Double charmonium production at B-factories and charmonium distribution amplitudes.

V.V. Braguta
Institute for High Energy Physics
Protvino, Russia

Content:

- Introduction
- The study of 1S and 2S charmonium distribution amplitudes *(Potential models, NRQCD, QCD sum rules approaches)*
- Properties of distribution amplitudes
- Application of distribution amplitudes to double charmonium production at B-factories

The results were obtained in papers:

- “**The study of leading twist light cone wave functions of eta_c meson**”
  V.V. Braguta, A.K. Likhoded, A.V. Luchinsky
  Phys.Lett.B646:80-90,2007

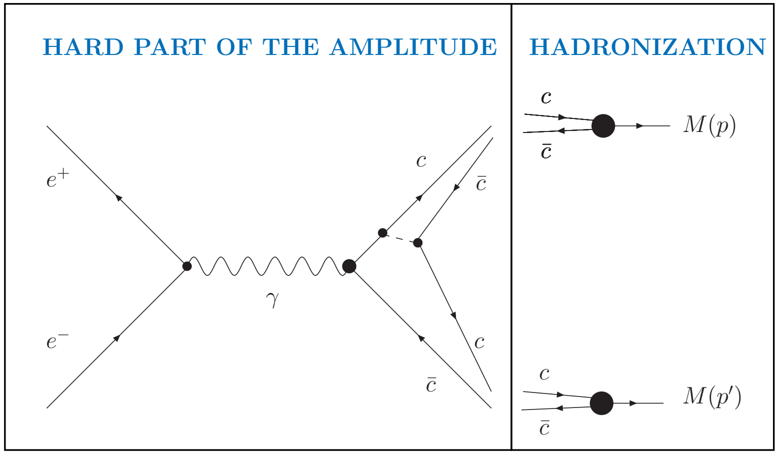
- “**The study of leading twist light cone wave functions of J/Psi meson**”
  V.V. Braguta
  Phys.Rev.D75:094016,2007

- “**The study of leading twist light cone wave functions of 2S state charmonium mesons**”
  V.V. Braguta, to be published

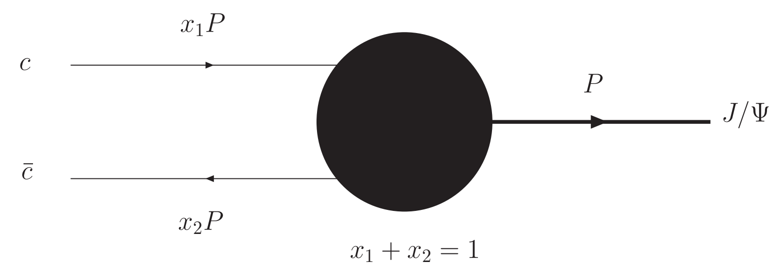
Introduction

Light cone formalism

The amplitude is divided into two parts:



Hadronization



Twist-2	**2-distribution amplitudes**
Twist-3	**4-distribution amplitudes**
…	**…**

$$\text{The cross section}:\ \sigma = \frac{\sigma_0}{s^n} + \frac{\sigma_1}{s^{n+1}} + \frac{\sigma_2}{s^{n+2}} + \ldots$$

Light cone formalism can be considered as an alternative to NRQCD

Advantages

1. Light cone formalism resums relativistic corrections, if DA is known

2. DA resums leading logarithmic radiative correction to the amplitude $\sim \alpha_s\, Log(Q)$

$$\mathrm{M} = \int_{-1}^{1} \mathrm{d}\xi\, \mathrm{H}(\xi)\, \phi(\xi,\mathrm{Q}), \qquad \xi = x_1 - x_2$$

DA is key ingredient of light cone formalism.

Definitions of leading twist DA

$$\langle 0|\bar{Q}(z)\gamma_\alpha\gamma_5[z,-z]Q(-z)|P(p)\rangle_\mu = if_P p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_P(\xi,\mu)$$

$$\langle 0|\bar{Q}(z)\gamma_\alpha[z,-z]Q(-z)|V(\epsilon_{\lambda=0},p)\rangle_\mu = f_L p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_L(\xi,\mu)$$

$$\langle 0|\bar{Q}(z)\sigma_{\alpha\beta}[z,-z]Q(-z)|V(\epsilon_{\lambda=\pm1},p)\rangle_\mu = f_T(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha) \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_T(\xi,\mu)$$

$$[z,-z] = P\exp[ig\int_{-z}^{z} dx^\mu A_\mu(x)]$$

DAs $\phi_P(\xi,\mu), \phi_L(\xi,\mu), \phi_T(\xi,\mu)$ are ξ - even

Evolution of DA

- DA can be parameterized through the coefficients of conformal expansion a_n :

$$\phi_{P,L,T}(\xi,\mu) = \frac{3}{4}(1-\xi^2)\left[1+\sum_{n=2,4..} a_n^{P,L,T}(\mu) C_n^{3/2}(\xi)\right]$$

$$a_n^{P,L,T}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\epsilon_n^{P,L,T}/b_0} a_n^{P,L,T}(\mu_0)$$

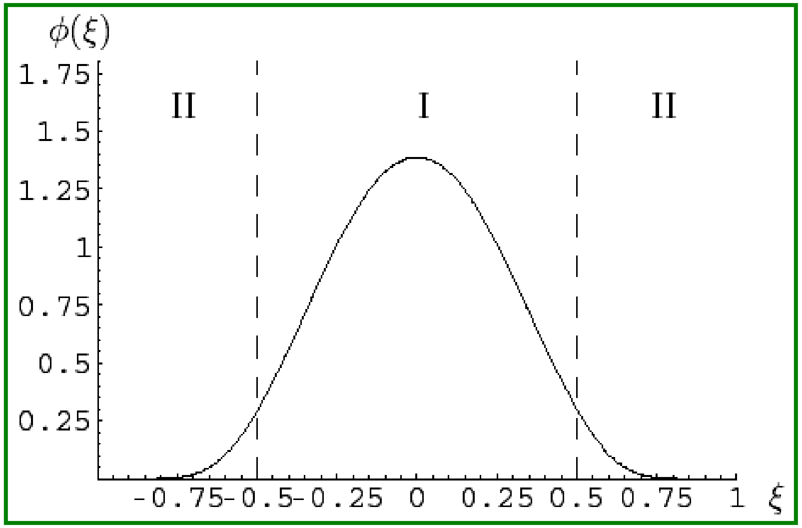
- Alternative parameterization through the moments:

$$\begin{aligned}
\langle \xi^n \rangle_\mu &= \int_{-1}^{1} d\xi\ \xi^n \phi(\xi,\mu) \\
i f_P p_\nu (zp)^n \langle \xi_P^n \rangle_\mu &= \langle 0|\bar{Q}\gamma_\nu\gamma_5 (iz^\sigma \overleftrightarrow{D}_\sigma)^n Q|P\rangle_\mu \\
f_L p_\nu (zp)^n \langle \xi_L^n \rangle_\mu &= \langle 0|\bar{Q}\gamma_\nu (iz^\sigma \overleftrightarrow{D}_\sigma)^n Q|V(\epsilon_{\lambda=0},p)\rangle_\mu \\
f_T(\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)(zp)^n \langle \xi_T^n \rangle_\mu &= \langle 0|\bar{Q}\sigma_{\mu\nu} (iz^\sigma \overleftrightarrow{D}_\sigma)^n Q|V(\epsilon_{\lambda=\pm 1},p)\rangle_\mu
\end{aligned}$$

DA of nonrelativistic system

Properties:

1. The width of DA is $\xi^2 \sim v^2$
2. The motion in Region I ($\xi^2 \sim v^2$) is nonerlativistic
3. The motion in the end point Region II ($\xi^2 \sim 1$) is relativistic



At leading order approximation in relative velocity

$$\phi_P(\xi) = \phi_L(\xi) = \phi_T(\xi) = \phi(\xi)$$

The study of charmonium distribution amplitudes

Different approaches to the study of DA

1. **Functional approach**

   - Bethe-Salpeter equation

2. **Operator approach**

   - NRQCD

   - QCD sum rules

Potential models

Brodsky-Huang-Lepage procedure:

- Solve Schrodinger equation
- Get wave function in momentum space: $\psi(\vec{k}^2)$
- Make the substitution in the wave function:

$$\vec{k}_\perp \to \vec{k}_\perp, \; k_z \to (x_1 - x_2)\frac{M_0}{2}, \; M_0^2 = \frac{M_c^2 + \vec{k}_\perp^2}{x_1 x_2}$$

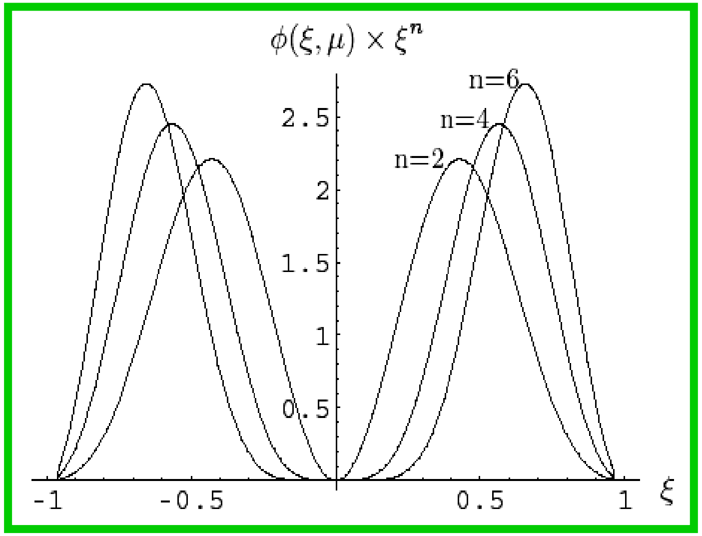
- Integrate over transverse momentum:

$$\phi(\xi,\mu) \sim \int^{\mu^2} d^2k_\perp \, \psi(\xi,\vec{k}_\perp), \qquad \mu \sim M_c$$

The moments within Potential Models

$$<\xi^n>_\mu = \int_{-1}^{1} d\xi\, \xi^n\, \phi(\xi,\mu)$$

- The larger the moment, the larger the contribution of relativistic motion
- Only few moment can be calculated



Higher moments contain information about relativistic motion in quarkonium

Property of DA

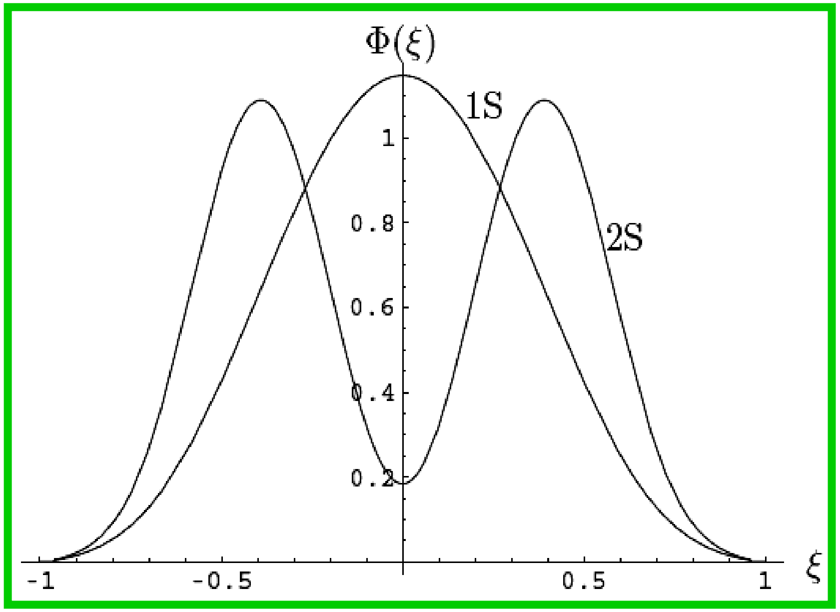
$$\phi(\xi,\mu) \sim (1-\xi^2)\,\Phi(\xi)$$

$$\Phi(\xi) = \int dt\, \psi\left(t + \frac{\xi^2}{1-\xi^2} M_c^2\right)$$

one zero in $\psi(\vec{k}^2)$

$\Downarrow$

two extremums in $\Phi(\xi)$



DA of nS state has 2n+1 extremums

The moments within NRQCD

LEADING ORDER APPROXIMATION IN RELATIVE VELOCITY

$$i f_{\eta_c} p_\nu (zp)^{2k} \langle \xi^{2k} \rangle = \langle 0|\bar{Q}\gamma_\nu\gamma_5 (iz^\sigma \overleftrightarrow{D}_\sigma)^n Q|\eta_c(p)\rangle \to \langle 0|\chi^+((i\overleftrightarrow{\mathbf{D}})^2)^k \psi|\eta_c(p)\rangle$$

$$\langle \xi^n \rangle = \frac{\langle v^n \rangle}{n+1}, \quad \text{where } \langle v^{2k} \rangle = \frac{\langle 0|\chi^+((i\overleftrightarrow{\mathbf{D}})^2)^k \psi|\eta_c(p)\rangle}{\langle 0|\chi^+\psi|\eta_c(p)\rangle}$$

Derivation of the formula

$$<\xi^n>=\frac{<\mathrm{v}^n>}{n+1}$$

Suppose $c\bar{c}$ pair has WF $\psi(\vec{\mathrm{p}}^2)$

At leading order approximation

$$\mathrm{i}\,\vec{\mathrm{D}} \to \vec{\mathrm{p}}$$

$$<\xi^n>\sim\int d^3\mathrm{p}\,(\mathrm{p_z})^n\,\psi(\mathrm{p}^2)\sim$$

$$\int d\cos\theta\,(\cos\theta)^n \times \int d\mathrm{p}\,\mathrm{p}^{\mathrm{n+2}}\psi(\mathrm{p}^2)\sim\frac{1}{\mathrm{n+1}}\times<\mathrm{v}^\mathrm{n}>

The moments within NRQCD

The values of $\langle v^n \rangle$ were calculated in paper

G. Bodwin, **Phys.Rev.D74:014014,206**

$$\boxed{< \mathrm{v}^{\mathrm{n}} >= \gamma^{n}}$$

The constant γ can be expressed through the $\langle v^2 \rangle$

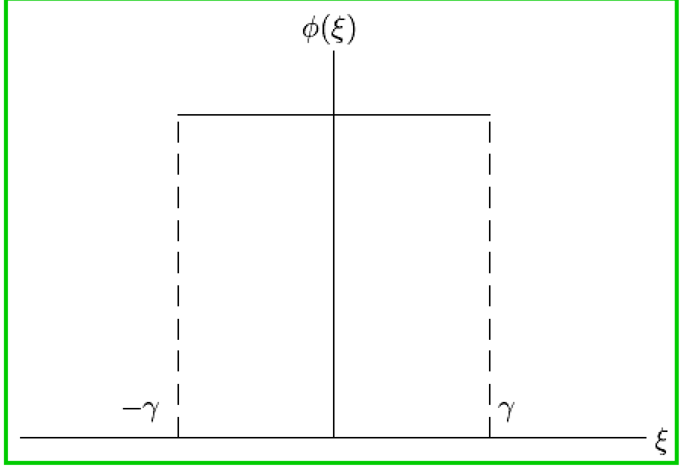
$$\boxed{\begin{array}{l} \text{For 1S states} \quad < \mathrm{v}^2 >= 0.25 \pm 0.08 \\ \text{For 2S states} \quad < \mathrm{v}^2 >= 0.65 \pm 0.42 \end{array}}$$

The model for DA within NRQCD

LEADING ORDER APPROXIMATION IN RELATIVE VELOCITY

$$<\xi^n>=\frac{\gamma^n}{n+1}, \quad \gamma^2=<v^2>$$

$$\boxed{\phi(\xi)=\frac{1}{\gamma}\theta(\gamma-|\xi|)}$$



At leading order approximation γ is the only parameter

The model for DA within QCD sum rules

Advantage:

The results are free from the uncertainty due to the relativistic corrections

Disadvantage:

The results are sensitive to the uncertainties in QCD sum rules parameters:

$$m_c, \ < G^2 >, \ S_0$$

QCD sum rules is the most accurate approach

The results of the calculation

The results for 1S states

$\langle \xi^n \rangle$	Buchmuller-Tye model	Cornell model	NRQCD	QCD sum rules
$n = 2$	0.086	0.084	0.075 ± 0.011	0.070 ± 0.007
$n = 4$	0.020	0.019	0.010 ± 0.003	0.012 ± 0.002
$n = 6$	0.0066	0.0066	0.0017 ± 0.0007	0.0032 ± 0.0009

The results for 2S states

$\langle \xi^n \rangle$	Buchmuller-Tye model	Cornell model	NRQCD	QCD sum rules
$n = 2$	0.16	0.16	0.22 ± 0.14	$0.18\ ^{+0.05}_{-0.07}$
$n = 4$	0.042	0.046	0.085 ± 0.110	$0.051\ ^{+0.031}_{-0.031}$
$n = 6$	0.015	0.016	0.039 ± 0.077	$0.017\ ^{+0.016}_{-0.014}$

The models of DAs

1S states

$$\phi(\xi,\mu\sim m_c)\sim(1-\xi^2)\,\mathrm{Exp}\left(-\frac{\beta}{1-\xi^2}\right)$$

$$\beta=3.8\pm0.7,$$

$$\text{characteristic velocity } \mathrm{v}^2\sim\frac{1}{\beta}\sim 0.25$$

2S states

$$\phi(\xi,\mu\sim m_c)\sim(1-\xi^2)\,(\alpha+\xi^2)\,\mathrm{Exp}\left(-\frac{\beta}{1-\xi^2}\right)$$

$$\alpha=0.03^{+0.32}_{-0.03},\;\; \beta=2.5^{+3.2}_{-0.8},$$

$$\text{characteristic velocity } \mathrm{v}^2\sim\frac{1}{\beta}\sim 0.4$$

The properties of distribution amplitudes

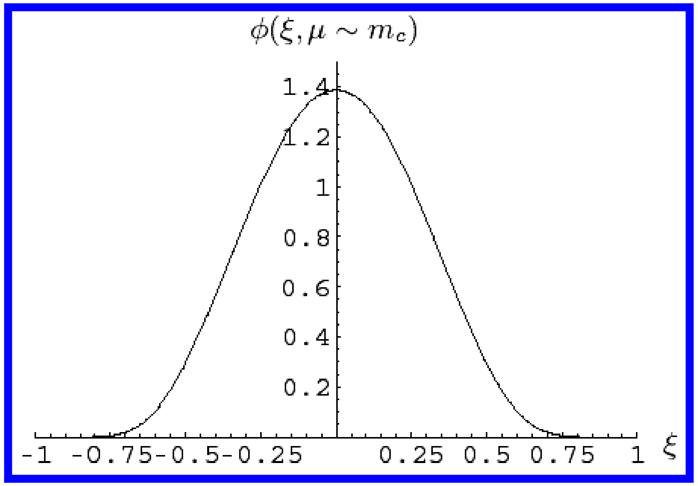
Relativistic tail

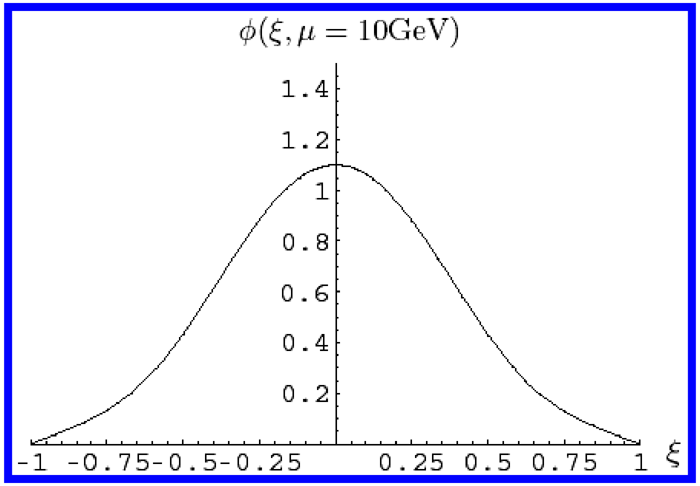
- At $\mu \sim m_c$ DA is suppressed in the region $|\xi| > 0.75$

- This suppression can be achieved if there is fine tuning of a_n

$$\phi(\xi,\mu) \sim (1-\xi^2)\left(\sum_{n} a_n(\mu)\, G_n^{3/2}(\xi)\right)$$

- Fine tuning is broken at $\mu > m_c$ due to evolution





The violation of NRQCD scaling rules

At larger scales the fine tuning of the coefficients a_n is broken
and
NRQCD scaling rules are violated

$$\begin{aligned}
\langle \xi^2 \rangle_\mu &= \frac{1}{5} + a_2(\mu)\frac{12}{35} \sim v^2 \\
\langle \xi^4 \rangle_\mu &= \frac{3}{35} + a_2(\mu)\frac{8}{35} + a_4(\mu)\frac{8}{77} \sim v^4 \\
\langle \xi^6 \rangle_\mu &= \frac{1}{21} + a_2(\mu)\frac{12}{77} + a_4(\mu)\frac{120}{1001} + a_6(\mu)\frac{64}{2145} \sim v^6
\end{aligned}$$

NRQCD velocity scaling rules are violated in hard processes

Improvement of the model for DA

The evolution of the second moment

$$\boxed{<\xi^2>_\mu = \frac{1}{5} + a_2(\mu)\frac{12}{35}}$$

The coefficients $a_2(\mu)$ decreases as μ increases

$$\Downarrow$$

The error in $<\xi^2>$ decreases as μ increases

$1S$ state

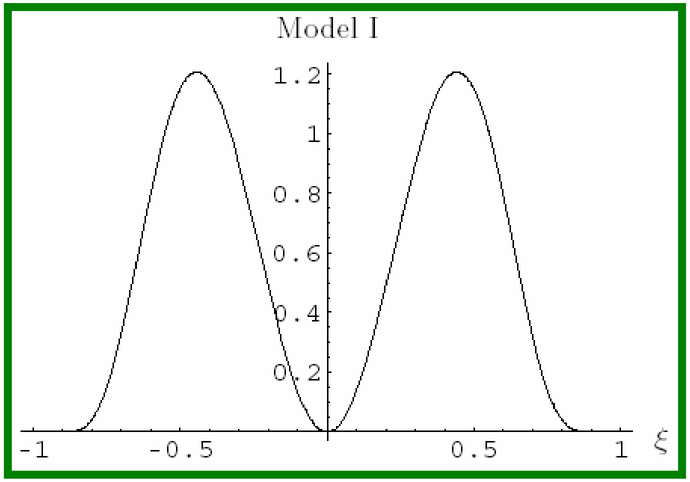
$<\xi^2>_{\mu\sim m_c} = 0.070 \pm 0.007$ $\qquad$ $<\xi^2>_{\mu=10\ \mathrm{GeV}} = 0.123 \pm 0.005$

$2S$ state

$<\xi^2>_{\mu\sim m_c} = 0.18^{+0.5}_{-0.7}$ $\qquad$ $<\xi^2>_{\mu=10\ \mathrm{GeV}} = 0.19^{+0.3}_{-0.4}$

The accuracy of the model for DA becomes better at larger scales

Models for 2S states

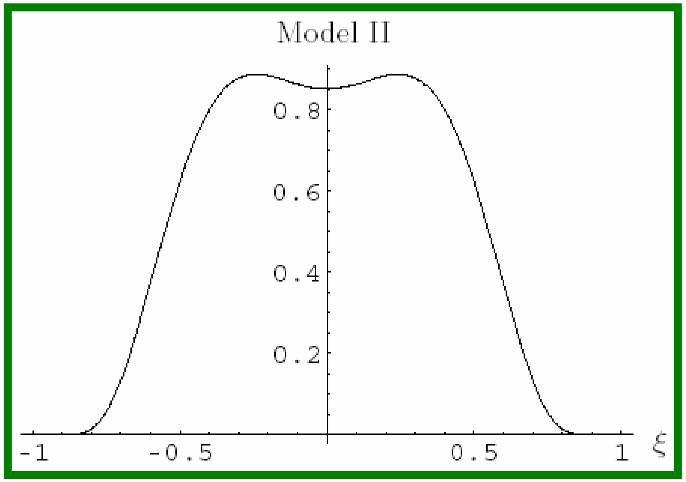


$\alpha = 0 \quad \beta = 2.5$

$< \xi^2 >= 0.21$

$< \xi^4 >= 0.061$

relative momentum $\sim |\xi| p \sim \frac{1}{2} p$



$\alpha = 0.2 \quad \beta = 2.5$

$< \xi^2 >= 0.12$

$< \xi^4 >= 0.031$

relative momentum ~ 0

At leading order approximation of NRQCD
the relative momentum of quark-antiquark pair is zero

Double charmonium production at B-factories

The study of the process $e^+e^- \to J/\Psi\,\eta_c$

Experimental results

$$\sigma(e^+e^- \to J/\Psi\,\eta_c)\times Br(\eta_c > 2\,\text{charged}) = 25.6 \pm 2.8 \pm 3.4\ \text{fb} \qquad \text{BELLE}$$

$$\sigma(e^+e^- \to J/\Psi\,\eta_c)\times Br(\eta_c > 2\,\text{charged}) = 17.6 \pm 2.8^{+1.5}_{-2.1}\ \text{fb} \qquad \text{BABAR}$$

Leading order NRQCD predictions

$$\sigma(e^+e^- \to J/\Psi\,\eta_c) = 3.78 \pm 1.26\ \text{fb} \qquad \text{Braaten and Lee, Phys.Rev. D67}$$

$$\sigma(e^+e^- \to J/\Psi\,\eta_c) = 5.5\ \ \text{fb} \qquad \text{Liu et al., Phys. Lett. B557}$$

How it is possible to get agreement between the theory and the experiment?

Relativistic and radiative corrections

NRQCD formalism

Relativistic corrections
K = 2.1 Bodwin et al., hep-ph/0611002
K = 1.7 He et al., Phys. Rev. D75

One loop radiative corrections
K = 1.96 Zhang et al., Phys. Rev. Lett. 96

$\sigma(e^+e^- \to J/\Psi\,\eta_c) = 17.5 \pm 5.7$ fb Bodwin et al., hep-ph/061102
$\sigma(e^+e^- \to J/\Psi\,\eta_c) = 20$ fb He et al., Phys. Rev. D75

Light cone formalism

Relativistic corrections
K = 1.8-2.1
Leading logarithmic radiative corrections
K = 1.9 – 2.1

$$\sigma(e^+e^- \to J/\Psi\,\eta_c) = 25\,\text{fb}$$

The amplitude was derived in paper Bondar, Chernyak, Phys.Lett. B612

The other processes

Preliminary results

$H_1 H_2$	$\sigma_{BaBar} \times Br_{H_2 \to charged>2}(fb)$	$\sigma_{Belle} \times Br_{H_2 \to charged>2}(fb)$	$\sigma_{Light\ Cone}(fb)$	$\sigma_{NRQCD}(fb)$
$\psi(1S)\eta_c(1S)$	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$25.6 \pm 2.8 \pm 3.4$	25^{+1}_{-1}	3.78 ± 1.26
$\psi(2S)\eta_c(1S)$	$-$	$16.3 \pm 4.6 \pm 3.9$	18^{+5}_{-6}	1.57 ± 0.52
$\psi(1S)\eta_c(2S)$	$16.4 \pm 3.7^{+2.4}_{-3.0}$	$16.5 \pm 3.0 \pm 2.4$	28^{+5}_{-6}	1.57 ± 0.52
$\psi(2S)\eta_c(2S)$	$-$	$16.0 \pm 5.1 \pm 3.8$	17^{+5}_{-8}	0.65 ± 0.22

The uncertainties in light cone predictions are due to the uncertainties in DAs