

Light glueball spectrum in a holographic description of QCD

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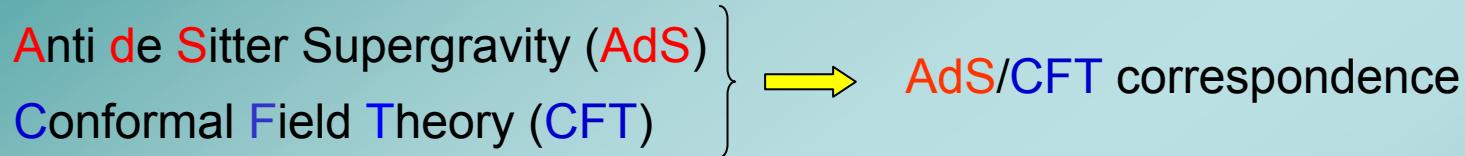
In collaboration with P. Colangelo, F. De Fazio, S. Nicotri

19/06/2007
QCD@Work '07

Contents

- AdS/CFT correspondence

(Maldacena '98, Witten '98, Gubser, Klebanov, Polyakov '98,...)



- AdS/QCD : Holographic Model of Glueballs

(P. Colangelo, F. de Fazio, S. Nicotri, F.J., '07)

- Scalar 0^{++} and Vector 1^{--} mass spectra (pseudoscalar 0^{-+} , hybrid mesons)
- Construction of the dual theory of QCD (if exists...)

(C. Csaki, H. Ooguri, Y. Oz & J. Terning '99,
N. Evans, J.P. Shock & T. Waterson '05,
N. Braga & H. Boschi-Filho '06,
C. Csaki & M. Reece '06,...)

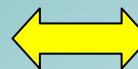
- Conclusion and Outlook

AdS/CFT Correspondence

(Maldacena '98)

Supergravity limit of M-theory/Superstring Theory in

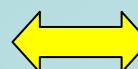
$$AdS_5 \times S^5$$



Large N limit of Superconformal SU(N) gauge theory in ∂AdS_4

Anti de Sitter space \times compact space

Holographic spacetime / bulk
(no physical extra dimensions)



Minkowski spacetime

$$\mathcal{M}^4$$

Anti-de Sitter AdS_5 (d=5) :

$$ds^2 = g_{MN} dx^M dx^N$$

(M,N=0,1,2,3,4)
(-,+,+,+,+)

- Solution of vacuum Einstein equation :

$$R_{MN} - \frac{1}{2}g_{MN}R = \frac{1}{2}g_{MN}\Lambda$$

cosmological constant $\Lambda > 0$

- Isometry group $SO(2,4)$
(preserves distances, $\sim SO(1,3)$)

on the boundary

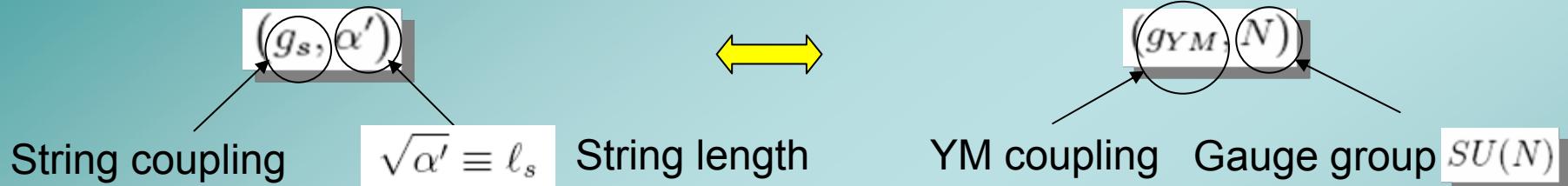


Conformal $SO(2,4)$ group
acting on \mathcal{M}^4

Supergravity limit of M-theory/
Superstring Theory in AdS_5

Large N limit of Superconformal
SU(N) gauge theory in \mathcal{M}^4

Parameter correspondence



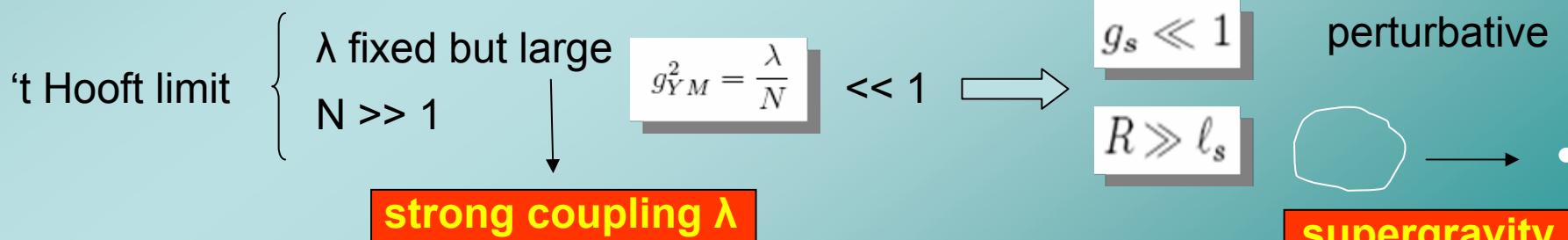
$$g_s = g_{YM}^2$$

$$\frac{R^4}{(\alpha')^2} = 4\pi N g_{YM}^2$$

R : AdS radius (AdS typical size)

't Hooft coupling

$$\lambda \equiv N g_{YM}^2$$



Classical
Perturbative } string theory in AdS_5



Strongly coupled gauge theory in \mathcal{M}^4

Symmetry correspondence

Local (gauged) symmetry \longleftrightarrow Global symmetry

Ex. : chiral sym.

$$(SU(3)_L \times SU(3)_R)_{local}$$

$$(SU(3)_L \times SU(3)_R)_{global}$$

Operator/field correspondence (Witten '98, Gubser, Klebanov, Polyakov '98)

Bulk field (p-form)

$$A(x^M)$$



Operator (scaling dim. Δ)

$$e^{iS_5^{eff}[A(x^M)]} = \langle e^{i \int d^4x A_0(x) O(x)} \rangle_{CFT}$$

Bulk field

$$A(x^M)$$



Source field

$$A_0(x^\mu)$$

of operator

$$O(x^\mu)$$

boundary coord.
($\mu, v = 0, 1, 2, 3$)

Bulk-to-boundary propagator :

$$A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu)$$

AdS mass of the bulk field :

$$R^2 m_{AdS}^2 = (\Delta - p)(\Delta + p - 4)$$

$$\partial AdS_4$$

Conformally flat metric :

$$g_{MN} = e^{2A(x)} \eta_{MN}$$

$$A(x) = -\ln \frac{z}{R}$$

where

$$z \equiv x^4$$

Holographic spacetime :

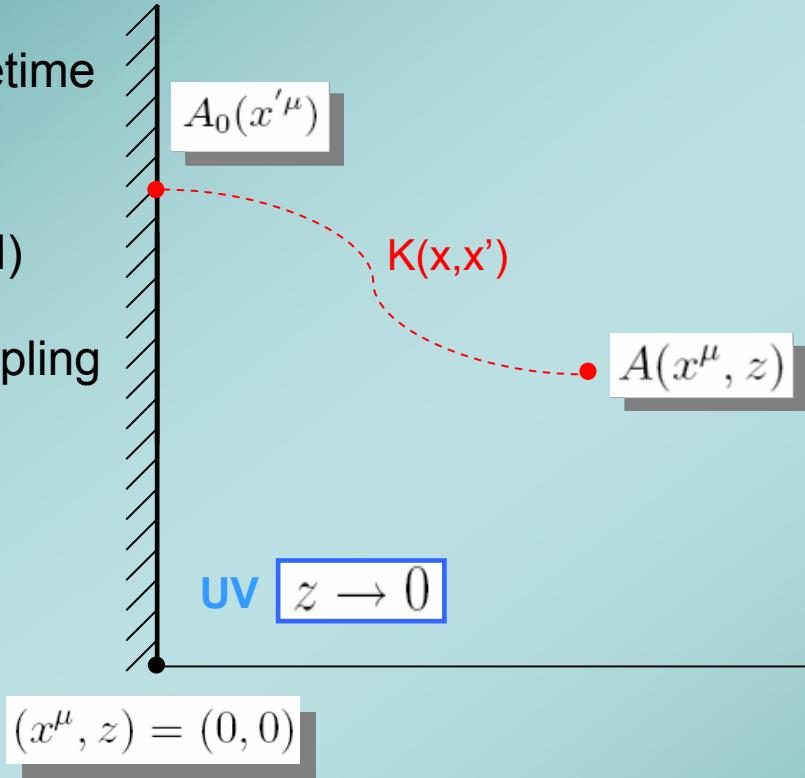
$$ds^2 = g_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2)$$

Our spacetime

$$\mathcal{M}^4$$

$$SU(N)$$

- strong coupling
- SUSY
- conformal



Bulk : holographic spacetime

$$AdS_5$$

M-Theory/Superstring

- weak coupling
- classical

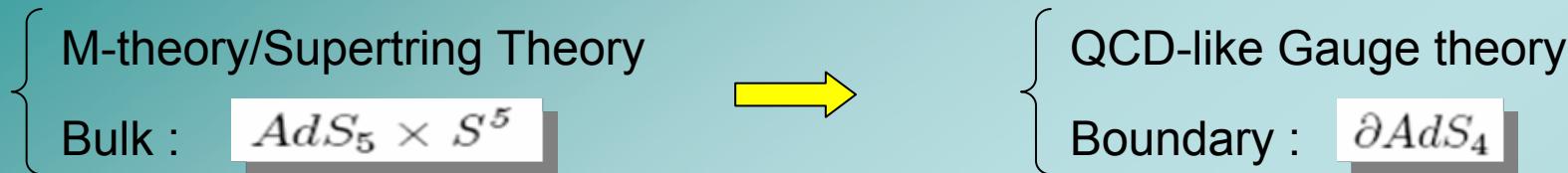
$$IR \quad z \rightarrow \infty$$

holographic
coordinate

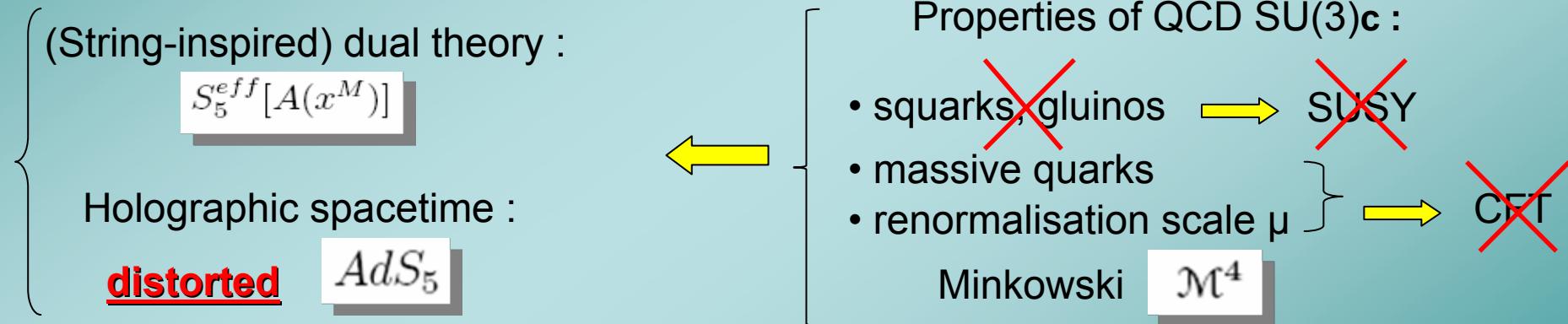
Energy scale

Holographic Models of mesons

I) Top-to-bottom approach :



II) Bottom-up approach (**AdS/QCD**) :



Confinement, Chiral symmetry breaking, masses, decay constants, form factors, etc...



Glueball Spectroscopy

(Colangelo, de Fazio, Nicotri, F.J. '07)

-
- A diagram showing the results of Glueball Spectroscopy. A brace on the right groups two bullet points: "• Scalar 0^{++} and vector 1^{--} mass spectrum (pseudoscalar 0^{-+} , hybrid mesons)" and "• Dual theory of QCD (if exists...)".

AdS/QCD Model of light glueballs

Glueballs : Bound-states of gluons (gg...)

QCD in \mathcal{M}^4

Scalar glueball

$$\text{Tr} F^2$$

Vector glueball

$$\text{Tr}(F(DF)F)_\mu$$

boundary operators

Dual theory in AdS_5

Scalar field :

$$X(x, z)$$

Vector field :

$$A_M(x, z)$$

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$

Dilaton : $\phi(z)$

(AdS radius : $R=1$)

bulk fields

Operators / fields of the model

4D : $\mathcal{O}(x)$

5D : $\phi(x, z)$

p

Δ

m_{AdS}^2

$\text{Tr} F^2$

$X(x, z)$

0

4

0

} massless

$\text{Tr}(F(DF)F)_\mu$

$A_M(x, z)$

1

7

24

} massive

boundary

bulk

J^{PC}

Scalar glueball 0^{++}

$Tr F^2$ ($\Delta=4$)



$X(x, z)$ ($p=0$)

$m_5^2 = 0$

Vector glueball 1^{--}

$Tr(F(DF)F)_\mu$ ($\Delta=7$)



$A_M(x, z)$ ($p=1$)

$m_5^2 = 24$

$$\text{AdS/CFT} \left\{ \begin{array}{l} A(x^M) = \int_{M^4} d^4x' K(x^M, x'^\mu) A_0(x'^\mu) \\ m_5^2 = (\Delta - p)(\Delta + p - 4) \end{array} \right. \quad \rightarrow \quad \text{AdS/QCD} \left\{ \begin{array}{l} A(x^M) \stackrel{?}{=} A_0(x^\mu) \\ m_5^2 = m_{AdS}^2 \end{array} \right.$$

• **Scalar bulk field :**

$$S_5^{eff} = -\frac{1}{2} \int d^5x \sqrt{-g} e^{-\phi(z)} g^{MN} (\partial_M X)(\partial_N X)$$

• **Vector bulk field :**

$$S_5^{eff} = -\frac{1}{2} \int d^5x \sqrt{-g} e^{-\phi(z)} \left[\frac{1}{2} g^{MN} g^{ST} [F_{MS} F_{NT} + m_{AdS}^2 g^{ST} A_S A_T] \right]$$

5-dim. bulk

Dilaton

$$\phi(z) = a^2 z^2$$

Bulk field mass

- **Broken** AdS isometries/conformal sym. (energy scale $[a]=1$)
- **Regge behaviour** of the mass spectrum

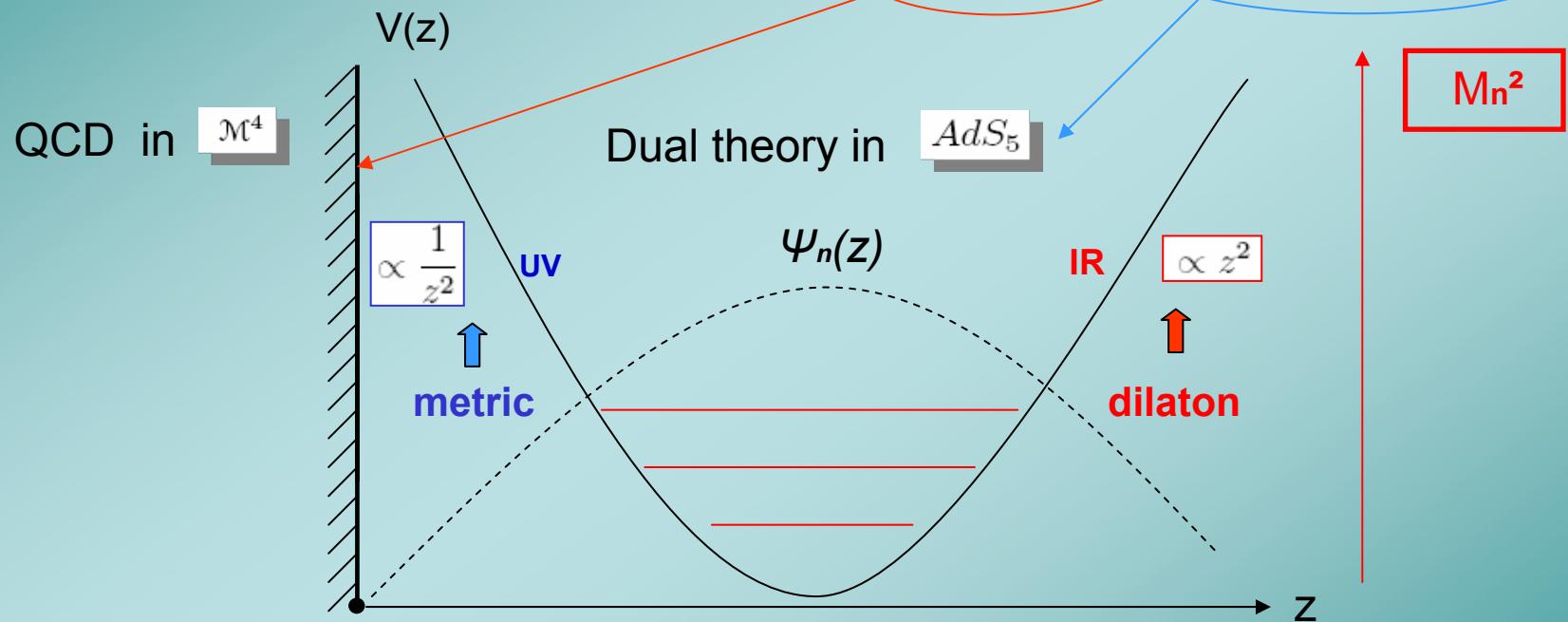
$$F_{MS} = \partial_M A_S - \partial_S A_M$$

- (Classical) eq. of motion :

$$\partial_N (\sqrt{-g} e^{-\phi} F^{NM}) - \sqrt{-g} e^{-\phi} m_5^2 A^M = 0$$

- Bulk field decomposition (mode) :

$$A_\mu(x, z) = \underbrace{\epsilon_\mu e^{iq \cdot x}}_{\text{plane wave}} \underbrace{\psi(z)}_{\text{holo. wave function}}$$



- Schrödinger eq. :

$$-\psi'' + V(z)\psi = m_n^2\psi(z)$$

with

$$V(z) = a^4 z^2 + \underbrace{\frac{4m_5^2 + (c+2)c}{4z^2}}_{\text{IR}} + (c-1)a^2$$

$$\begin{cases} c = 1 : A_M(x, z) \\ c = 3 : X(x, z) \end{cases}$$

dilaton $\phi(z) = a^2 z^2$

(IR : $z \rightarrow \infty$)

metric $g_{MN} = \frac{1}{z^2} \eta_{MN}$

(UV : $z \rightarrow 0$)

- Mass spectrum :

$$m_n^2 = \left(4n + 1 + c + \sqrt{(c+1)^2 + 4m_5^2} \right) a^2$$

- Holo. wave function :

$$\psi_n(z) = A_n e^{-a^2 z^2/2} {}_1F_1 \left(-n, g(c, m_5^2) + 1, a^2 z^2 \right)$$

$\rightarrow 0 \begin{cases} z \rightarrow \infty \\ z \rightarrow 0 \end{cases}$

$$g(m_5, c) = \sqrt{m_5^2 + \frac{(c+1)^2}{4}}$$

Kummer confluent hypergeometric function
($-n < 0$: polynomial)

Scalar glueball Vector glueball

Vector ρ meson (Son et al. '05)

J^{PC} :

0^{++}

1^{--}

1^{--}

Boundary

$$Tr F^2$$

($\Delta=4$)

$$Tr(F(DF)F)_\mu$$

($\Delta=7$)

$$j_L^a(x)$$

$$j_R^a(x)$$

($\Delta=3$)

Bulk

$$X(x, z)$$

($p=0$)

$$m_5^2 = 0$$

$$A_M(x, z)$$

($p=1$)

$$m_5^2 = 24$$

$$A_L^a(x, z)$$

$$A_R^a(x, z)$$

($p=0$)

$$m_5^2 = 0$$

Spectra

$$m_n^2 = (4n + 8)a^2$$

$$m_n^2 = (4n + 12)a^2$$

$$m_n^2 = (4n + 4)a^2$$

• Scalar glueball : $m_n^2 = (4n + 8)a^2$

• Vector glueball : $m_n^2 = (4n + 12)a^2$

→ Regge behaviour ($n \gg 1$) : $m_n^2 \propto n$ (dilaton) $\phi(z) = a^2 z^2$)

\neq • Top-to-bottom approach
• AdS slice $0 \leq z < z_c$ (z_c : IR brane) } $m_n^2 \propto n^2$ (KK spectrum)

→ Ground states : $m_{G_1}^2 - m_{G_0}^2 = m_\rho^2$

AdS/QCD	QCDSR		Lattice QCD		
(hep-ph/0703316)	Dominguez, Paver ('86)	Narison (hep-ph/9612457)	Hang, Zhang (hep-ph/9801214)	Morningstar (hep-lat/9901004)	Meyer (hep-lat/0508002)
m_{G_0}	1.096	< 1	1.5 (0.2)	1.580(150)	1.730(50)(80)
m_{G_1}	1.342	too light ?		3.850(50)(190)	3.240(330)(150)

→ { Hybrid meson : $m_n^2 = (4n + 6)a^2$
Pseudoscalar glueball : $m_n^2 = (4n + 8)a^2$ (degenerated with scalar → puzzle)

Perturbed background

Background : $\left\{ \begin{array}{l} \bullet \text{ AdS dual spacetime : } ds^2 = e^{2A(z)} \eta_{MN} ds^M dx^N = \frac{1}{z^2} (dx^2 + dz^2) \\ \bullet \text{ Dilaton : } \phi(z) = a^2 z^2 \end{array} \right.$

Regge behaviour : $m_n^2 \propto n$  connection dilaton/metric

• $z \rightarrow 0$: asymptotic AdS

$$\phi - A \xrightarrow{z \rightarrow 0} -\ln(z)$$

• $z \rightarrow \infty$: harmonic-like potential

$$\phi - A \xrightarrow{z \rightarrow \infty} z^2$$

• Higher spin meson spectrum

$$A(z) \not\propto z^{2+\beta} \quad \beta > 0$$

Perturbation :

$$\phi - A \sim z^\alpha$$

$$0 \leq \alpha < 2$$

$$\alpha = 1$$

Modification of the background

modification of the **dilaton**

$$\begin{aligned}\phi(z) &= a^2 z^2 + \lambda a z \\ g_{MN} &= \frac{1}{z^2} \eta_{MN}\end{aligned}$$

IR subleading

modification of the **metric**

$$\begin{aligned}\phi(z) &= a^2 z^2 \\ g_{MN} &= \frac{e^{-\lambda a z}}{z^2} \eta_{MN}\end{aligned}$$

UV subleading

$$m_n^2 = m_{n,(0)}^2 + \lambda m_{n,(1)}^2$$

- modification of the **dilaton** :

$$V(z) = a^4 z^2 + \underbrace{\frac{(c+2)c}{4z^2}}_{\text{dilaton}} + \underbrace{\frac{4m_5^2}{4z^2}}_{\text{metric}} + (c-1)a^2 + O(\lambda)$$

- modification of the **metric** :

$$V(z) = a^4 z^2 + \underbrace{\frac{(c+2)c}{4z^2}}_{\text{dilaton}} + \underbrace{\frac{4m_5^2}{4z^2} e^{-2\lambda a z}}_{\text{metric}} + (c-1)a^2 + O(\lambda)$$

mass dependent modification :
Scalar \neq Vector

Mass splitting

- **dilaton** :
- **metric** :

$$m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right)a^2$$

$$m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right)a^2$$

Mass splitting increases
Maximum effect : **metric A(z)**

→ $\lambda < 0$

Conclusion



Mass spectra

- scalar glueball : $m_n^2 = (4n + 8)a^2$
- vector glueball : $m_n^2 = (4n + 12)a^2$



Regge behaviour ($n \gg 1$) : $m_n^2 \propto n$

AdS slice $0 \leq z < z_c$ ($z_c \sim 1/\Lambda_{\text{QCD}}$: IR brane) $m_n^2 \propto n^2$



Ground states : $m_{G_1}^2 - m_{G_0}^2 = m_\rho^2$



Perturbation
of the background
(metric/dilaton)

- **dilaton** modification: $m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{3\sqrt{\pi}}{128}\lambda\right)a^2$
- **metric** modification : $m_{G_1}^2 - m_{G_0}^2 = \left(4 - \frac{1899\sqrt{\pi}}{128}\lambda\right)a^2$

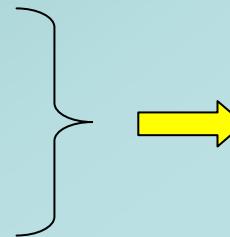
$\lambda < 0$: $\begin{cases} \text{Mass splitting} \nearrow \\ \text{Metric} : \text{maximal effect} \end{cases}$

Outlook

- Hybrid mesons
- Pseudoscalar glueball 0^{-+}

Operator $\text{Tr} F \tilde{F}$ ($\Delta=4$)

Scalar bulk field $X(x, z)$ ($p=0$) $m_5^2 = 0$



$$m_n^2 = (4n + 8)a^2$$

degenerate spectrum
with scalar glueball !

Lattice QCD (Meyer '05)
 Top-to-bottom approach

$$\frac{m_0^2(0^{-+})}{m_0^2(0^{++})} = 2.33 \pm 0.26$$

→ pseudoscalar **heavier** than
scalar glueballs !

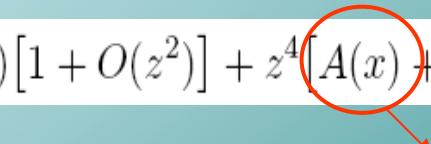
- Gluon condensate $\langle \frac{\alpha_s}{\pi} \text{Tr } F^2 \rangle$

Bulk-to-boundary propagator

$$X(x, z) = \int_{M^4} d^4x' K(x, z; x', 0) X_0(x')$$

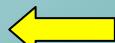
UV limit $z \rightarrow 0$

$$X(x, z) \sim X_0(x)[1 + O(z^2)] + z^4[A(x) + O(z^2)] \quad (\text{Witten '99})$$



AdS prediction of
gluon condensate

$$\langle \frac{\alpha_s}{\pi} \text{Tr } F^2 \rangle_{AdS}$$



physical fluctuation

$$\langle \mathcal{O}(x) \rangle = 4 A(x)$$

A widely studied topic with many applications

- Exotic states (glueball, etc.) :

- P. Colangelo, F. De Fazio, F.J., S. Nicotri, hep-ph/0703316
- C.Csáki et al., hep-th/9806021, hep-ph/0608266
- E. Katz et al., hep-ph/0510388
- N.R. Constable and R.C. Myers, hep-th/9905081
- H. Boschi-Filho and N.R.F. Brage, hep-th/0207071, 0209080
- N. Evans et al., hep-th/0505250

- Chiral symmetry breaking mechanism, light mesons :

- J. Erlich et al., hep-ph/0501128, 0602229
- N. Evans, hep-th/0306018
- Da Rold and Pomarol, hep-ph/0510268
- J. Hirn, V. Sanz and N. Ruis, hep-ph/0507049, 0512240

- Wilson loop and Heavy quarkonium $Q\bar{Q}$ potential :

- J. Maldacena, hep-th/9803002
- O. Andreev and V.I. Zakharov, hep-ph/0604204
- C.D. White, hep-ph/0701157

- Quark-gluon plasma :

- S.-J. Rey and J.-T. Yee, hep-ph/9803001
- D.T. Son et al., hep-th/0405231

- Heavy-light mesons :

- N. Evans et al., hep-th/0605241
- M. Bando et al., hep-ph/0602203
- R.C. Myers, hep-th/0304032

- Baryons :

- D.K. Hong et al., hep-ph/0609270

Backup Slides

More about the operator/field correspondence

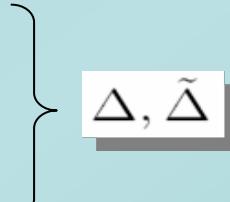
- Bulk field $A(x,z)$: **p** form (totally antisymmetric tensor with p indexes)

$\left\{ \begin{array}{l} \text{0-form : } \\ \text{1-form : } \\ \text{2-form : } \end{array} \right.$	ϕ	(scalar)
	A_M	(vector)
	$A_{[M,N]}$	(strength field F_{MN})

• eq. of motion $\mathcal{D}A(x^M) = 0$  mass term  $A(x^M)$

- Superconformal gauge theory : conformal group invariant

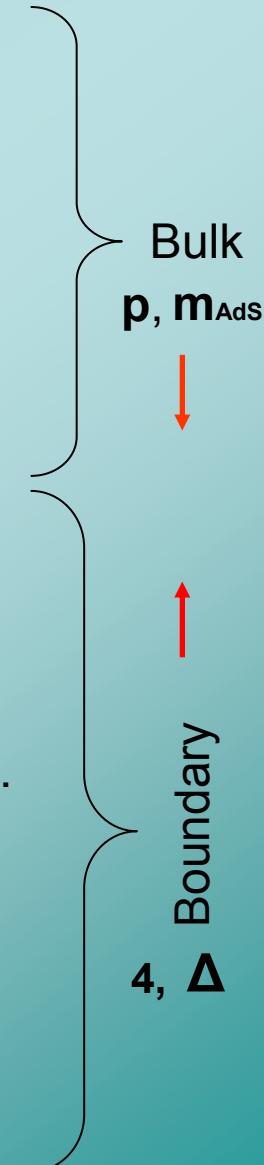
Scale transf. : $x^\mu \rightarrow \lambda x^\mu$

Field $A_0(x^\mu) \rightarrow \lambda^{-\tilde{\Delta}} A_0(x^\mu)$ Operator $O(x^\mu) \rightarrow \lambda^{-\Delta} O(x^\mu)$  $\Delta, \tilde{\Delta}$: scaling dim. = canonical dim. (without anomalous dim.)

$\langle e^{i \int d^4x A_0(x) O(x)} \rangle_{CFT} \rightarrow \langle e^{i \int d^4x \lambda^4 \tilde{\Delta} A_0(x) \lambda^{-\Delta} O(x)} \rangle_{CFT}$

 $4 - \tilde{\Delta} - \Delta = 0$ or $\tilde{\Delta} = 4 - \Delta$

$R^2 m_{AdS}^2 = (\Delta - p)(\Delta + p - 4)$



AdS \longleftrightarrow CFT

Holographic space : Bulk AdS_5

\longleftrightarrow Our spacetime \mathcal{M}^4

String theory $\left\{ \begin{array}{l} \cdot \text{weakly coupled} \\ \cdot \text{classical} \end{array} \right.$

\longleftrightarrow SU(N) $\left\{ \begin{array}{l} \cdot \text{strongly coupled } \lambda \\ \cdot \text{SUSY} \\ \cdot \text{conformal} \end{array} \right.$

Bulk field $A(x^\mu, z)$ $\left\{ \begin{array}{l} \cdot p\text{-form} \\ \cdot \text{massive} \end{array} \right.$

\longleftrightarrow Operator $O(x^\mu)$ scaling dimension Δ

$A_0(x'^\mu)$

Source $A_0(x'^\mu)$ scaling dimension $\tilde{\Delta}$

$A(x^\mu, z)$

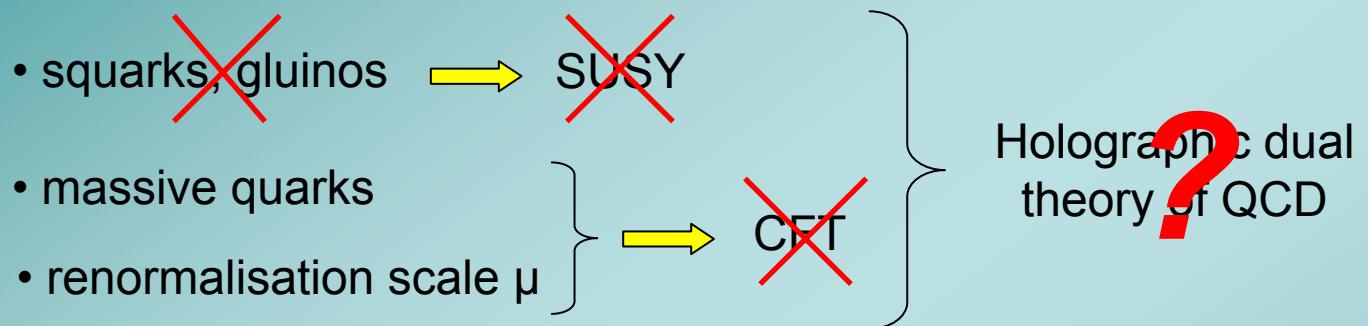
$$R^2 m_{AdS}^2 = (\Delta - p)(\Delta + p - 4)$$

z

AdS/QCD Correspondence

(Witten '98)

QCD SU(3)c



How modifying AdS/CFT towards AdS/QCD ?



QCD could be nearly conformal (**UV**) (Brodsky '02; Alkofer et al. '04)

QCD could have **IR fixed point**

Dimensionless renormalized Green function :

$$G^{(n)}(p, m, \lambda, \mu)$$

Renormalization :

$$\mu \rightarrow \bar{\mu}(t) = e^t \mu$$

$$\left. \begin{array}{l} \bullet \text{ effective coupling} \\ \bullet \text{ effective mass} \end{array} \right\} \quad \begin{array}{l} \lambda \rightarrow \bar{\lambda}(t, \lambda) \\ m \rightarrow \bar{m}(t, m) \end{array} \quad \begin{array}{l} \text{with} \\ \text{with} \end{array} \quad \begin{array}{l} \frac{d\bar{\lambda}(t)}{dt} = \beta(\bar{\lambda}) \\ \frac{d \ln \bar{m}(t)}{dt} = \gamma_m(\bar{\lambda}) \end{array}$$

Homogeneous RGE :

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma_m(\lambda) m \frac{\partial}{\partial m} \right) G^{(n)} = 0$$

Scale transf. :

$$G^{(n)}(p, m, \lambda, \mu) \rightarrow G^{(n)}(e^t p, m, \lambda, \mu) = G^{(n)}(p, \bar{\lambda}(t), e^{-t} \bar{m}(t), \mu)$$

Chiral limit $m=0$: $\lambda(t)$ breaks scale invariance

Classical theory or fixed point : $\beta=0$ and $\lambda(t) = \lambda = \text{const.}$

scale invariant theory



$\left\{ \begin{array}{l} \text{Chiral QCD : } m=0 \\ \text{IR fixed point } \lambda^* : \beta(\lambda^*)=0 \end{array} \right.$



QCD nearly conformal invariant

AdS/CFT



AdS/QCD



Effective bulk field action

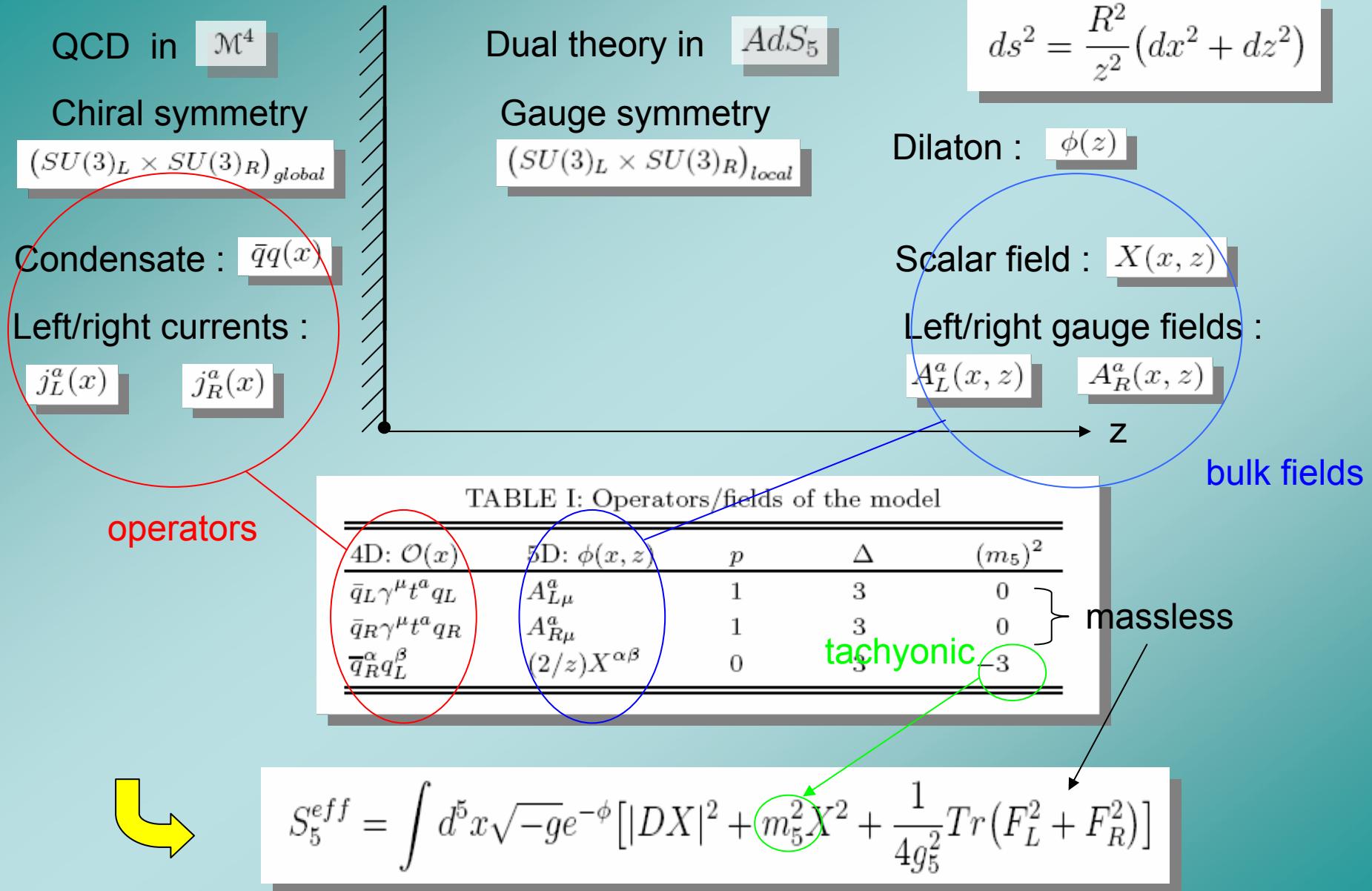
$$S_5^{eff}[A(x^M)]$$



Deformation of the geometry

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2)$$

AdS/QCD spectrum of p meson (Son et al. '05)



(Classical) eq. of motion : $\partial_M (\sqrt{-g} e^{-\phi} [\partial^M V^N - \partial^N V^M]) = 0$

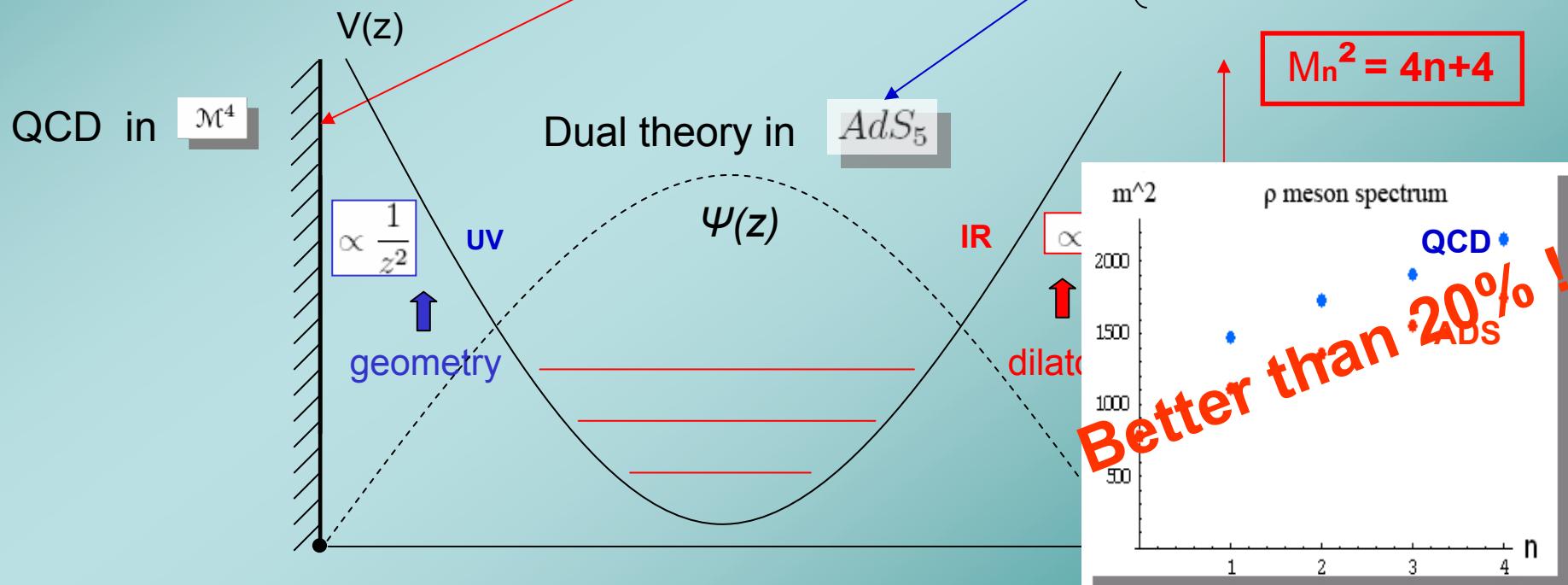
ρ meson vector field : $V = \frac{A_R + A_L}{2}$ → $V_\mu(x, z) = \underbrace{\epsilon_\mu}_{\text{plane wave}} e^{iq \cdot x} \underbrace{\psi(z)}_{\text{holo. wave function}}$

Schrödinger eq. : $-\psi'' + V(z)\psi = m_n^2 \psi(z)$

Regge behaviour : $m_n^2 \propto n$

connection
dilaton/geometry

$$\left. \begin{array}{l} \phi - A \xrightarrow{z \rightarrow 0} -\ln\left(\frac{z}{R}\right) \\ \phi - A \xrightarrow{z \rightarrow \infty} \frac{z^2}{R^2} \end{array} \right\}$$



The particular case of the Pseudoscalar glueball

Lattice QCD (Meyer '05)

Top-to-bottom approach

$$\frac{m_0^2(0^{-+})}{m_0^2(0^{++})} = 2.33 \pm 0.26$$

pseudoscalar **heavier** than scalar glueballs !

Scalar glueball 0^{++}

$$Tr F^2$$

$$\Delta = 4$$

$$X(x, z) \left\{ \begin{array}{l} p = 0 \\ m_5^2 = 0 \end{array} \right.$$

$$m_n^2 = 4n + 8$$

Hybrid states ($q\bar{q}'g$)

$$Tr(\bar{q}Dq)_\mu$$

$$\Delta = 4$$

$$A_M(x, z)$$

$$\left\{ \begin{array}{l} p = 1 \\ m_5^2 = 3 \end{array} \right.$$

$$m_n^2 = 4n + 6$$

Pseudoscalar glueball 0^{-+}

$$Tr F \tilde{F}$$

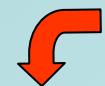
$$\Delta = 4$$

$$A_{\bar{M}}(x, z) \left\{ \begin{array}{l} pp=10 \\ m_5^2 = 3 \end{array} \right.$$

$$m_n^2 = 4n + 6$$

Top-to-bottom approach (Evans et al. '05, Csaki et al. '06)

(massless) 1-form A_M



Modification of the background



modification of the dilaton

$$\begin{aligned} \phi(z) &= z^2 + \lambda z \\ g_{MN} &= \frac{1}{z^2} \eta_{MN} \end{aligned}$$



$$m_n^2 = m_{n,(0)}^2 + \lambda m_{n,(1)}^2$$

modification of the geometry

$$\begin{aligned} \phi(z) &= z^2 \\ g_{MN} &= \frac{e^{-2\lambda z}}{z^2} \eta_{MN} \end{aligned}$$

- modification of the dilaton :



cannot switch hierarchy

- modification of the geometry :



can switch hierarchy

$$V(z) = z^2 + \frac{(c^2 + 2c)}{4z^2} + \frac{4m_5^2}{4z^2} + (c - 1) + O(\lambda)$$

dilaton geometry

$$V(z) = z^2 + \frac{(c^2 + 2c)}{4z^2} + \frac{4m_5^2 e^{-2\lambda z}}{4z^2} + (c - 1) + O(\lambda)$$

dilaton geometry mass dependent modification

Scalar glueball 0^{++}

$$Tr F^2$$

$$\Delta = 4$$

$$X(x, z)$$

$$\left\{ \begin{array}{l} p = 0 \\ m_5^2 = 0 \end{array} \right.$$

$$m_n^2 = 4n + 8$$

$$\begin{aligned} m_0^2 &= 8 + \lambda \frac{9\sqrt{\pi}}{2} \\ m_1^2 &= 12 + \lambda \frac{81\sqrt{\pi}}{16} \\ m_2^2 &= 16 + \lambda \frac{711\sqrt{\pi}}{128} \end{aligned}$$

Perturbative
modification
of the geometry

Pseudoscalar glueball 0^{-+}

$$Tr F \tilde{F}$$

$$\Delta = 4$$

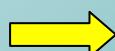
$$A_M(x, z)$$

$$\left\{ \begin{array}{l} p = 1 \\ m_5^2 = 3 \end{array} \right.$$

$$m_n^2 = 4n + 6$$

$$\begin{aligned} m_0^2 &= 6 - \lambda \frac{9\sqrt{\pi}}{8} \\ m_1^2 &= 10 - \lambda \frac{23\sqrt{\pi}}{32} \\ m_2^2 &= 14 - \lambda \frac{407\sqrt{\pi}}{1024} \end{aligned}$$

$$m_0^2(0^{-+}) \geq m_0^2(0^{++})$$



$$\lambda \leq -\frac{16}{45\sqrt{\pi}} \simeq -0.2$$

small value of λ :
modified spectrum

Decay constants of glueballs

Operator/field correspondence :

$$e^{iS_5^{eff}[X(x,z)]} = \langle e^{i \int d^4x X_0(x) \mathcal{O}(x)} \rangle_{CFT}$$

2-points correlator function

$$\Pi(q^2)$$



Decay constant

$$f_n = \langle 0 | \mathcal{O}(0) | n \rangle$$

$$\Pi_{QCD}(q^2) = \Pi_{AdS}(q^2)$$

• **QCD :** $\Pi_{QCD}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0 | T[\mathcal{O}(x)\mathcal{O}(0)] | 0 \rangle$

Completeness in the 2 chronological order :

$$\Pi_{QCD}(q^2) = \sum_n \frac{f_n^2}{q^2 + m_n^2}$$

• **AdS :** $\Pi_{AdS}(q^2) = \left(\tilde{X}(q, z), \partial_z \tilde{X}(q, z) \right) \Big|_{z \rightarrow 0}$

Fourier transf. of $X(x, z)$



Bulk-to-boundary
propagator

Bulk-to-boundary propagator (massless scalar bulk field) :

$$X(x, z) = \int_{M^4} d^4x' K(x, z; x', 0) X_0(x')$$

Boundary translation invariance : $K(x - x'; z, 0) \xrightarrow{z \rightarrow 0} \delta^4(x - x')$

$$\tilde{X}(q, z) = \tilde{K}(q, z) \tilde{X}_0(q) \quad \text{with} \quad \tilde{K}(q, z) \xrightarrow{z \rightarrow 0} 1 \quad (\text{massless scalar})$$



$$\Pi_{AdS}(q^2) = \tilde{K}(q, z) \left(\frac{e^{-\phi(z)}}{z^3} \right) \partial_z \tilde{K}(q, z) \Big|_{z=0}$$

- $q^2 = -m_n^2$ normalizable bulk mode $\tilde{K}_n(z)$ dual to particle states

$$z \rightarrow 0 \quad \tilde{K}_n(z) \sim A_n z^4$$
- $q^2 > 0$ non-normalizable bulk mode $\tilde{K}(q, z)$ dual to currents (virtuality)

$$z \rightarrow 0 \quad \tilde{K}(q, z) \sim 1$$

 (deep inelastic limit : $q^2 \rightarrow \infty$)

eq. of motion :

$$\mathcal{D}\tilde{K}_n(z) = \left[\partial_z \left(\frac{e^{-\phi}}{z^3} \partial_z \right) + m_n^2 \frac{e^{-\phi}}{z^3} \right] \tilde{K}_n(z) = 0$$

$$q^2 = -m_n^2$$

Sturm-Liouville operator

completeness

Green's function :

$$\mathcal{D}G(q^2; z, z') = -\delta(z - z')$$



$$G(q^2; z, z') = \sum_n \frac{\tilde{K}_n(z)\tilde{K}_n(z')}{q^2 + m_n^2}$$

Green's theorem :

$$\tilde{K}(q, z) = \tilde{K}(q, z') \left(\frac{e^{-\phi(z')}}{z'^3} \right) \partial_{z'} G(q^2, z', z) \Big|_{z' \rightarrow 0}$$

$$\Pi_{AdS}(q^2) = \sum_n \frac{1}{q^2 + m_n^2} \left[\underbrace{\tilde{K}(q, z)}_{1} \underbrace{\frac{e^{-\phi(z)}}{z^3}}_{1/z^3} \underbrace{\partial_z \tilde{K}_n(z)}_{4A_n z^3} \right]^2 \Big|_{z \rightarrow 0}$$

→ $f_n = 4A_n \sim \sqrt{8(n+1)(n+2)}$

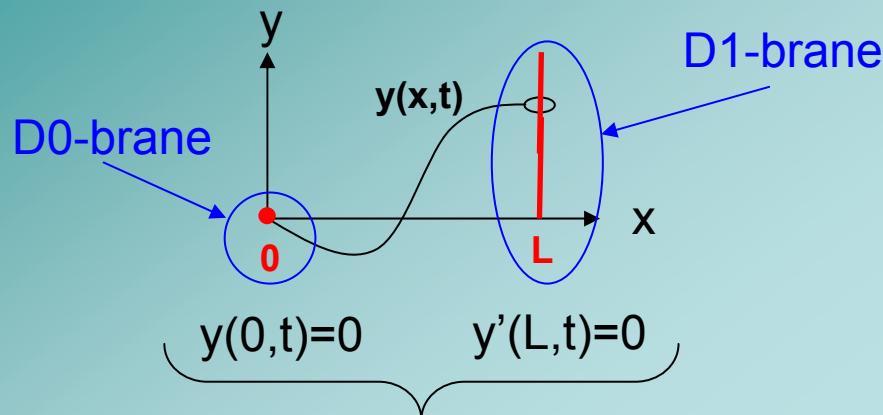
Heavy-light meson spectrum

(Evans et al. '06)

$Q\bar{q}$ mesons { $D=c\bar{q}$ $B=b\bar{q}$ ($q=u,d,s$) } \rightarrow

$D_{(\text{irichlet})}$ p-brane model of spacetime :

- p spatial-dim. object
- $(p+1)$ -dim. spacetime



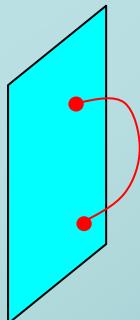
D3-brane in 4-dim. Spacetime : \mathcal{M}^4 ?

D_p -branes : boundary conditions \rightarrow

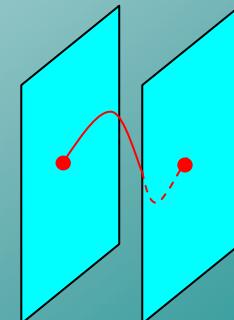
Open string endpoints attached to D_p -branes

Open string spectrum

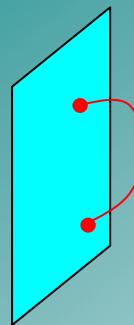
D_3 -brane :



D_3 - D_3 -branes :



D3-brane :

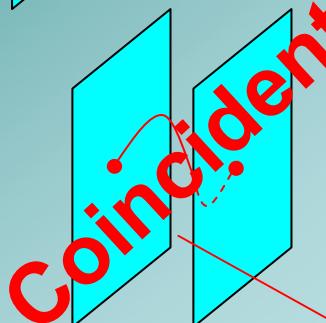


$$M^2 = \frac{1}{\alpha'}(N - 1)$$

1 massless vector
(tachyon, massless scalars)

(harm. osc. $E = \hbar\omega(N+1/2)$)

D3-D3-branes :



$$M^2 = \underbrace{\frac{1}{\alpha'}(N - 1)}_{\text{quantum osc.}} + \underbrace{[T_0(\bar{x}_2 - \bar{x}_1)]^2}_{\text{classical energy of the stretched string}}$$

: (energy/length)
x (length)

\bar{x}_1

\bar{x}_2

1 massive vector
(tachyon, massive scalars)

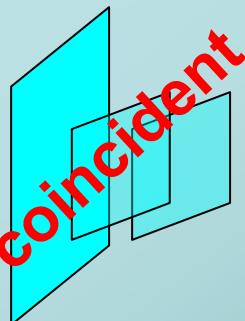
$$M^2 = [T_0(\bar{x}_2 - \bar{x}_1)]^2$$

1 massless vector

$$M^2 = 0$$

Standard Model
(QCD)

3 D3-branes



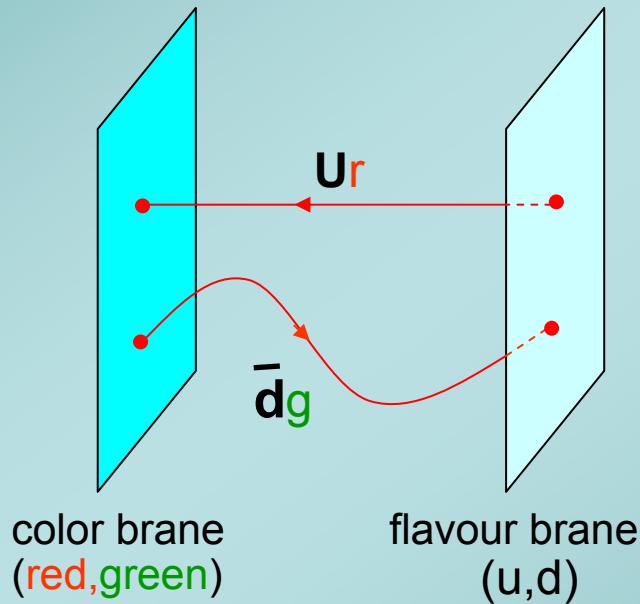
3 x 3 massless vectors : 9 gauge fields : $SU(3) \times U(1)$
in (3+1) spacetime

$SU(3)c$

Boundary of the bulk

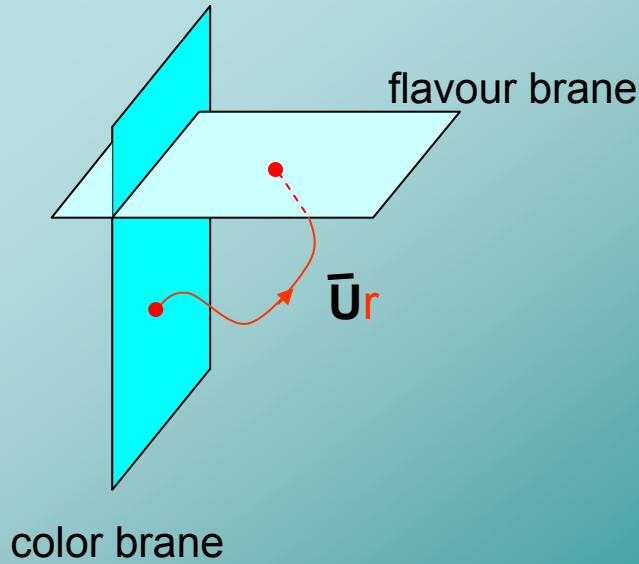
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- Gluons : open strings with the **2** endpoints attached on the 3 (colored) D3-branes
 - Quarks : open strings with 
 - 1** endpoint attached on the 3 (colored) D3-brane
 - 1** endpoint attached to a flavour Dp-brane (D7-brane)



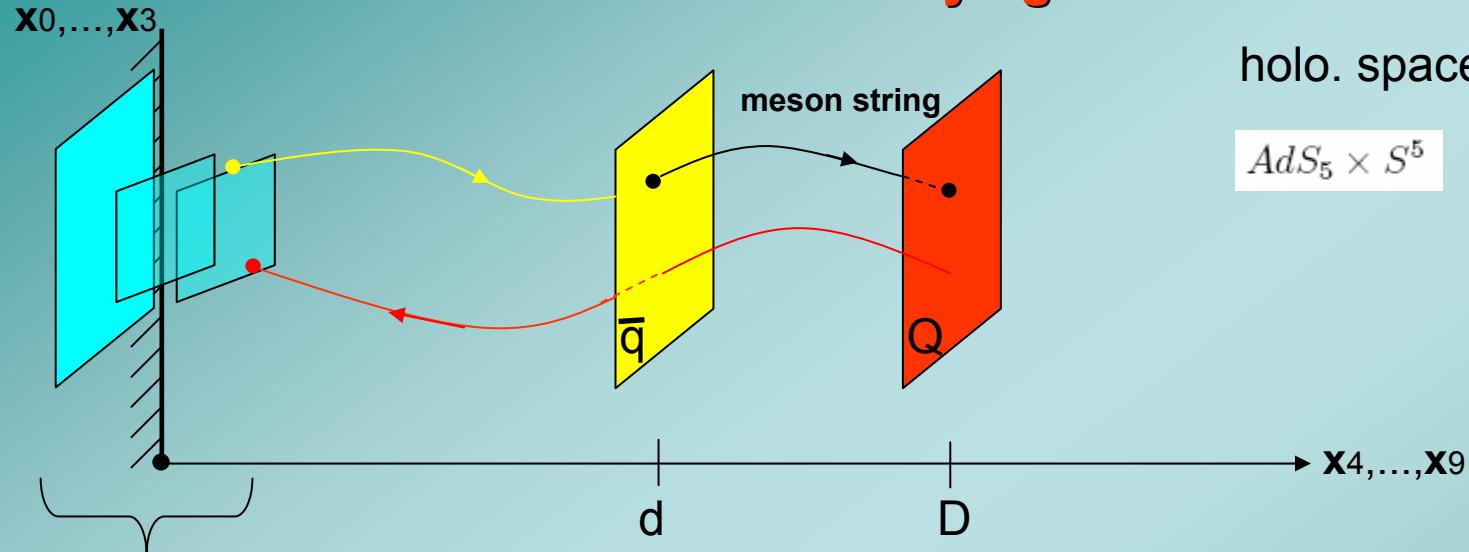
Massive quarks

$$M^2 = [T_0(\bar{x}_2 - \bar{x}_1)]^2$$



Massless (chiral) quarks

D3-D7-brane model of heavy-light mesons



D7-D3 open string spectrum :

$$M^2 = \frac{1}{\alpha'} \left(N - 1 + \frac{1}{4} \right) + [T_0(\bar{x}_2 - \bar{x}_1)]^2$$

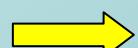


semi-classical string limit $\longrightarrow D \gg d$ (B meson)

Heavy-light meson spectrum :

$$M^2 = [T_0(D - d)]^2$$

$$\left. \begin{array}{l} M_\rho = 770 \text{ MeV} : d \\ M_Y = 9.4 \text{ GeV} : D \end{array} \right\}$$



B meson : $M_B = 6529 \text{ MeV} (5279 \text{ MeV})$

better than 20% !

holo. spacetime

$$AdS_5 \times S^5$$