairing
   (Alford, Bowers, Rajagopal, `hep-ph/0008208`)
3. Gravitational waves: r-mode instability, shear and bulk viscosity
   (Madsen, `astro-ph/9912418`; Manuel, Dobado, Llanes-Estrada, `hep-ph/0406058`, Alford, Schmitt `nucl-th/0608019`, Alford, Braby, Reddy, Schäfer `nucl-th/0701067`, Manuel, Llanes-Estrada `arXiv:0705.3909`)

r-modes: gravitational spin-down of compact stars

An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



The Lindblom group at Caltech has made a movie of r-mode evolution.

`http://www.cacr.caltech.edu/projects/hydrligo/rmode.html`

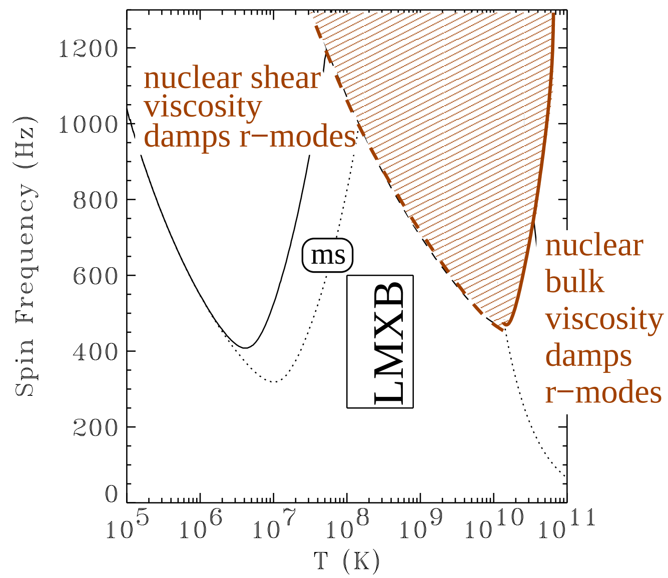
r-modes are unstable if rotation rate $\Omega > \Omega_{\rm crit}(T)$, and they can spin the star down within months (Andersson gr-qc/9706075; Friedman and Morsink gr-qc/9706073; Lindblom astro-ph/0101136).

Once we measure T and Ω for a star, we can put an upper limit on $\Omega_{\rm crit}(T)$.

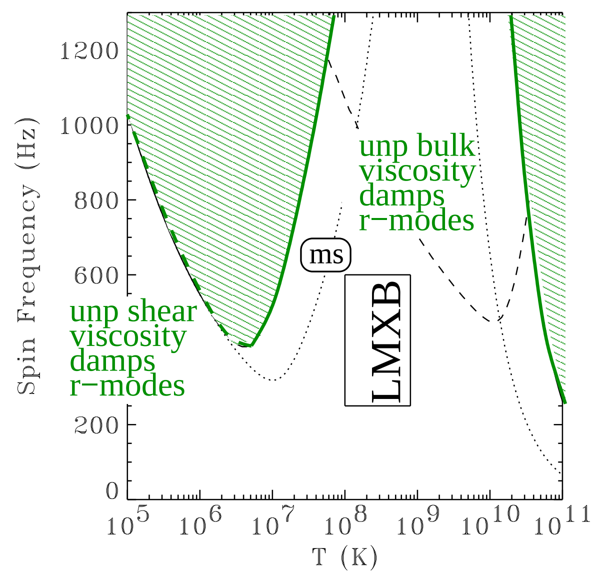
Constraints from r-modes (Madsen, astro-ph/9912418)

Predicted $\Omega_{\rm crit}(T)$ for various phases. Shaded regions above curves are unstable: viscosity is too low to hold back the r-modes.

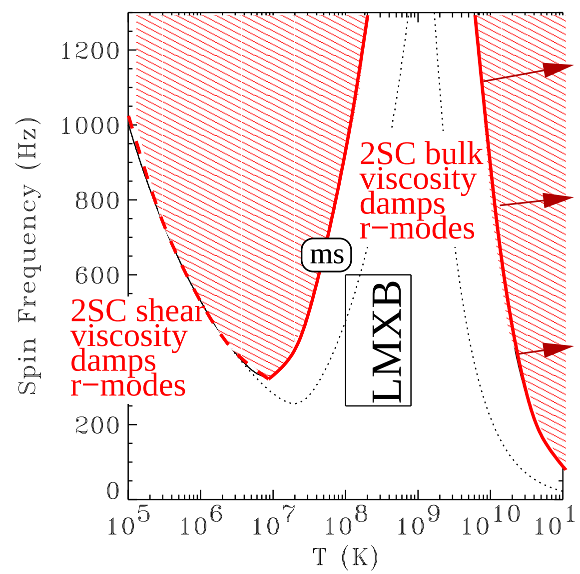
Nuclear matter



Unpaired ($m_s = 200$)



2SC ($m_s = 200$)



Dotted lines: $m_s = 100$ MeV; $\boxed{\text{ms}}$ = millisecond pulsars

According to Madsen's original calculation, pairing always lowers bulk viscosity, making 2SC more vulnerable to r-modes.

We find that actually $\zeta_{\rm 2SC} > \zeta_{\rm unp}$ at high T. We expect this will move the $\Omega_{\rm crit}$ line outward (dark red arrows).

What is bulk viscosity?

(L. *viscum* = mistletoe; It. vischio, Jp. 宿り木(やどりぎ), Gm. Mistelzweig, Sp. muérdago, Fr. gui, Ru. omela)
A sticky glue was made from mistletoe berries and coated onto small tree branches to catch birds.

Energy consumed in a compression cycle:

$$\begin{aligned} V(t) &= \bar{V} + \mathrm{Re}[\delta V \exp(i\omega t)] \\ p(t) &= \bar{p} + \mathrm{Re}[\delta p \exp(i\omega t)] \end{aligned}$$

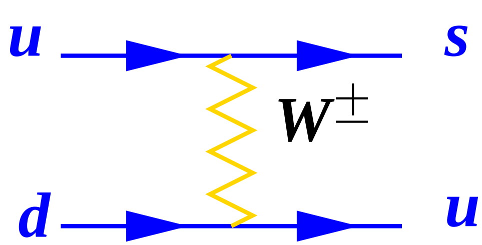
$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau} \int_0^\tau (\mathrm{div}\, \vec{v})^2 dt = \frac{\zeta}{2} \omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}} \int_0^\tau p(t) \frac{dV}{dt} dt$$

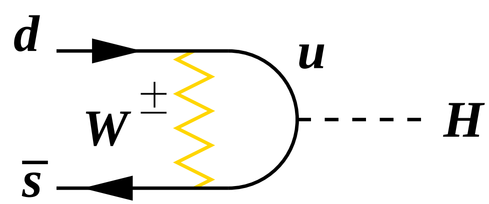
$$\Rightarrow \zeta(\omega, T) = -\frac{\bar{V}}{\delta V} \frac{\mathrm{Im}(\delta p)}{\omega}$$

Physically, bulk viscosity arises from re-equilibration processes. If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to τ, then pressure gets out of phase with volume and energy is consumed. (Just like V and Q in a R-C circuit.)

Flavor re-equilibration processes

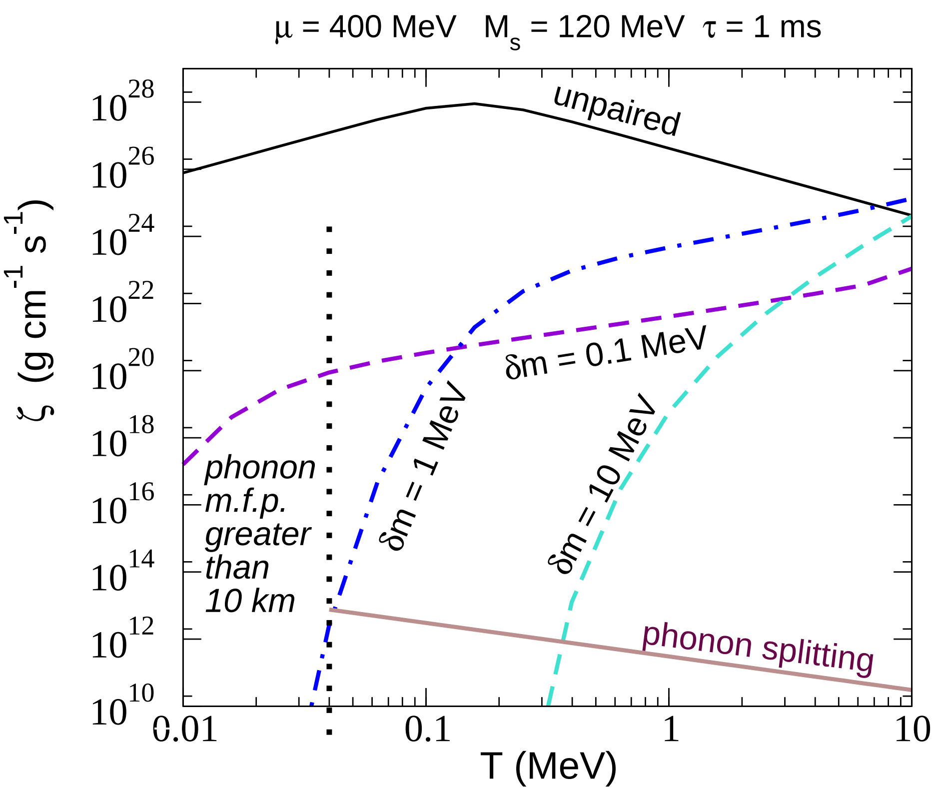
phase:	2SC	CFL (not CFL-K^0)
lightest modes:	unpaired ("blue") quarks	H, K^0
flavor equilibration:	$u + d \leftrightarrow s + u$	$K^0 \leftrightarrow H\ H$ $K^0\ H \leftrightarrow H$





CFL thermal kaon bulk viscosity

(Alford, Braby, Reddy, Schäfer `nucl-th/0701067`)



K^0 dispersion relation:

$$
\begin{aligned}
E(p) &= -\frac{M_s^2}{2\mu} + \sqrt{\tfrac{1}{3}p^2 + m_{K^0}^2} \\
&\approx \underbrace{m_{K^0} - \frac{M_s^2}{2\mu}}_{\delta m} + \frac{\frac{1}{3}p^2}{2m_{K^0}}
\end{aligned}
$$

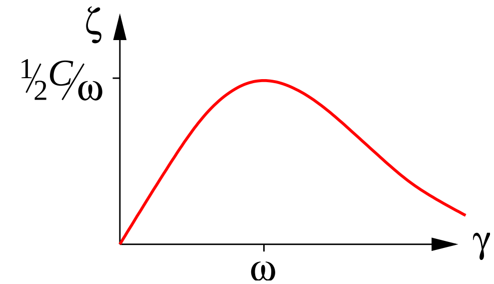
Thermal kaon density $\sim \exp(-\delta m/T)$, drops rapidly for $T \ll \delta m$.

Kaons dominate bulk viscosity for $T \gtrsim \delta m/30$.

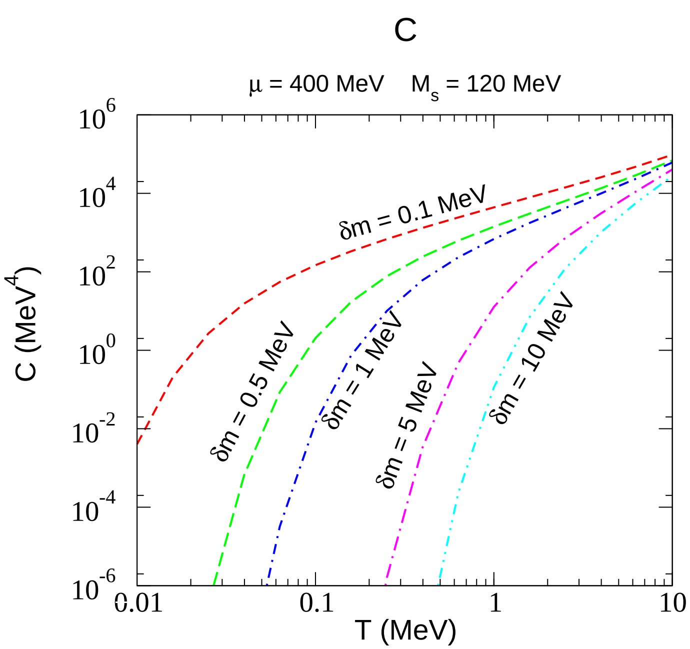
Superfluid mode ("phonon") splitting dominates in some temp range if $\delta m \gtrsim 2$ MeV (Manuel, Llanes-Estrada `arXiv:0705.3909`)

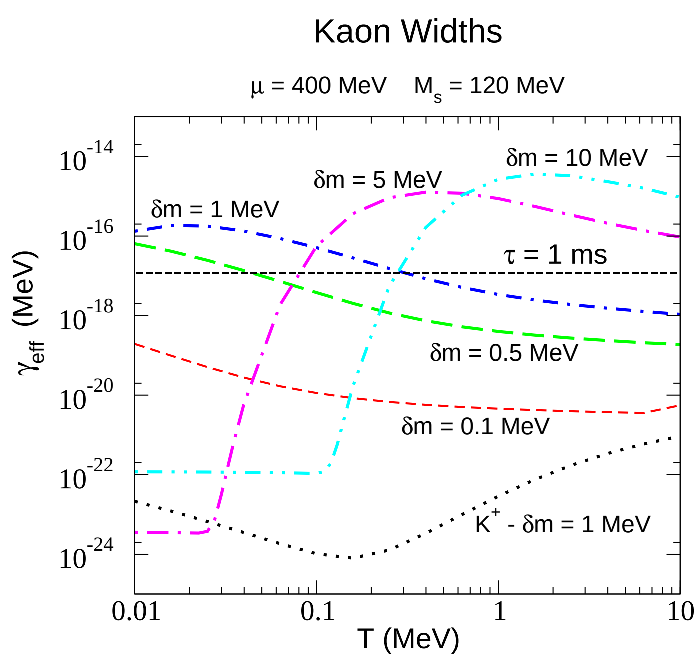
How bulk viscosity depends on equilibration rate

$$\zeta(\omega, T) = C(T)\frac{\gamma_K(T)}{\gamma_K(T)^2 + \omega^2}$$

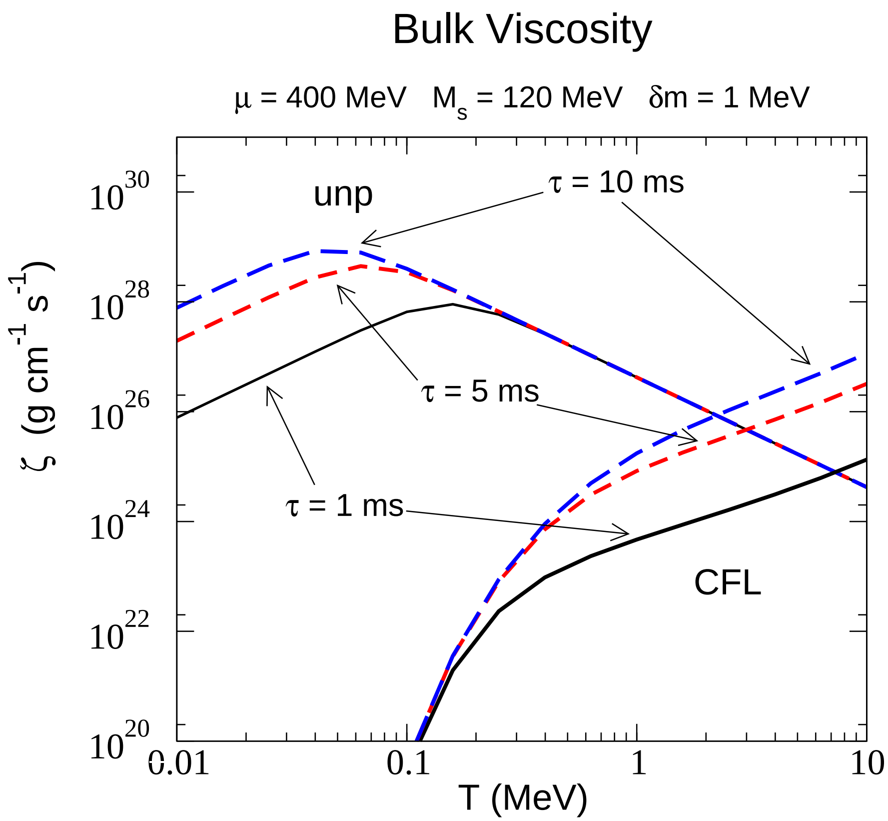


- ω is angular frequency of applied compression cycle.
- C measures the sensitivity of n_K and n_q to changes in μ_K and μ.
- γ_K is the average kaon width, from $K^0 \leftrightarrow H$.





CFL kaonic bulk viscosity: dependence on ω



$$\zeta(\omega, T) = C(T)\frac{\gamma_K(T)}{\gamma_K(T)^2 + \omega^2}$$

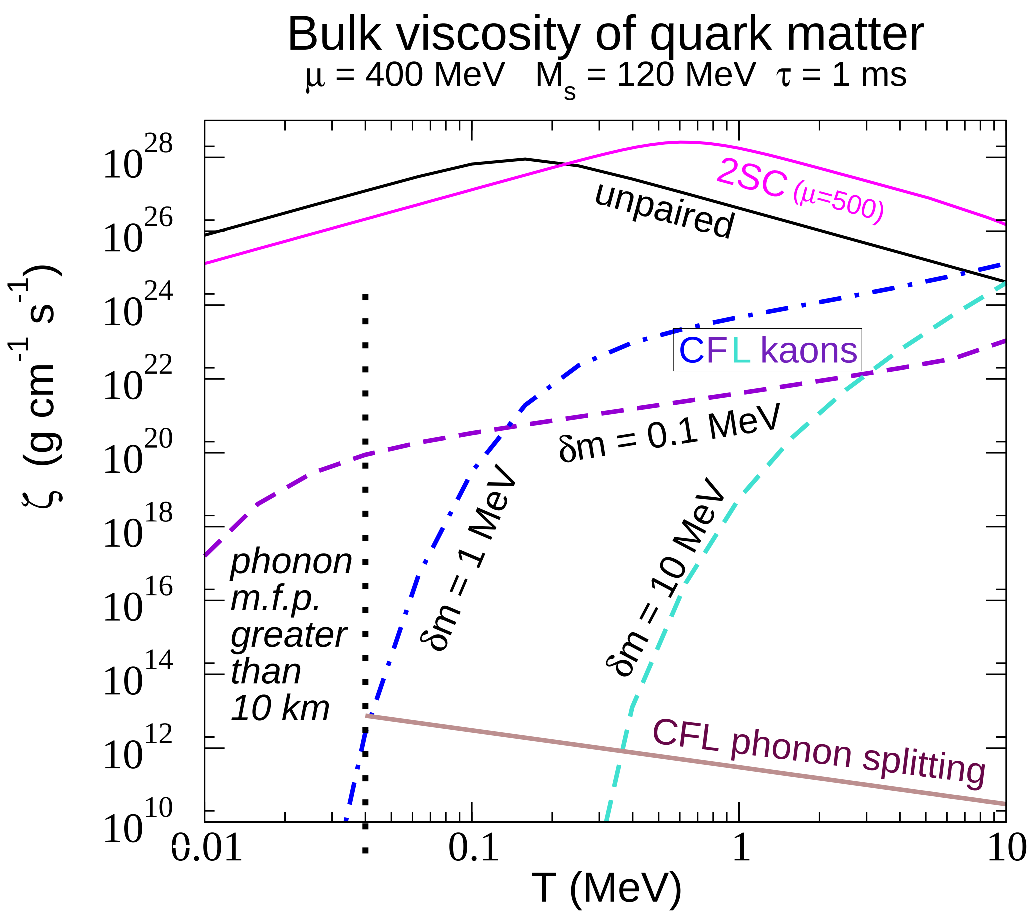
At high temp, $\gamma_K(T)$ rises, and ω becomes negligible.

For unpaired quark matter, C is indp of T, and the resonance peak at $\gamma_K(T) = \omega$ is clear.

As the frequency of compression drops,

- The peak in ζ_{unp}, which occurs where $\gamma_K(T) = \omega$, drops to lower temp.
- The peak value rises: $\zeta_{\max} = \frac{1}{2}C/\omega$.

Quark matter bulk viscosity: Summary



Alford, Schmitt `nucl-th/0608019`; Alford, Braby, Reddy, Schäfer `nucl-th/0701067`;
Manuel, Llanes-Estrada `arXiv:0705.3909`

- Unpaired and 2SC have the largest bulk viscosity, because they have unpaired modes at Fermi surface (large phase space).
- K^0 density $\sim \exp(-\delta m/T)$ drops rapidly for $T \lesssim \delta m/10$.
- $\delta m = m_{K^0} - M_s^2/(2\mu)$ could be anything from negative (kaon condensation) to ~ 10 MeV.
- Superfluid modes ("phonons") alone contribute some bulk viscosity.

Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - shear and bulk viscosity of CFL-K^0, other phases...
  - detailed analysis of r-mode profiles in hybrid star
  - heat capacity, conductivity and emissivity (neutrino cooling)
  - structure: nuclear-quark interface (gravitational waves?)
  - crystalline phase (glitches)
  - CFL: vortices but no flux tubes
- More general questions:
  - magnetic instability of gapless phases
  - better weak-coupling calculations, include vertex corrections
  - go beyond mean-field, include fluctuations
  - solve the sign problem and do lattice QCD at high density.

Calculating bulk viscosity for a known equilibration rate

Suppose the equilibrating quantity is n_y (this will be $n_d - n_s$).
Corresponding chemical potential $\mu_y = \delta\mu_y \exp(i\omega t)$.

We want $\mathrm{Im}(\delta p) = \mathrm{Im}\Big(\frac{dp}{d\mu_y}\delta\mu_y\Big) = n_y \mathrm{Im}(\delta\mu_y)$.

Write $\dot{n}_y$ two ways: $\dfrac{dn_y}{d\mu_y}\dot{\mu}_y = -\dfrac{\bar{n}_y}{\bar{V}}\dot{V} - (n_y - \bar{n}_y)\Gamma$ $\left\{\begin{array}{l}\text{equilibration}\\ \text{rate is } \Gamma\end{array}\right.$

$$\Rightarrow \frac{dn_y}{d\mu_y}(i\omega + \gamma)\delta\mu_y = -\frac{\bar{n}_y}{\bar{V}} i\omega\delta V \qquad \text{writing } \Gamma \equiv \gamma\frac{dn_y}{d\mu_y}$$

$$\Rightarrow \delta\mu_y = \frac{-i\omega}{i\omega + \gamma}\bar{n}_y\left(\frac{dn_y}{d\mu_y}\right)^{-1}\frac{\delta V}{\bar{V}}$$

$$\Rightarrow \mathrm{Im}(\delta p) = \bar{n}_y \mathrm{Im}(\delta\mu_y) = \frac{-\omega\gamma}{\omega^2 + \gamma^2}\frac{\delta V}{\bar{V}}\bar{n}_y^2\left(\frac{dn_y}{d\mu_y}\right)^{-1}$$

$$\Rightarrow \zeta = -\frac{\bar{V}}{\delta V}\frac{\mathrm{Im}(\delta p)}{\omega} = \boxed{\bar{n}_y^2\left(\frac{dn_y}{d\mu_y}\right)^{-1}\frac{\gamma}{\gamma^2 + \omega^2}}$$