Confining properties in finite density QCD

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In collaboration with:
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based on arXiv:0705.3698

plus a brief summary of recent lattice results (arXiv:0705.3814) obtained in collaboration with F. Di Renzo (Parma) and M.P. Lombardo (Frascati) about the strongly interacting QGP

OUTLINE

- Confinement, Dual Superconductivity of the vacuum and the QCD Phase Diagram

- Lattice QCD at finite temperature and density: the sign problem and the two-color theory

- Exploring the fate of Dual Superconductivity at finite density: Deconfinement and Chiral Symmetry restoration
  A few remarks on fermion saturation on the lattice

- QCD at finite density via an imaginary chemical potential

- Some recent lattice results concerning the QGP

1 – CONFINEMENT AND THE QCD PHASE DIAGRAM

The QCD vacuum state is characterized by a few fundamental non-perturbative properties ruling the phenomenology of strongly interacting matter at the low energy scale: Color Confinement, Chiral Symmetry Breaking, Fate of $U_A(1)$ symmetry

The relation among these phenomena and their interpretation in terms of the symmetries of the QCD vacuum is not yet completely understood.

At temperatures $T \gg \Lambda_{\mathrm{QCD}}$ the theory is expected to be perturbative and these non-perturbative properties are expected to disappear. That could imply one or more phase transitions, whose nature and relative positions are strictly related to the underlying vacuum symmetries.

As a matter of fact, lattice Monte Carlo simulations show that, at least in ordinary QCD, a single transition takes place and that deconfinement and chiral symmetry restoration coincide (or at least are not distinguishable within statistical errors).

Several phenomenological issues make it necessary to enlarge our view and consider deconfinement and its relation to other transitions in presence of a finite density of baryonic matter.

- Heavy Ion Collision Experiments
- Structure of Compact astrophysical objects.

We are in particular interested in addressing the following questions:

- Can a finite density of baryonic matter induce deconfinement?
- Does deconfinement coincide with the chiral transition also at finite density?
- Is there any deconfinement transition at $T \sim 0$?

These questions have been addressed in the past mostly by looking at the Polyakov loop (S. Hands, S. Kim and J. I. Skullerud, 2006, B. Alles, M. D'Elia and M.P. Lombardo, 2006. M. D'Elia and M.P. Lombardo, 2003.) which however is not an order parameter for confinement in full QCD.

Our plan is to investigate the issue by means of order parameters directly related to mechanisms of Color Confinement and which may be valid also in presence of dynamical fermions.

One such mechanism is that based on Dual Superconductivity of the QCD vacuum ('t Hooft, Mandelstam) $\Longrightarrow$ Confinement is related to the spontaneous breaking of an abelian magnetic symmetry induced by the condensation of magnetic charge.

The magnetic condensate filling the QCD vacuum repels electric fields out of the medium (dual Meissner effect), thus leading to the formation of flux tubes between colored charges, to the linearly rising potential, and to confinement.

A related order parameter is the vacuum expectation value of a magnetically charged operator $\langle\mu\rangle$, which has been successfully tested both in the quenched theory and in presence of dynamical fermions
($\longrightarrow$ talk by A. Di Giacomo).

L. Del Debbio, A. Di Giacomo and G. Paffuti, Phys. Lett. B 349, 513 (1995)

A. Di Giacomo, B. Lucini, L. Montesi and G. Paffuti, Phys. Rev. D 61, 034503 (2000); Phys. Rev. D 61, 034504 (2000)

M. D'E., A. Di Giacomo, B. Lucini, C. Pica and G. Paffuti, Phys. Rev. D 71, 114502 (2005)

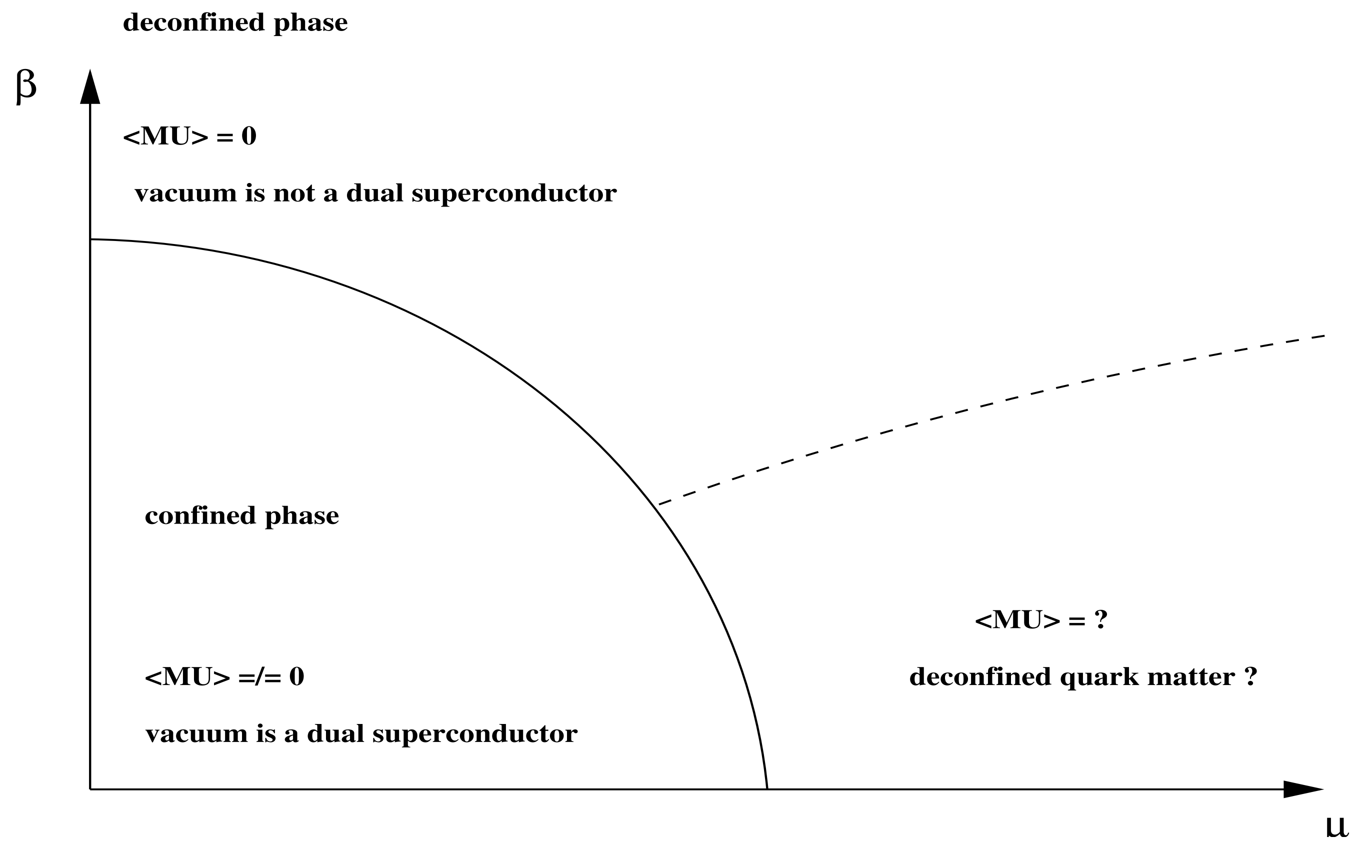
P. Cea and L. Cosmai, JHEP 0111, 064 (2001), P. Cea, L. Cosmai and M. D'Elia, JHEP 0402, 018 (2004).

A slight change of notation in the following: $\langle\mu\rangle \longrightarrow \langle\mathcal{M}\rangle$ in order to avoid confusion with the quark chemical potential μ.

$\langle\mathcal{M}\rangle \neq 0 \implies$ Dual Superconductivity, Confinement

$\langle\mathcal{M}\rangle = 0 \implies$ Normal conducting, deconfined state of matter.

Our goal is to study the behaviour of $\langle \mathcal{M} \rangle$ in the whole $T - \mu$ plane, in order to characterize the confining properties of the various phases in the QCD phase diagram.



Notice: $T = \frac{1}{L_t a(\beta, m_q)} \Longrightarrow \frac{\partial T}{\partial \beta} > 0 \Longrightarrow T \leftrightarrow \beta$

2 – Lattice QCD at finite density

The QCD partition function can be given a path integral formulation and discretized on a lattice

$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-(S_G+S_F)} = \int \mathcal{D}U e^{-S_G} \det M[U]$$

$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N_c} \mathrm{Tr}\Pi_{\mu\nu}(x)\right)$ $\qquad \Pi_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x)$

$S_F = \frac{1}{2} \sum_{x,\mu} \bar{\psi}(x) \gamma^E_\mu \left[U_\mu(x)\psi(x+\hat{\mu}) - U^\dagger_\mu(x-\hat{\mu})\psi(x-\hat{\mu})\right] + \sum_x m\bar{\psi}(x)\psi(x) \equiv \bar{\psi}_i M_{ij} \psi_j$

The thermal expectation value of a generic operator O is written as

$$\langle O \rangle = \frac{\int DU \, \det M[U] \, e^{-S_G[U]} \, O[U]}{\int DU \, \det M[U] \, e^{-S_G[U]}}$$

at zero baryonic chemical potential $\det M[U] \, e^{-S_G[U]} > 0$ and this has a probabilistic interpretation: Monte Carlo methods can be applied to numerically determine it.

A non-zero baryonic density can be realized by introducing a chemical potential coupled to the quark (baryonic) number operator $N = \int d^3x \psi^\dagger \psi = \int d^3x \bar{\psi}\gamma_0\psi$.

$$\bar{\psi}\left(\gamma_\mu(\partial_\mu + iA_\mu) + m\right)\psi \to \bar{\psi}\left(\gamma_\mu(\partial_\mu + iA_\mu) + m + \mu\gamma_0\right)\psi$$

The correct lattice discretization consists in considering μ as part of the covariant derivative, like the temporal component of a $U(1)$ imaginary background field P. Hasenfratz F. Karsch, Phys. Lett. B125 (1983) 308; J.B. Kogut et al., Nucl. Phys. B225 (1983) 93

$$U_\mu \to e^{a\mu} U_\mu\,; \qquad U_\mu^\dagger \to e^{-a\mu} U_\mu^\dagger$$

However the fermion determinant $\det M[U]$ is in general complex for $\mu \neq 0 \implies$ sign problem. Usual Monte-Carlo simulations are not feasible.

Several approaches exists to (partially) circumvent the sign problem. We have chosen to work in QCD with two colors, where the sign problem is absent since the gauge group is real (all gauge invariant observables, including the determinant, are real). That differs from ordinary QCD in several aspects: for instance baryons are degenerate with mesons. However we expect that some features of two color QCD, like those related to confinement, may be relevant also for real QCD.

3 – The disorder parameter at finite density

$\langle \mathcal{M} \rangle$ **is defined in the continuum as the operator which creates of a magnetic monopole**

$$\mathcal{M}(\vec{x},t) = \exp\left[i \int d\vec{y}\ \vec{E}_{\perp\ diag}(\vec{y},t)\vec{b}_{\perp}(\vec{x}-\vec{y})\right]$$

by shifting the quantum field by the monopole vector potential $\vec{b}_{\perp}(\vec{x}-\vec{y})$**.**

Its expectation value appears as the ratio of two partition functions

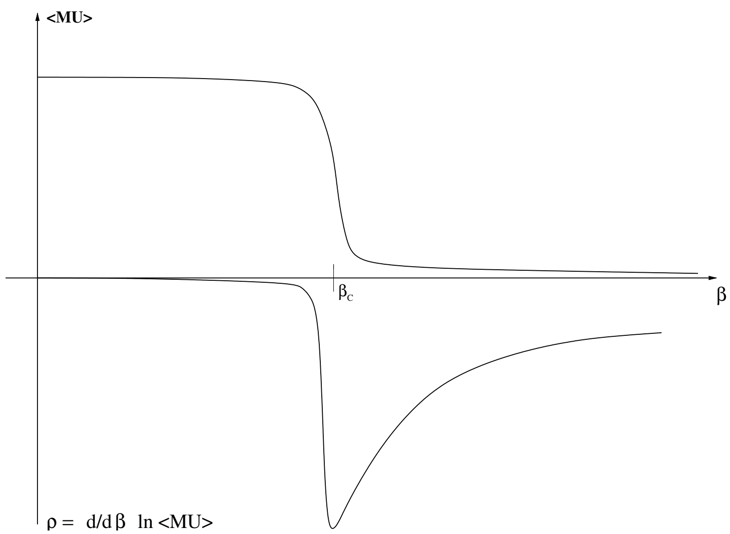
$$\langle \mathcal{M} \rangle = \frac{\tilde{Z}}{Z} \quad Z = \int (\mathcal{D}U) \det M(\mu) e^{-\beta S_G} \quad \tilde{Z} = \int (\mathcal{D}U) \det M(\mu) e^{-\beta \tilde{S}_G}$$

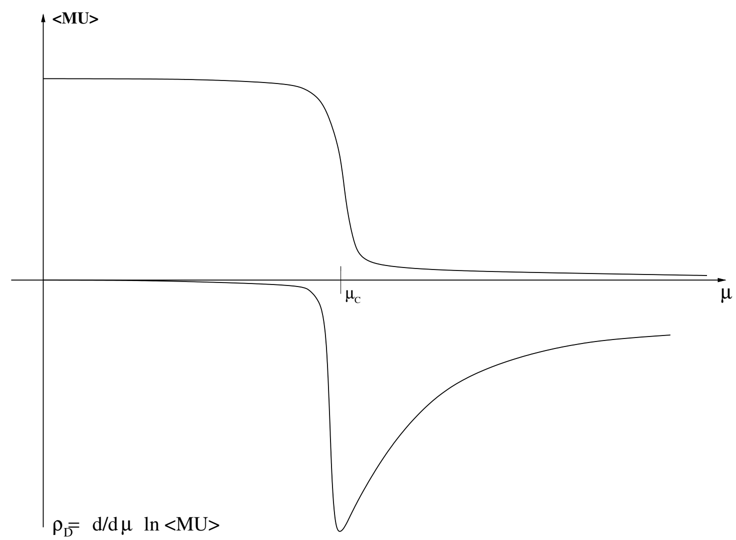
Z **and** $\tilde{Z}$ **differ by the addition of a monopole field in the pure gauge action,** $S_G \rightarrow \tilde{S}_G$

Determining the ratio of two partition functions is numerically difficult, it is better to study the susceptibilities of $\langle \mathcal{M} \rangle$

$$\rho = \frac{\partial}{\partial \beta} \ln \langle \mathcal{M} \rangle = \frac{\partial}{\partial \beta} \ln \tilde{Z} - \frac{\partial}{\partial \beta} \ln Z = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}$$

$$\rho_D = \frac{\partial}{\partial \hat{\mu}} \ln \langle \mathcal{M} \rangle = \frac{\partial \ln \tilde{Z}}{\partial \hat{\mu}} - \frac{\partial \ln Z}{\partial \hat{\mu}} = \langle N_q \rangle_{\tilde{S}} - \langle N_q \rangle_S$$





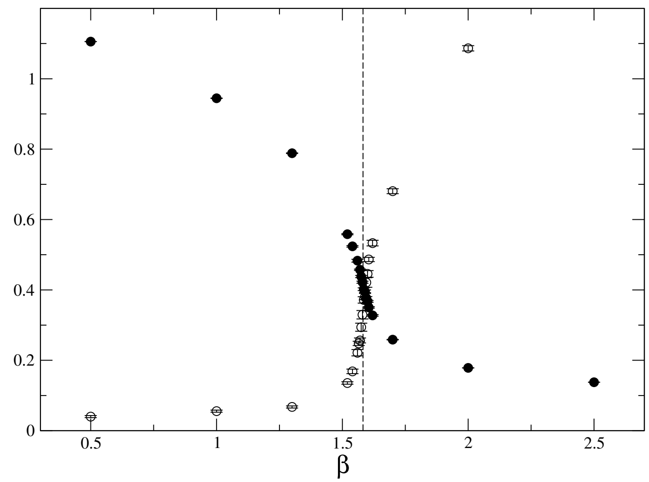
The disorder parameter can then be reconstructed in terms of its susceptibilities

$$\langle \mathcal{M} \rangle (\beta, 0) = \exp \left(\int_0^{\beta} \rho(\beta', 0) \mathrm{d}\beta' \right) \qquad \langle \mathcal{M} \rangle (\beta, \hat{\mu}) = \langle \mathcal{M} \rangle (\beta, 0) \exp \left(\int_0^{\hat{\mu}} \rho_D(\beta, \hat{\mu}') \mathrm{d}\hat{\mu}' \right)$$

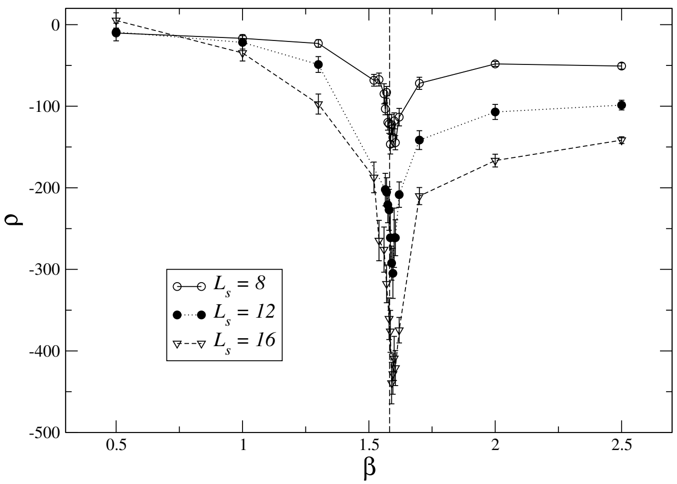
RESULTS

We present results for 2-color QCD with 8 staggered flavors and $am_q = 0.07$

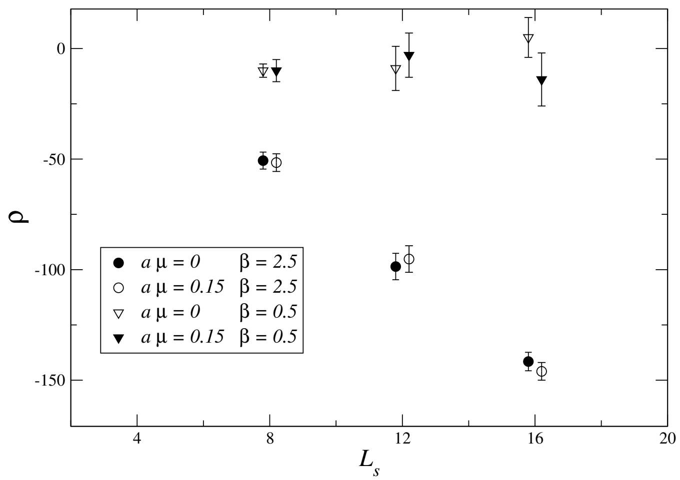
Exact HMC algorithm, $L_t = 6$, $L_s = 8, 12, 16$.



$\langle\bar\psi\psi\rangle$ and the Polyakov loop



ρ peaks at various L_s



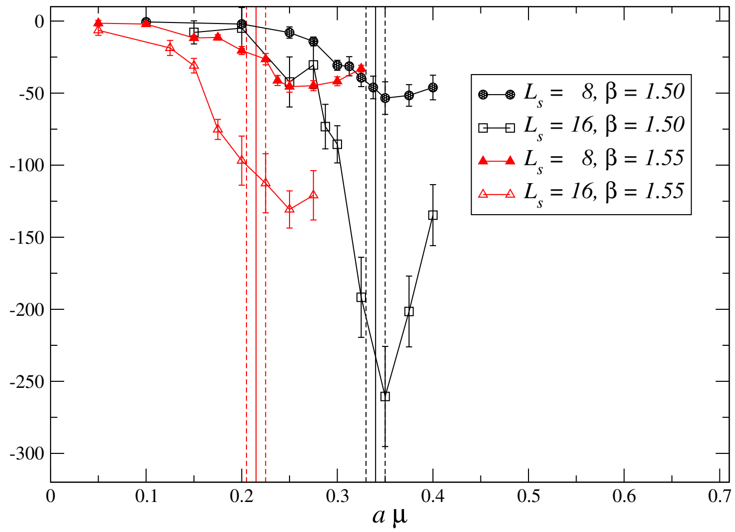
ρ at low and high β.

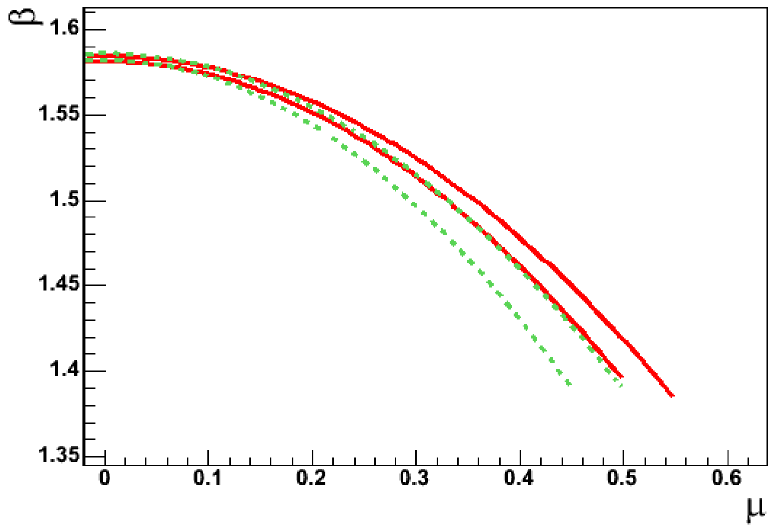
At zero density:

- **A negative peak is observed for ρ at $\beta \simeq 1.58$, deepening in the thermodynamical limit, in correspondence of the drop of $\langle\bar\psi\psi\rangle$ and of the rise of the Polyakov loop.**
- **$\rho \sim 0$ for $T < T_c$ and diverges linearly with L_s for $T > T_c$**

$\Longrightarrow$ **Dual Superconductivity disappears in correspondence of the chiral transition.**

At finite density: We have studied ρ_D as a function of the chemical potential for temperatures $T < T_c$ ($\beta = 1.55,\ 1.50$) and down to $T \sim 0.4T_c$





Clear peaks for ρ_D in correspondence of the locations of the chiral transition.

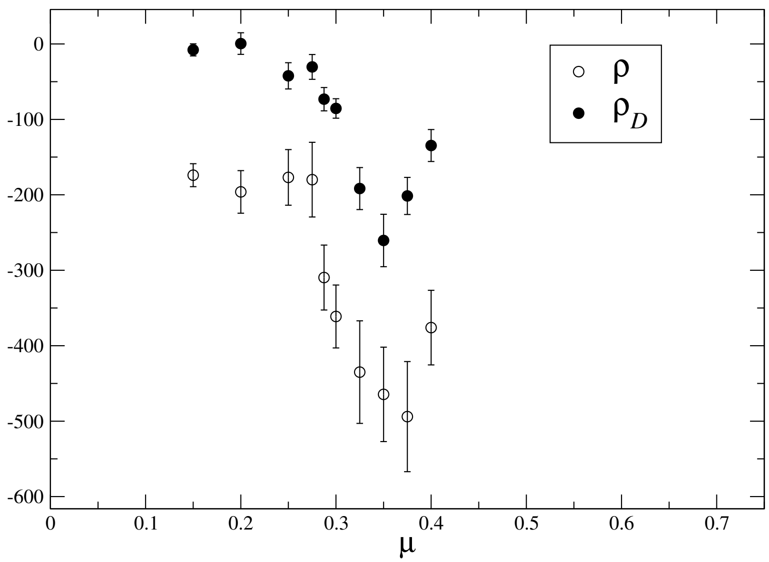
- **Disappearance of dual superconductivity (deconfinement) can be induced by a finite density of baryonic matter**
- **Chiral symmetry breaking (red curve in the figure on the right) and deconfinement (green curve) seem to coincide also at finite density**
- **Eigenvalue distribution analysis shows that we are right above the low temperature region where superfluidity sets in. Can we get to lower T? See later ...**

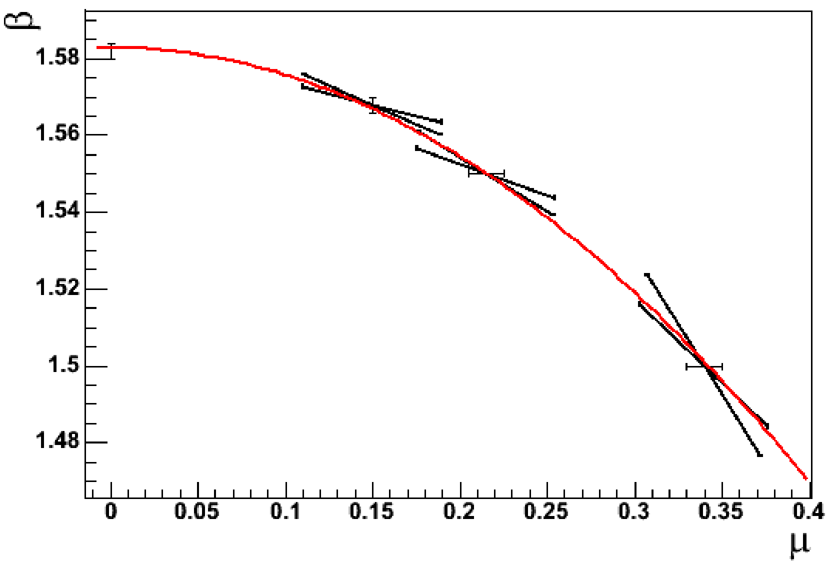
The two susceptibilities ρ and ρ_D can be used not only to locate the position of the line, but also to compute its slope.

Indeed, it is quite natural to assume that the gradient of the disorder parameter,

$$\vec{\nabla}\langle\mathcal{M}\rangle = \left(\frac{\partial\langle\mathcal{M}\rangle}{\partial\beta}, \frac{\partial\langle\mathcal{M}\rangle}{\partial\hat{\mu}}\right) = (\rho,\, \rho_D)\,\langle\mathcal{M}\rangle, \tag{1}$$

be orthogonal, in the $\beta, \hat{\mu}$ plane, to the critical line, whose slope is then equal to $-\rho_D/\rho$.



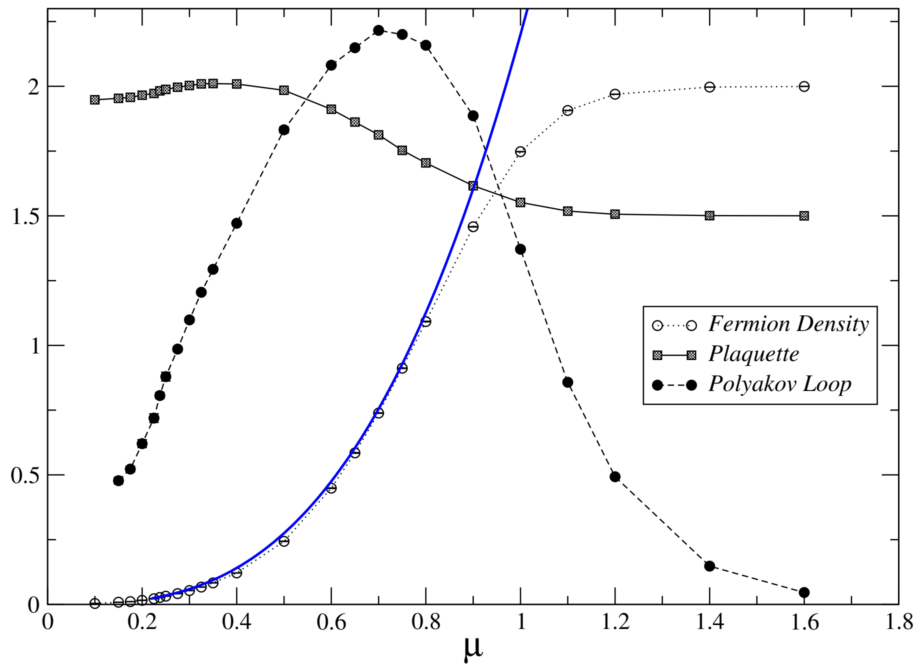


That is nice agreement, within errors, with the critical curve obtained according to a quadratic fit in $\hat{\mu}^2$.

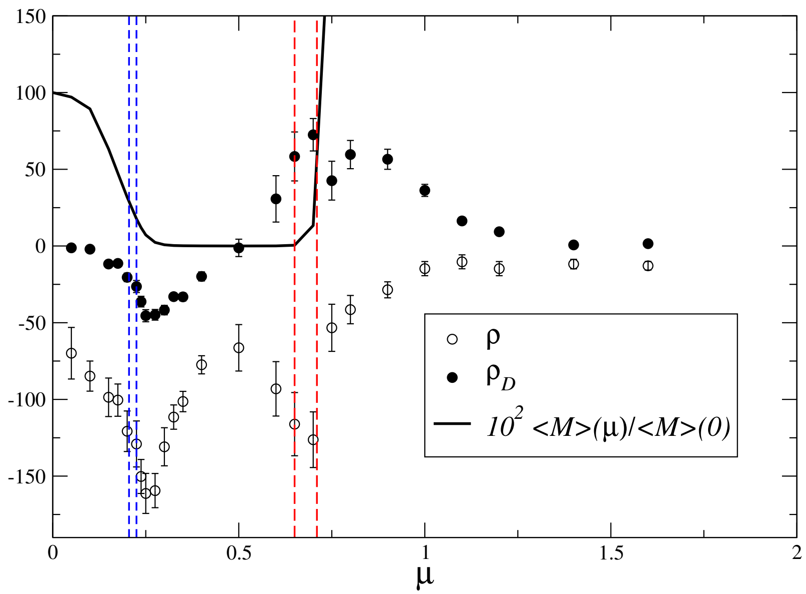
A short detour: Saturation effects

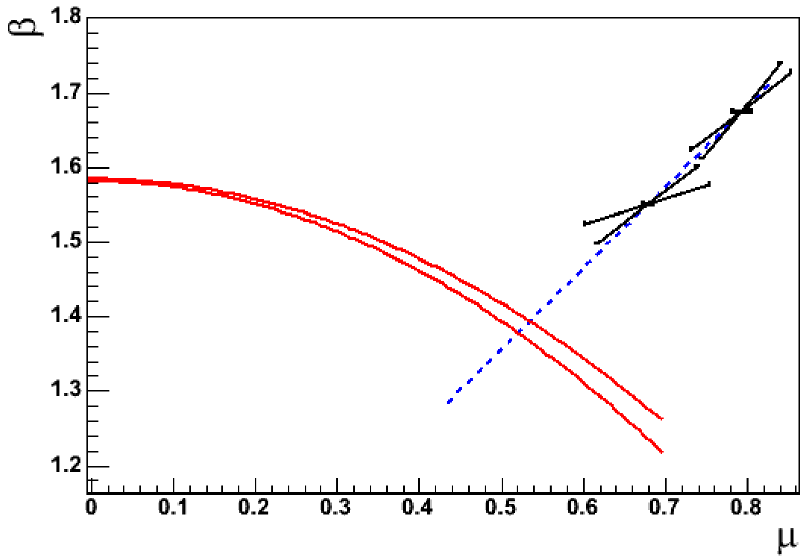
Saturation is a lattice artifact related to the finite number of available fermion levels, which places an upper limit on the reachable lattice densities.

As all available levels are filled, fermion dyamics gets quenched, the theory is modified at the ultraviolet scale and becomes equivalent to a pure gauge theory.



This phenomenon may in principle obscure other interesting physical phenomena.

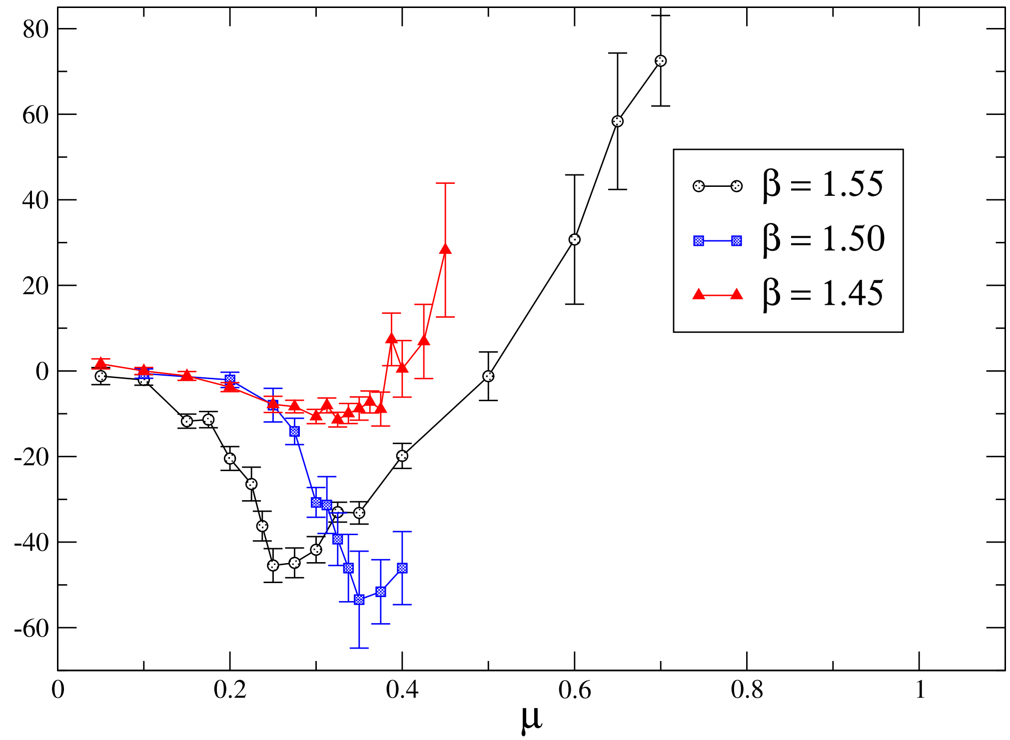




- Unphysical saturation effects are also clearly visible in the behaviour of the disorder parameter: dual superconductivity (confinement) is unphysically restored as fermion dynamics gets quenched ($\beta = 1.55$ in the figure is deep in the confined region for the quenched theory)

- Comparing ρ and ρ_D we learn about the slope of the saturation transition: it is opposite to that of the physical transition. Therefore the two transitions move towards each other as β is lowered!

- That is no good news for the low β region: the saturation transition may completely obscure the physical transition as the strong coupling regime is approached.



Unfortunately our fears reveal to be well founded as we move to $\beta = 1.45$. The negative peak of ρ_D corresponding to disappearance of dual superconductivity seems to be completely washed out by the nearby large positive peak.

- **We cannot reach, on our present lattice size ($L_t = 6$) the low temperature region relevant for compact astrophysical objects.**
- **Simulation on finer lattices or using different lattice discretizations will be needed in the future.**

4 – CONCLUSIONS

- **We have investigated the fate of dual superconductivity two color QCD at finite temperature and density**

- **We have shown that deconfinement (disappearance of dual superconductivity) can be induced by a finite density of baryonic matter**

- **We have shown, for temperatures down to $T \sim 0.4 T_c$, that deconfinement is coincident with the restoration of chiral symmetry.**

- **We have not been able on our present lattices to go to lower temperatures because of saturation effects. We plan to repeat our investigation on finer lattices in the future.**

LATTICE RESULTS ABOUT THE SQGP

FROM AN IMAGINARY CHEMICAL POTENTIAL

in collaboration with F. Di Renzo and M.P. Lombardo (arXiv:0705.3698)

5 – Finite density QCD via an imaginary chemical potential

Consider a purely imaginary chemical potential, $\mu = i\mu_I$

$$U_t \longrightarrow e^{ia\mu_I} U_t \qquad\qquad U_{-t} \longrightarrow e^{-ia\mu_I} U_{-t} = (e^{ia\mu_I} U_t)^\dagger$$

this is like adding a constant and real $U(1)$ **background field.** $\det M[U] > 0$ **, Monte Carlo simulations are feasible, which can be then used in several ways**

Reconstruction of the canonical partition function

$$Z(n) = \mathrm{Tr}\left((e^{-\frac{H_{\rm QCD}}{T}} \delta(N-n)\right) = \frac{1}{2\pi} \mathrm{Tr}\left(e^{-\frac{H_{\rm QCD}}{T}} \int_0^{2\pi} \mathrm{d}\theta e^{i\theta(N-n)}\right) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta e^{-i\theta n} Z(i\theta T)$$

A. Hasenfratz and D. Toussaint, 1990; Alford *et al.*, 1992; de Forcrand, Kratochvila, 2004, 2006.

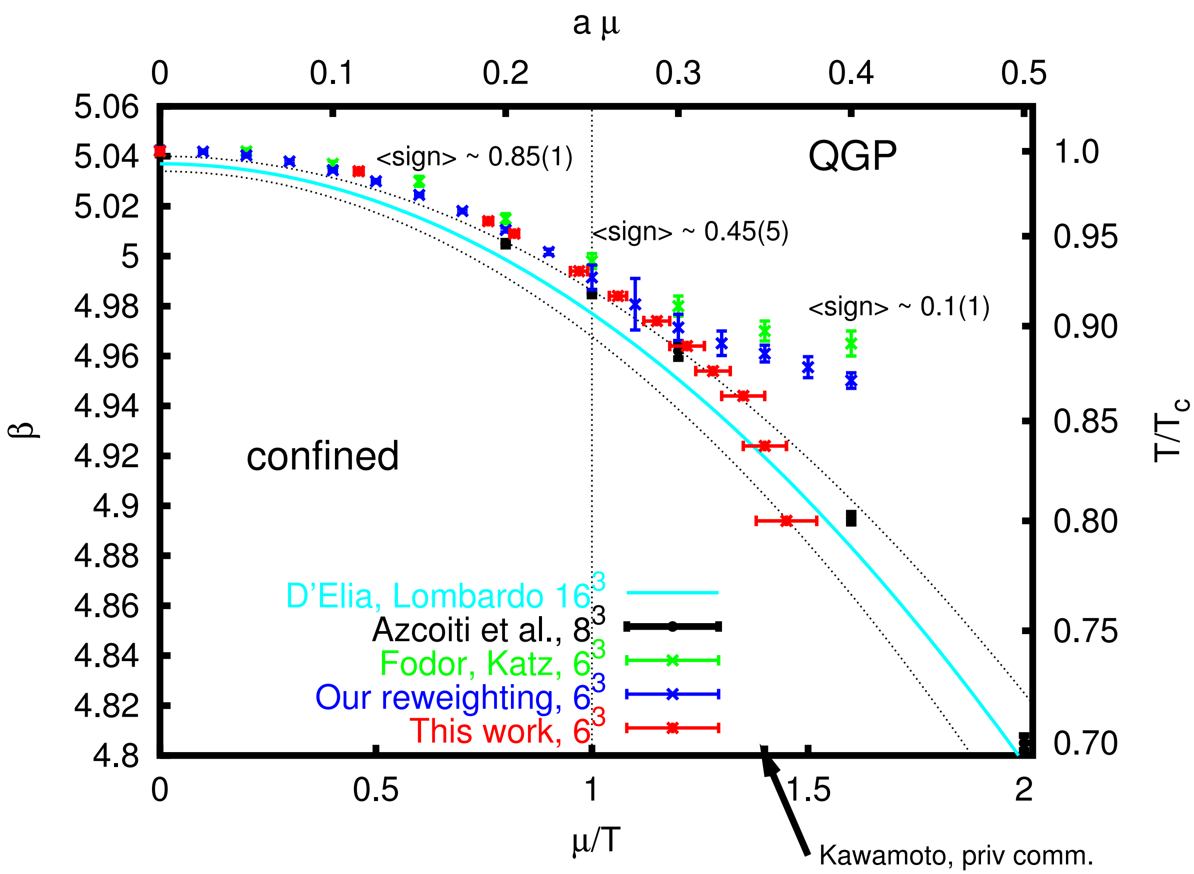
Analytic continuation to real μ

Away from critical points $Z(T,\mu)$ **is a regular function of** μ^2**. Results at** μ_I **(**$\mu^2 < 0$**) can be used to fit the expected dependence, as continued from real values of** μ **(**$\mu^2 > 0$**).**

Ph. De Forcrand, O. Philipsen, 2002, 2003, 2006; M. D'E., M.P. Lombardo, 2003, 2004; P. Giudice and A. Papa, 2004; V. Azcoiti *et al.*, 2004; H. S. Chen and X. Q. Luo, 2005.

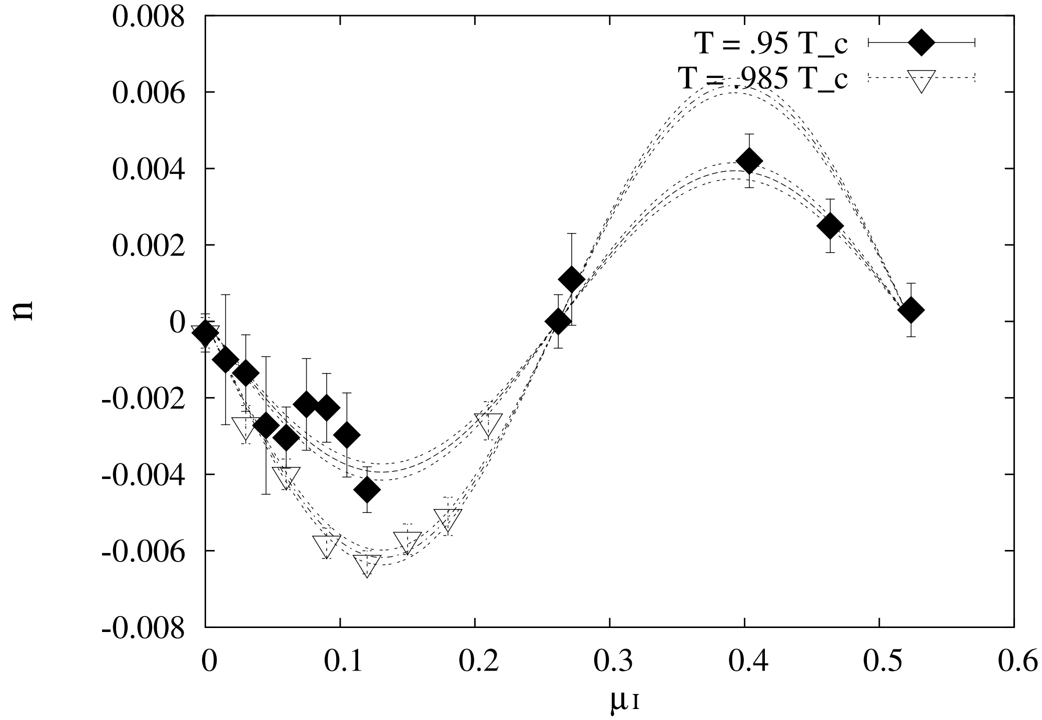
Results obtainable by an imaginary chemical potential

The critical line can be fitted for imaginary chemical potentials and continued to real (small) chemical potentials.



In the figure the critical line obtained via imaginary chemical potential is compared to other methods (from de Forcrand, Kratochvila, hep-lat/0409072)

Testing the HRG model: The behavior observed right below T_c is in agreement with a simple trigonometric behaviour corresponding to the analytic continuation of the hadron resonance gas model prediction.



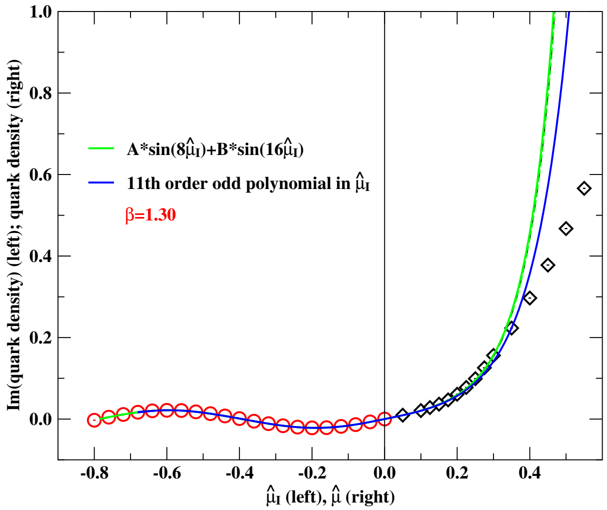
We show in particular results regarding the fermion number density

$$n(\mu) = A\sinh(3\mu/T) \rightarrow n(\mu_I) = iA\sin(3\mu_I/T)$$

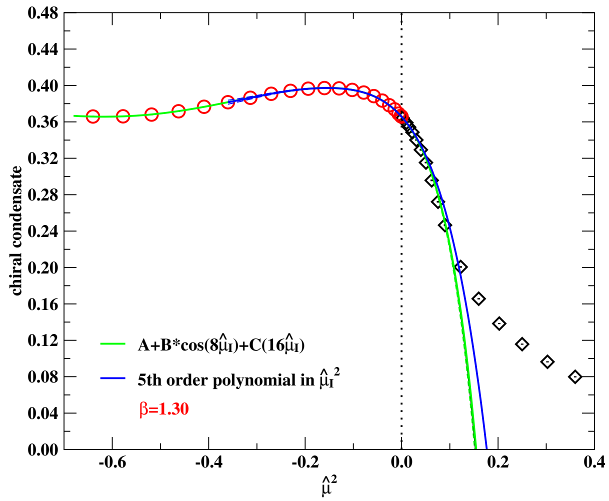
from M. D'E. and M. P. Lombardo, Phys. Rev. D 67 (2003) 014505, Phys. Rev. D 70 (2004) 074509

High precision tests of the method can be performed in theories free of the sign problem, like 2-color QCD.

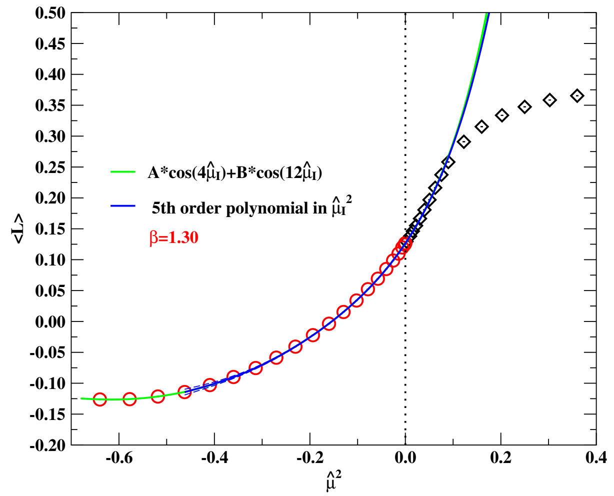
Low Temperature Region $T < T_c$



quark density



chiral condensate



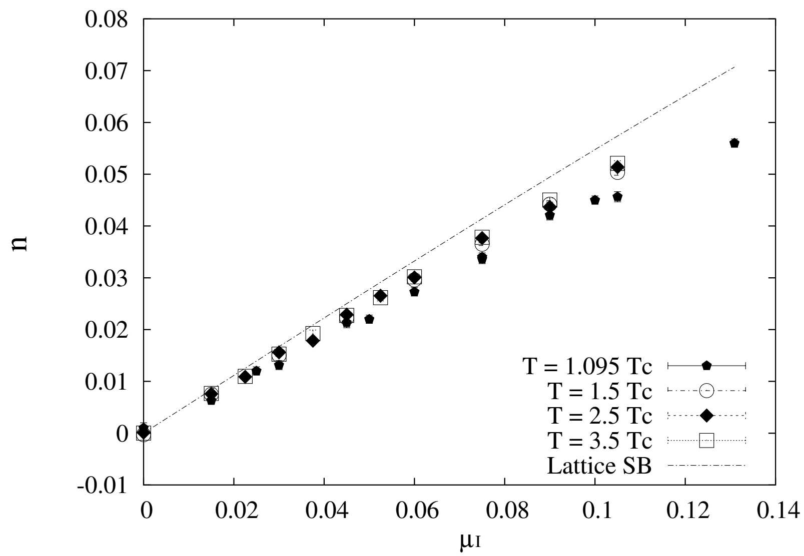
Polyakov loop

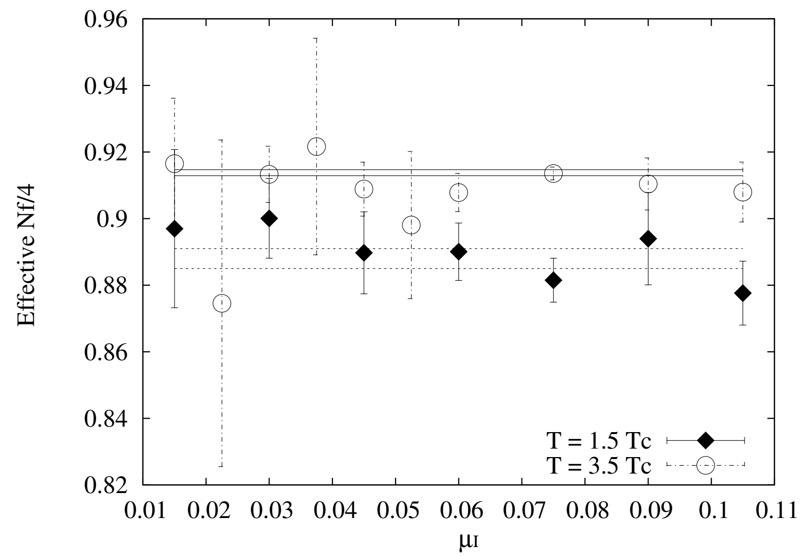
From P. Cea, L. Cosmai, M. D'E., A. Papa, JHEP 0702 (2007) 066 [arXiv:hep-lat/0612018]

THIS WORK

Can we obtain information about the properties of the QGP at high temperatures and, more interesting, close to T_c?

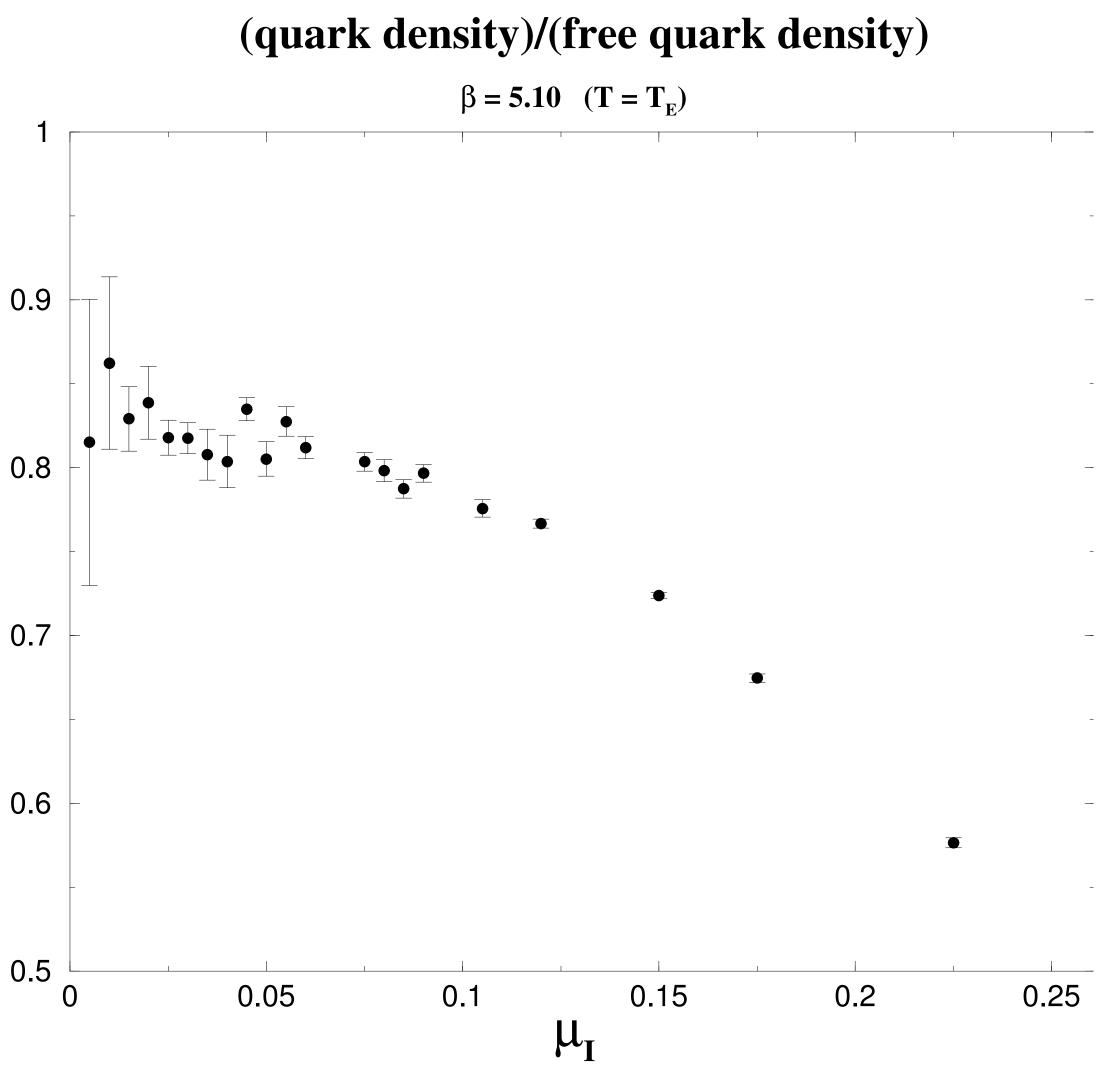
Results at high temperatures well reproduce what expected for a gas of free particles, in particular for the fermion density:





Results at high T can be reproduced in terms of an effective number of free flavors.

The situation is different closer to T_c, where strong deviations are present.

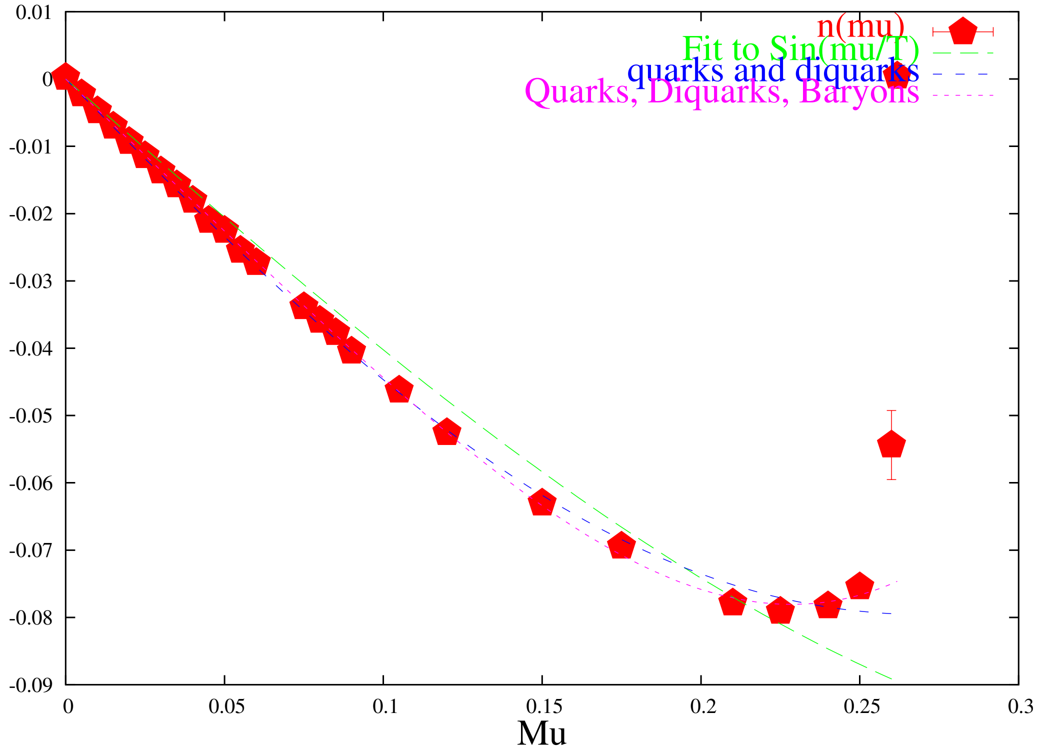


Results refer to $T \sim 1.1T_c$

Can the results be interpreted in terms of bound states populating the QGP?

Assuming a HRG model-like behaviour ($\sim$ non interacting resonances), we fit the fermion number density

$$n(i\mu, T) = A_q(T) sin(\mu/T) + 2B_{qq}(T) sin(2\mu/T) + 3C_{qqq}(3\mu/T)$$

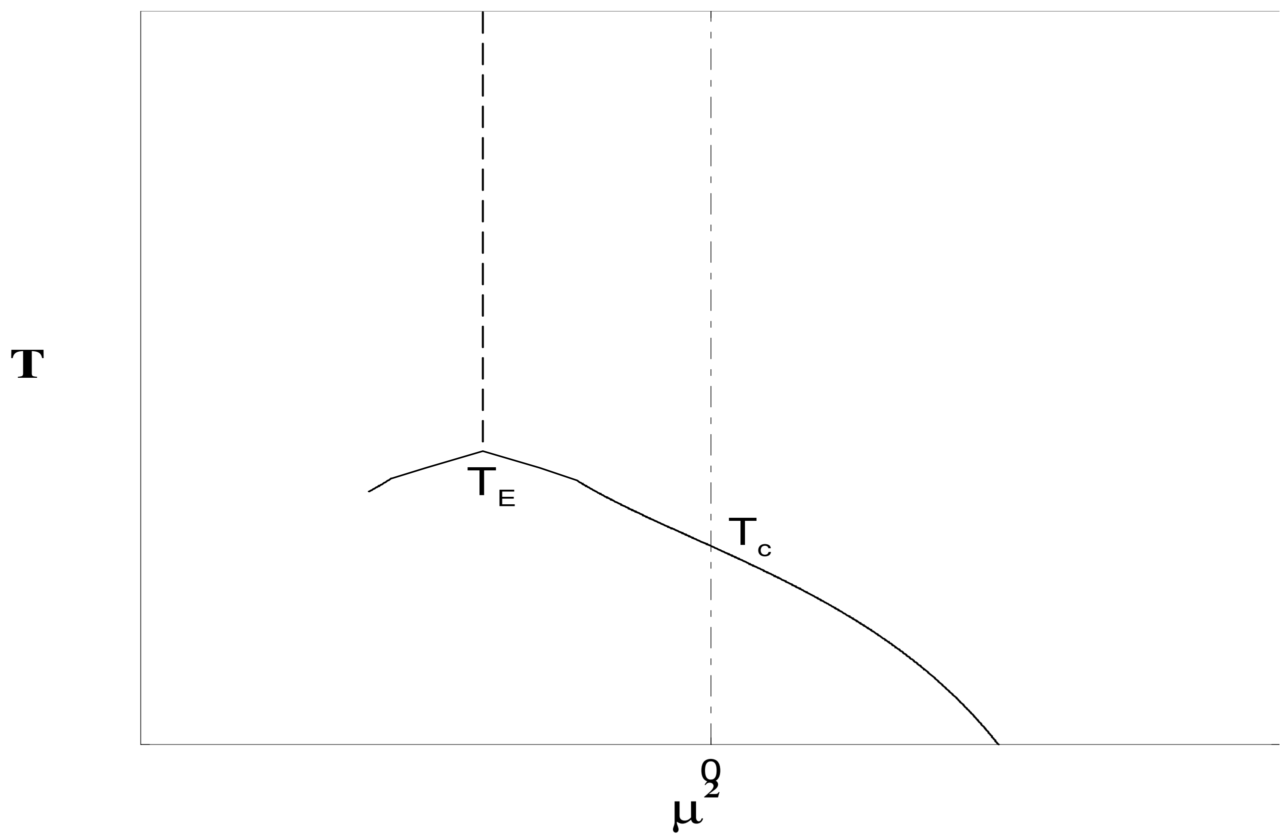


We cannot get values of the reduced χ^2 better than ~ 3. Fit parameters unstable ...

$\Longrightarrow$ **hypothesis on interactions not justified, possible dependence of masses on μ?**

A different hypothesis:

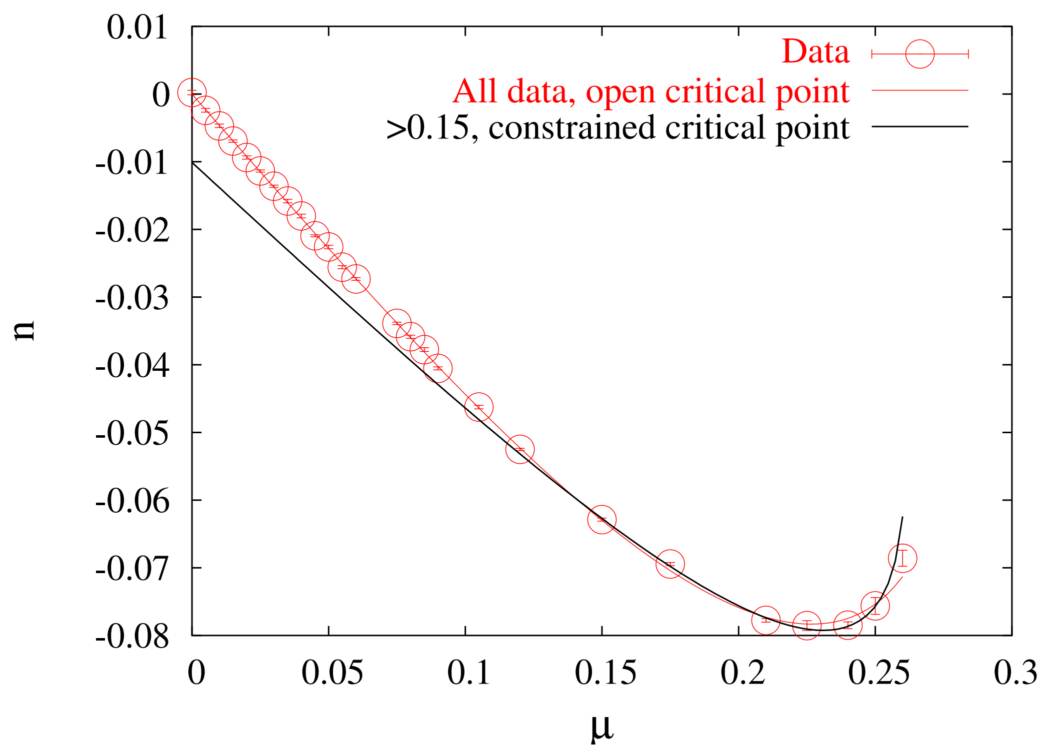
Can we interpret results in terms of a critical behaviour induced by the nearby end-point of the Roberge-Weiss transition line happening at $\mu_I/T = \pi/3$, which is related to the dynamics of the Polyakov loop?



A critical behaviour for the fermion number density (imaginary part) like

$$n(\mu_I) = A\mu_I(\mu_I^{c\,2} - \mu_I^2)^\alpha \rightarrow n(\mu) = A\mu(\mu_I^{c\,2} + \mu^2)^\alpha$$

with $\alpha \sim 0.3$, $\mu_I^{c\,2} \sim 0.08$ and $\tilde{\chi}^2 \sim 1.8$ well reproduces the numerical data.



Can we interpret the strongly interacting Quark-Gluon Plasma in terms of this critical behaviour?