Nonperturbative Effects in QCD near the deconfinement phase transition

D. Antonov, S. Domdey, H.-J. Pirner

Institut für Theoretische Physik, Universität Heidelberg

MARIE CURIE ACTIONS

QCD@Work, June 19, 2007

Contents

Introduction

• Motivation: Experimental data from RHIC $\Rightarrow$ the quark-gluon plasma behaves as a perfect fluid:

$$\frac{L_{\rm mfp}}{\beta} \ll 1,$$

$L_{\rm mfp}$ is a particle's mean free path, $\beta \equiv 1/T$ is the inter-particle distance.

Introduction

• Motivation: Experimental data from RHIC $\Rightarrow$ the quark-gluon plasma behaves as a perfect fluid:

$$\frac{L_{\rm mfp}}{\beta} \ll 1,$$

$L_{\rm mfp}$ is a particle's mean free path, $\beta \equiv 1/T$ is the inter-particle distance. For a dilute gas

$$L_{\rm mfp} \sim \frac{1}{n\sigma_t},$$

where $n \sim T^3$ is the particle-number density, σ_t is the Coulomb transport cross section:

$$\sigma_t = \int d\sigma(1 - \cos\theta) \sim g^4 \int_{(gT)^2} \frac{d^2p_\perp}{p_\perp^4}\frac{p_\perp^2}{T^2} \sim \frac{g^4}{T^2}\ln\frac{1}{g}.$$

Introduction

• Motivation: Experimental data from RHIC $\Rightarrow$ the quark-gluon plasma behaves as a perfect fluid:

$$\frac{L_{\rm mfp}}{\beta} \ll 1,$$

$L_{\rm mfp}$ is a particle's mean free path, $\beta \equiv 1/T$ is the inter-particle distance. For a dilute gas

$$L_{\rm mfp} \sim \frac{1}{n\sigma_t},$$

where $n \sim T^3$ is the particle-number density, σ_t is the Coulomb transport cross section:

$$\sigma_t = \int d\sigma(1-\cos\theta) \sim g^4 \int\limits_{(gT)^2} \frac{d^2p_\perp}{p_\perp^4}\frac{p_\perp^2}{T^2} \sim \frac{g^4}{T^2}\ln\frac{1}{g}.$$

$\Rightarrow$ a strong contradiction with the experiment:

$$\frac{L_{\rm mfp}}{\beta} \sim \frac{1}{g^4 \ln\frac{1}{g}} \gg 1.$$

Introduction

On the lattice, the two-point correlation function of Wilson lines (Polyakov loops) $L(\mathbf{R}) = \mathcal{P}\,\exp\left[ig\int_0^\beta dt A_4(\mathbf{R},t)\right]$ in the singlet channel was measured (F. Karsch, O. Kaczmarek, P. Petreczky, F. Zantow, '05):

$$\frac{1}{3}\,\mathrm{Tr}\,\left\langle L(\mathbf{R})L^\dagger(\mathbf{0})\right\rangle = \frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)} =$$

$$= \frac{1}{\mathcal{Z}(T)}\int \mathcal{D}A_\mu^a\mathcal{D}\bar{\psi}\mathcal{D}\psi\,\frac{1}{3}\,\mathrm{Tr}\,L(\mathbf{R})L^\dagger(\mathbf{0})\,\exp\left[-\int_0^\beta dt\int d^3x\mathcal{L}_{\mathrm{QCD}}(\mathbf{x},t)\right].$$

Introduction

On the lattice, the two-point correlation function of Wilson lines (Polyakov loops) $L(\mathbf{R}) = \mathcal{P}\ \exp\left[ig\int_0^\beta dtA_4(\mathbf{R},t)\right]$ in the singlet channel was measured (F. Karsch, O. Kaczmarek, P. Petreczky, F. Zantow, '05):

$$\frac{1}{3}\,\mathrm{Tr}\,\left\langle L(\mathbf{R})L^\dagger(\mathbf{0})\right\rangle = \frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)} =$$

$$= \frac{1}{\mathcal{Z}(T)}\int \mathcal{D}A_\mu^a\mathcal{D}\bar{\psi}\mathcal{D}\psi\,\frac{1}{3}\,\mathrm{Tr}\,L(\mathbf{R})L^\dagger(\mathbf{0})\ \exp\left[-\int_0^\beta dt\int d^3x\mathcal{L}_{\mathrm{QCD}}(\mathbf{x},t)\right].$$

The free energy of the static $Q\bar{Q}$-pair at a fixed large separation $|\mathbf{R}| \geq 1.5\,\mathrm{fm}$:

$$F(T) = -T\ln\left.\frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)}\right|_{\mathbf{R}\ \mathrm{fixed}}.$$

Introduction

On the lattice, the two-point correlation function of Wilson lines (Polyakov loops) $L(\mathbf{R}) = \mathcal{P}\exp\left[ig\int_0^\beta dtA_4(\mathbf{R},t)\right]$ in the singlet channel was measured (F. Karsch, O. Kaczmarek, P. Petreczky, F. Zantow, '05):

$$\frac{1}{3}\,\mathrm{Tr}\,\left\langle L(\mathbf{R})L^\dagger(\mathbf{0})\right\rangle = \frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)} =$$

$$= \frac{1}{\mathcal{Z}(T)}\int \mathcal{D}A_\mu^a\mathcal{D}\bar{\psi}\mathcal{D}\psi\,\frac{1}{3}\,\mathrm{Tr}\,L(\mathbf{R})L^\dagger(\mathbf{0})\exp\left[-\int_0^\beta dt\int d^3x\mathcal{L}_{\mathrm{QCD}}(\mathbf{x},t)\right].$$

The free energy of the static $Q\bar{Q}$-pair at a fixed large separation $|\mathbf{R}| \geq 1.5\,\mathrm{fm}$:

$$F(T) = -T\ln\frac{\mathcal{Z}_{Q\bar{Q}}(\mathbf{R},T)}{\mathcal{Z}(T)}\Bigg|_{\mathbf{R}\ \mathrm{fixed}}.$$

The corresponding entropy $S(T) = -\frac{\partial F(T)}{\partial T}$ and the internal energy $U(T) = F(T) + TS(T)$ exhibit maxima at $T \to T_c$, which cannot be explained by perturbation theory alone.

Introduction

The first part of this talk is devoted to an attempt to explain these data theoretically.

- Strategy and models:

- to determine an effective string tension $\sigma_{\rm eff}(T)$ in quenched SU(3) QCD.
Model: gluon chain = the $Q\bar{Q}$-string with multiple valence gluons.

- with the use of $\sigma_{\rm eff}(T)$ extrapolated to the unquenched case, to calculate $S(T)$ and $U(T)$ at $T < T_c$ for heavy-light mesons and heavy-light-light baryons, which are formed upon the string breaking and hadronization.
Model: the relativistic quark model.

Introduction

The first part of this talk is devoted to an attempt to explain these data theoretically.

• Strategy and models:

• to determine an effective string tension $\sigma_{\rm eff}(T)$ in quenched SU(3) QCD.
Model: gluon chain = the $Q\bar{Q}$-string with multiple valence gluons.

• with the use of $\sigma_{\rm eff}(T)$ extrapolated to the unquenched case, to calculate $S(T)$ and $U(T)$ at $T < T_c$ for heavy-light mesons and heavy-light-light baryons, which are formed upon the string breaking and hadronization.
Model: the relativistic quark model.

The second part of the talk is devoted to the analysis of the QCD running coupling in the infra-red region at $T = 0$ and $T > T_c$.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

At low enough temperatures, the free energy of one string bit in the gluon chain $>$ thermal gluon mass, which grows linearly with T. This situation changes at a certain temperature T_0, which is smaller than T_c.

$T < T_0 \Rightarrow$ an elastic string, gluons move collectively with it;
$T > T_0 \Rightarrow$ a sequence of static nodes with adjoint charges, connected by independently fluctuating string bits.

To form the gluon chain, the string originating at Q performs a random walk to $\bar{Q}$ over the lattice of such nodes. The large entropy of such a random walk eventually leads to the deconfinement phase transition.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

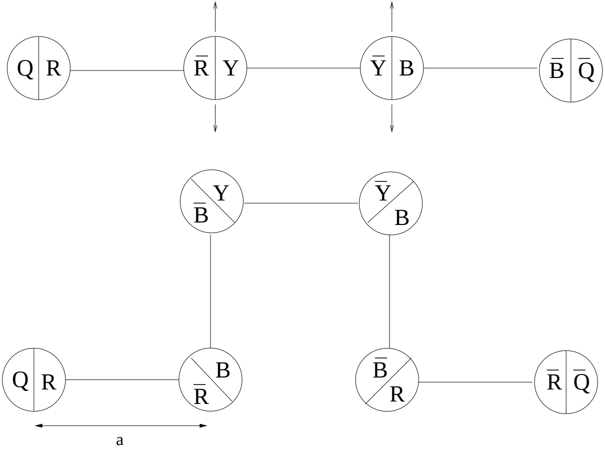

Figure: Gluon chain at $T < T_0$ and $T > T_0$. Below T_0, valence gluons move together with the string, while at $T > T_0$ they become static. Color may change from one string bit to another.

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

Every string bit may transport each of the N_c colors $\Rightarrow$ the total number of states of the gluon chain is $N_c^{L/a}$, where L is the length of the chain and a is the length of one bit.

The partition function of the random walk ($R \equiv |\mathbf{R}|$):

$$\mathcal{Z}(R,T) = \sum_{n=-\infty}^{+\infty} \int_0^\infty \frac{ds}{(4\pi s)^2} \exp\left[-\frac{R^2+(\beta n)^2}{4s} - \frac{s}{a}\left(\frac{\sigma}{T} - \frac{\ln N_c}{a}\right)\right].$$

The effective string tension and the critical temperature:

$$\sigma(T) = \sigma - \frac{T}{R} \ln \left.\frac{\mathcal{Z}(R,T)}{\mathcal{Z}(R,T_0)}\right|_{R\to\infty} =$$

$$= \sigma + \frac{T}{\sqrt{a}}\left[\sqrt{\frac{\sigma}{T} - \frac{\ln N_c}{a}} - \sqrt{\frac{\sigma}{T_0} - \frac{\ln N_c}{a}}\right]$$

$$\Rightarrow \quad T_c\Big|_{N_c>1} = \frac{\sigma a}{\ln N_c}.$$

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

$N_c = 3$, $T_c = 270\,\mathrm{MeV} \Rightarrow a \simeq 0.31\,\mathrm{fm}$. That is indeed larger than the vacuum correlation length, 0.22 fm, which defines the onset of a string-bit formation (A. Di Giacomo, M. D'Elia, E. Meggiolaro, H. Panagopoulos, '92–'03; G. Bali, N. Brambilla, A. Vairo, '97).
$\sigma(T_c) = 0 \Rightarrow$ the temperature below which the lattice of valence gluons does not exist:

$$T_0 = \frac{T_c}{\ln N_c + 1} \simeq 130\,\mathrm{MeV}.$$

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

$N_c = 3$, $T_c = 270\,\mathrm{MeV} \Rightarrow a \simeq 0.31\,\mathrm{fm}$. That is indeed larger than the vacuum correlation length, 0.22 fm, which defines the onset of a string-bit formation (A. Di Giacomo, M. D'Elia, E. Meggiolaro, H. Panagopoulos, '92–'03; G. Bali, N. Brambilla, A. Vairo, '97).
$\sigma(T_c) = 0 \Rightarrow$ the temperature below which the lattice of valence gluons does not exist:

$$T_0 = \frac{T_c}{\ln N_c + 1} \simeq 130\,\mathrm{MeV}.$$

An important result is the critical behavior

$$\sigma(T) \sim \sqrt{T_c - T} \quad \mathrm{at} \quad T \to T_c,$$

which is the same as in the Nambu-Goto model for the two-point correlation function of Polyakov loops (R.D. Pisarski and O. Alvarez, '82).

Static $Q\bar{Q}$-pair with gluons at $T < T_c$

Comparing to the limiting case when string bits cannot alter color:

$$\sigma(T) = \sigma + \sqrt{\frac{\sigma T}{a}}\left(1 - \sqrt{\frac{T}{T_0}}\right) \sim (T_c - T) \ \text{ at } T \to T_c \ \Rightarrow$$

the universality class of the 2d (!) Ising model, defined by the critical exponent $\nu = 1$, cannot be the right one for the 4d Yang-Mills theory.

The same linear fall-off of $\sigma(T)$ with $(T_c - T)$ one finds also in the

- Hagedorn phase transition: $S = \sigma R/T_H$, $F = \sigma R - TS$;

- deconfinement scenario based on the condensation of long closed strings: $S = \ln N$, $N = (2d-1)^{L/a}$ is the number of possibilities to realize on a hypercubic lattice a closed trajectory of length L, $F = \sigma L - TS \Rightarrow$ $T_c = \frac{\sigma a}{\ln(2d-1)}$, which yields 270 MeV only at $a = 0.54\,\text{fm} \simeq R/2$ (!).

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

In the unquenched case, the $Q\bar{Q}$-string breaks due to the production of a light $q\bar{q}$-pair. Hadronization $\Rightarrow$ formation of heavy-light mesons ($Q\bar{q}$), heavy-light-light baryons (Qqq), and their antiparticles. Considering the ($N_f = 2$)-case, with light u- and d-quarks, and using the value $T_c = 200\,\mathrm{MeV} \Rightarrow a = 0.23\,\mathrm{fm}$.

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

In the unquenched case, the $Q\bar{Q}$-string breaks due to the production of a light $q\bar{q}$-pair. Hadronization $\Rightarrow$ formation of heavy-light mesons $(Q\bar{q})$, heavy-light-light baryons (Qqq), and their antiparticles. Considering the $(N_f = 2)$-case, with light u- and d-quarks, and using the value $T_c = 200\,\mathrm{MeV} \Rightarrow a = 0.23\,\mathrm{fm}$.

Calculating the partition function of two noninteracting heavy-light mesons and two heavy-light-light baryons within the relativistic quark model, e.g.

$$H_{\bar{Q}q} = m_{\bar{Q}} + \sqrt{\mathbf{p}^2 + m_q^2} + V(r),$$

where $V(r) = \sigma(T)r - C\sqrt{\sigma(T)}$, m_q is the constituent mass of a light quark, $m_q \simeq 300\,\mathrm{MeV}$, and $C \simeq 1.65$ is fixed by the limit $T \to 0$.

Calculating further the entropies and the internal energies of these mesons and baryons together (D.A., S. Domdey, H.-J. Pirner, NPA '07).

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

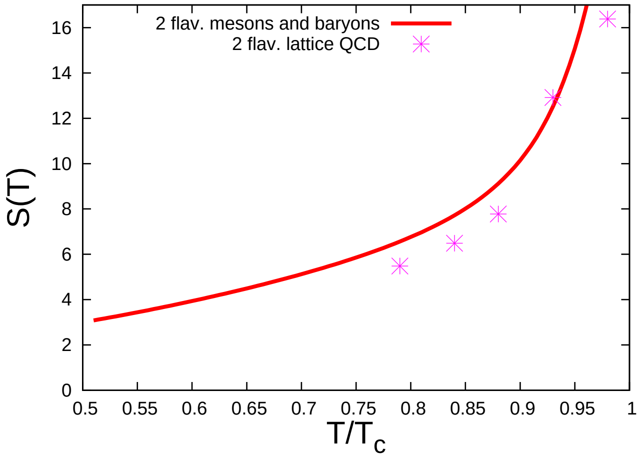

Figure: The calculated entropy $S(T)$ (full drawn curve) of two mesons and two baryons as a function of T/T_c with $T_c = 200\,\mathrm{MeV}$. The stars show the lattice data (O. Kaczmarek and F. Zantow, '05).

Static $Q\bar{Q}$-pair with light quarks at $T < T_c$

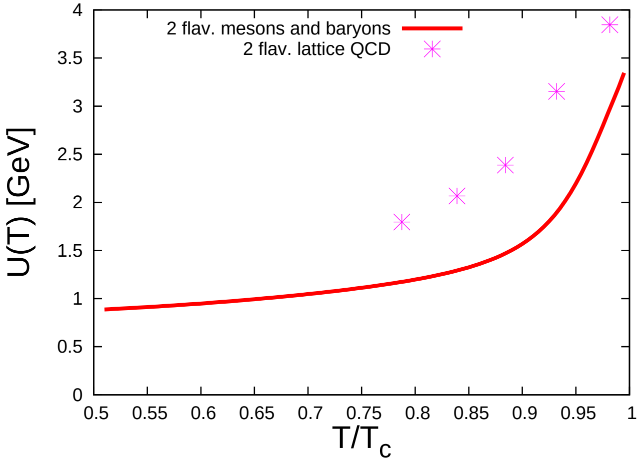

Figure: The calculated internal energy $U(T)$ (full drawn curve) of two mesons and two baryons as a function of T/T_c with $T_c = 200\,\mathrm{MeV}$. The stars show the lattice data (O. Kaczmarek and F. Zantow, '05).

QCD running coupling at $T = 0$

Motivation: To calculate the QCD running coupling in the infra-red region at $T = 0$ and $T > T_c$.

A reminder: calculation of $\alpha_s(p)$ through the integration over high-momentum gluonic fluctuations (A.M. Polyakov, '87),

$$A^a_\mu = \bar{A}^a_\mu + a^a_\mu, \quad p(\bar{A}^a_\mu) < p(a^a_\mu).$$

In the background Feynman gauge,

$$(D_\mu a_\nu)^a = \partial_\mu a^a_\nu + f^{abc}\bar{A}^b_\mu a^c_\nu = 0,$$

the kinetic term of the fluctuations is

$$S_{\rm kin} = -\frac{1}{2g_0^2}\int d^4x\, a^a_\mu \left\{\delta_{\mu\nu}(D^2)^{ac} + 2f^{abc}F^b_{\mu\nu}[\bar{A}]\right\} a^c_\nu.$$

QCD running coupling at $T = 0$

$\int \mathcal{D}a_\mu^a \Rightarrow$ two mutually competing effects:

Landau diamagnetic interaction of the $\bar{A}_\mu^a$-field with the orbital motion of the a_μ^a-gluons, which leads to the screening of charge and is present in the Abelian case too:

$$S^{\mathrm{dia}} = \mathrm{tr}\,\ln(-D^2);$$

Pauli paramagnetic interaction of the $\bar{A}_\mu^a$-field with the spin of the a_μ^a-gluons, which leads to the antiscreening of charge and is a specific property of the non-Abelian gauge theories:

$$S^{\mathrm{para}} = \mathrm{tr}\,\ln\left[\delta^{ac}\delta_{\mu\nu}\partial^2 + 2f^{abc}F_{\mu\nu}^b\right].$$

QCD running coupling at $T = 0$

The result:

$$S^{\left\{ \substack{\text{dia} \\ \text{para}} \right\}} = N_c \cdot \left\{ \begin{array}{c} \frac{1}{12} \\ (-1) \end{array} \right\} \cdot \int \frac{d^4 p}{(2\pi)^4} F^a_{\mu\nu}(p) F^a_{\mu\nu}(-p) \Pi_{\rm free}(p^2),$$

where

$$\Pi_{\rm free}(p^2) \equiv \frac{1}{16\pi^2} \ln \frac{\Lambda_0^2}{p^2}$$

is the free scalar polarization operator.

The paramagnetic effect is opposite in sign and is 12 times larger by an absolute value than the diamagnetic one. For this reason, a non-Abelian charge is antiscreened, and the Yang-Mills theory is asymptotically free.

QCD running coupling at $T = 0$

The full renormalized effective action

$$S = S_0 + S^{\mathrm{dia}} + S^{\mathrm{para}} = \int \frac{d^4p}{(2\pi)^4} \frac{1}{4g^2(p)} F^a_{\mu\nu}(p) F^a_{\mu\nu}(-p)$$

yields the running coupling $g(p)$ expressed through the bare one $g_0 = g(\Lambda_0)$:

$$\frac{1}{g^2(p)} = \frac{1}{g_0^2} - b\Pi_{\mathrm{free}}(p^2), \quad \text{where } b = 4\left(1 - \frac{1}{12}\right) N_c = \frac{11 N_c}{3}.$$

This leads to the standard result:

$$\alpha_s(p) = \frac{4\pi}{b \ln \frac{p^2}{\Lambda^2}}, \text{ where } \Lambda = \Lambda_0 \exp\left(-\frac{8\pi^2}{b g_0^2}\right)$$

is the renormalized cutoff.

QCD running coupling at $T = 0$

In reality, however, the a_μ^a-gluons are self-confined, rather than free $\Rightarrow$ two a_μ^a-gluons propagating along the loop form a colored bound state with the string tension in the octet channel $\sigma = \frac{9}{8}\sigma_{\rm fund}$.

The a_μ^a-gluons may be confined because of the stochastic background fields B_μ^a's (Yu.A. Simonov, '93):

$$A_\mu^a = B_\mu^a + \bar{A}_\mu^a + a_\mu^a, \quad p(B_\mu^a) < p(\bar{A}_\mu^a); \quad S = -\ln\left\langle \int \mathcal{D}a_\mu^a \mathrm{e}^{-S[A]} \right\rangle_B,$$

where $\langle \ldots \rangle_B$ is some gauge- and $O(4)$-invariant integration measure.

QCD running coupling at $T = 0$

Due to the background, $\partial^2 \to D^2[B] \Rightarrow$

$$\Pi_{\rm free}(R) = \frac{1}{(4\pi^2 R^2)^2}, \quad \text{where } R \equiv |x-y|,$$

goes over to

$$\Pi(x,y) \equiv \left\langle {\rm tr}\,(D^2[B])^{-1}_{xy}(D^2[B])^{-1}_{yx}\right\rangle_B \simeq$$

$$\simeq \int_0^\infty ds \int_0^\infty d\bar{s} \int (\mathcal{D}z_\mu)_{xy}(\mathcal{D}\bar{z}_\mu)_{yx} \exp\left(-\int_0^s d\lambda \frac{\dot{z}_\mu^2}{4} - \int_0^{\bar{s}} d\bar{\lambda}\frac{\dot{\bar{z}}_\mu^2}{4}\right)$$

$$\times \left\langle {\rm tr}\,\mathcal{P} \exp\left[i\left(\int_0^s d\lambda \dot{z}_\mu B_\mu^a(z)t^a + \int_0^{\bar{s}} d\lambda \dot{\bar{z}}_\mu B_\mu^a(\bar{z})t^a\right)\right]\right\rangle_B .$$

At $R \gtrsim 1\,{\rm fm}$, $\langle\ldots\rangle_B \simeq {\rm e}^{-\sigma S_{\rm min}}$, where $S_{\rm min}$ is the area of the minimal surface encircled by the paths $z_\mu(s)$ and $\bar{z}_\mu(\bar{s})$.

QCD running coupling at $T = 0$

The path integral becomes calculable through the Cauchy-Schwarz inequality:

$$S_{\min} = \int_0^R d\tau |\mathbf{r}(\tau)| \leq \left(R \int_0^R d\tau \mathbf{r}^2(\tau) \right)^{1/2},$$

where $|\mathbf{r}(\tau)|$ is the size of C in the direction transverse to τ.

(For a pair of linearly confined massive particles in 2d, the accuracy of this approximation for $S_{\min}$ is 9.6%.)

The path integral becomes calculable through the Cauchy-Schwarz inequality:

$$S_{\rm min} = \int_0^R d\tau |\mathbf{r}(\tau)| \le \left(R \int_0^R d\tau \mathbf{r}^2(\tau) \right)^{1/2},$$

where $|\mathbf{r}(\tau)|$ is the size of C in the direction transverse to τ.

(For a pair of linearly confined massive particles in 2d, the accuracy of this approximation for $S_{\rm min}$ is 9.6%.)

The result:

$$\Pi(R) = \frac{1}{16\pi^4 R^4} {\rm e}^{-A\sqrt{\sigma R^2}} \left(1 + B(\sigma R^2)^{5/4}\right),$$

where $A = 2\sqrt{3} \simeq 3.46$, $B = \frac{2^{3/2}\pi}{3^{1/4}} \simeq 6.75$.

The analytic value of the freezing mass $m_{\rm an} = A\sqrt{\sigma} = 1.64\,{\rm GeV}$. In d-dimensional space-time,

$$m_{\rm an} = 2\sqrt{(d-1)\sigma}.$$

QCD running coupling at $T = 0$

Fitting $\frac{d\Pi(p^2)}{dp^2} \equiv \frac{F(q)}{\sigma}$, where $q = |p|/\sqrt{\sigma}$, by the function $F_{\rm fit}(q) = -\frac{1}{16\pi^2}\frac{d_1}{q^2+d_2^2}$, we obtain: $d_1 = 0.585$, $d_2 = 2.491$.

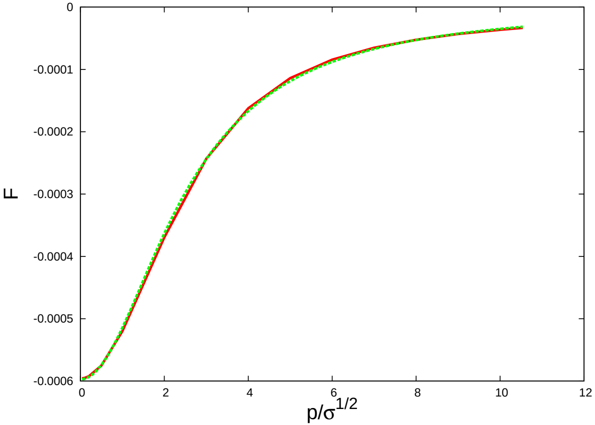

Figure: The function $F(q)$ (red) and a fit to it (green).

QCD running coupling at $T = 0$

The fit yields a smaller value of the freezing mass, $m = d_2\sqrt{\sigma} = 1.18\,\mathrm{GeV}$, which is closer to the phenomenological estimate $m \simeq 1\,\mathrm{GeV}$ (D. Ebert, R.N. Faustov, V.O. Galkin, '05, '06).

The coefficient b effectively decreases: $b \to \tilde{b} = d_1 b = 6.435$. Altogether,

$$\alpha_s(p) = \frac{4\pi}{b \ln \frac{p^2}{\Lambda^2}} \to \frac{4\pi}{\tilde{b} \ln \frac{p^2+m^2}{\Lambda^2}}.$$

QCD running coupling at $T > T_c$

At $T > T_c$, large spatial Wilson loops still exhibit the area law $\Rightarrow$ at $T_c < T < T_{\rm d.r.} \simeq 2T_c$ and Minkowskian $p^2 < 0$, freezing of $\alpha_s(p)$ takes place at the temperature-dependent momentum scale $m(T) = 2\sqrt{2\sigma_s(T)}$, where $\sigma_s(T)$ is the spatial string tension:

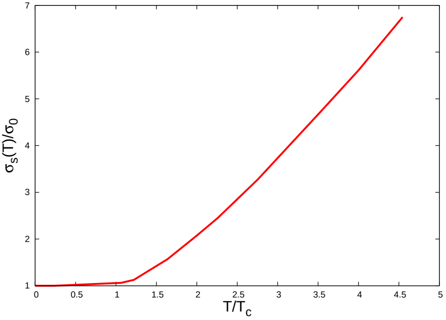

Figure: The curve interpolating the lattice data on the ratio of the spatial string tension to the zero-temperature one in the SU(3) quenched QCD as a function of T/T_c (G. Boyd *et al.*, '96).

QCD running coupling at $T > T_c$

Numerically, $d_1 = 0.711$, $d_2 = 2.289$. When T varies from T_c to $T_{\mathrm{d.r.}}$, $m(T)$ varies from 1.09 GeV to 1.56 GeV. The parameter $\tilde{b} = d_1 b = 7.821$ is larger than the one at zero-temperature.

Matching $\alpha_s(p)$ with freezing to the experimental value at a sufficiently large momentum scale, e.g. $\alpha_s(3\,\mathrm{GeV}) \simeq 0.25$,

QCD running coupling at $T > T_c$

Numerically, $d_1 = 0.711$, $d_2 = 2.289$. When T varies from T_c to $T_{\rm d.r.}$, $m(T)$ varies from 1.09 GeV to 1.56 GeV. The parameter $\tilde{b} = d_1 b = 7.821$ is larger than the one at zero-temperature.

Matching $\alpha_s(p)$ with freezing to the experimental value at a sufficiently large momentum scale, e.g. $\alpha_s(3\,\mathrm{GeV}) \simeq 0.25$, $\Rightarrow$ at a fixed p, one observes

- an increase of $\alpha_s(p)$ at $T = T_c$ with respect to $\alpha_s(p)$ at $T = 0$ due to the increase of $\tilde{b}$;
- a subsequent decrease of $\alpha_s(p)$ with the growth of T due to the increase of $m(T)$.

D.A. and H.-J. Pirner, EPJ C, '07.

QCD running coupling at $T > T_c$

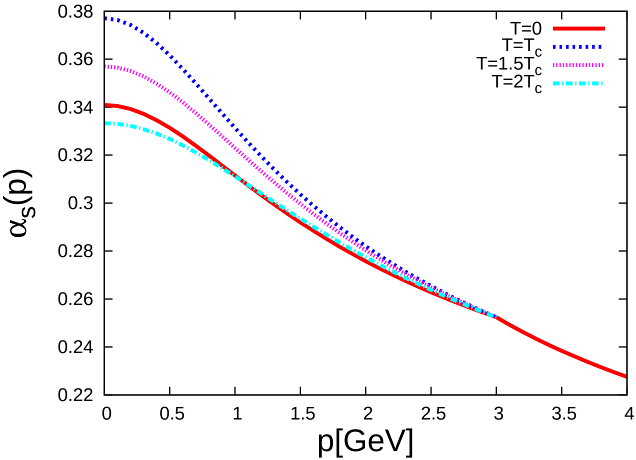

Figure: The running coupling with freezing at $0 \leq p \leq 3\,\mathrm{GeV}$ for $T = 0$, $T = T_c$, $T = 1.5\,T_c$, and $T = 2\,T_c$. The curve at $3\,\mathrm{GeV} \leq p \leq 4\,\mathrm{GeV}$ is the experimental $\alpha_s(p)$, according to the Particle Data Group.

A phenomenological test of $\alpha_s(p)$

The thrust variable

$$\mathcal{T} = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \mathbf{n}|}{\sum_i |\mathbf{p}_i|},$$

where $\mathbf{p}_i$'s are the momenta of the final-state hadrons and $\mathbf{n}$ is an arbitrary vector, which maximizes $\mathcal{T}$.

For an ideal pencil-like jet, $\mathcal{T} = 1$. Due to radiation of gluons, $\mathcal{T} \neq 1$. Hard-gluon radiation (R.K. Ellis, W.J. Stirling, B.R. Webber, '96) $\Rightarrow$

$$1 - \mathcal{T}\Big|_{\rm pert} = 0.334\alpha_s^{\rm pert}(p) + 1.02(\alpha_s^{\rm pert}(p))^2 + \mathcal{O}\left((\alpha_s^{\rm pert}(p))^3\right).$$

A correction due to soft-gluon radiation (Yu.L. Dokshitzer, G. Marchesini, B.R. Webber, '96) $\Rightarrow$

$$1 - \mathcal{T} = 1 - \mathcal{T}\Big|_{\rm pert} + \frac{2\lambda}{p}, \quad \text{where}$$

$$\lambda = \frac{C_F}{\pi}\int_0^{3\,{\rm GeV}} dp\alpha_s(p), \quad C_F = 4/3.$$

A phenomenological test of $\alpha_s(p)$

At the scale $p = M_Z$,

$$1 - \mathcal{T}\Big|_{\rm pert} \simeq 0.055,$$

which underestimates the experimental value measured at LEP (Z. Kunszt *et al.*, '89)

$$1 - \mathcal{T}\Big|_{\rm exp} \simeq 0.068.$$

Accounting for the soft-gluon radiation correction with $\alpha_s(p) = \frac{4\pi}{\tilde{b}\ln\frac{p^2+m^2}{\Lambda^2}}$:

$$1 - \mathcal{T}\Big|_{T=0.27\,{\rm GeV}} \simeq 0.064,$$

which is closer to the experimental value.

Conclusions

- Gluon-chain model in quenched SU(N_c) QCD below $T_c \Rightarrow$ $\sigma(T) \sim \sqrt{T_c - T}$ at $T \to T_c$; a correct estimate for T_c at $N_c = 3$.

- Unquenched case below T_c: the canonical partition function of two heavy-light mesons and two heavy-light-light baryons, which are formed after the string breaking, with $\sigma(T)$ for $N_f = 2 \Rightarrow$ entropy and internal energy reproduce well the corresponding lattice data for the static $Q\bar{Q}$-pair.

- A direct calculation of the path integral for the polarization operator of a valence gluon in the confining background fields $\Rightarrow$ infra-red freezing of $\alpha_s(p)$, also at $T_c < T < T_{\mathrm{d.r.}} \simeq 2T_c$ and $p^2 < 0$.

- The soft-gluon radiation contribution to the thrust variable, expressed through the first moment of $\alpha_s(p)$, makes the purely perturbative value of this variable closer to the experimental one.

Outlook

To explore the influence of magnetic confinement on the:

• jet quenching parameter (work in collaboration with D. Dietrich and H.-J. Pirner is in progress)

• rescattering of binary color bound states in the quark-gluon plasma $\Rightarrow$ a decrease of $L_{\rm mfp}$ is expected

• charm diffusion constant (its decrease with respect to the perturbative-QCD predictions is expected)

• spectra of collective excitations (plasmons and plasminos) in the quark-gluon plasma