

# Form factors of Baryons



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# Outline

## Form factors from Baryon DAs via LCSR

- Basic idea
- Example: Em form factors of the nucleon
- L.H.S. of the sum rule
- R.H.S. of the sum rule
- Combining L. & R.H.S. of the sum rule

## Current activities - including higher twist DAs:

- Literature - Overview
- Determining the DA
- Nucleon form factors
- Decays of Baryons

## Outlook



# Form factors from Baryon DAs via LCSR

To describe the transition of a baryon B to a baryon N via the current  $j_\mu$

$$B(P') \xrightarrow{j_\mu} N(P), \quad P' = P - q$$

Start with a correlation function

$$T_\mu(P, q) = \int d^4x e^{-ipx} \langle 0 | T\{\eta(0)j_\mu(x)\} | N(P) \rangle$$

■ Interpolating field  $\eta$ : “creating B from the vacuum”

e.g.  $\eta_{\text{CZ}}(x) = \varepsilon^{ijk} [u^i(x)(C\gamma^\mu) u^j(x)] (\gamma_5 \gamma^\mu) d_\delta^k(x) \Rightarrow B = \text{Proton}$

or  $\eta_{\text{Ioffe}}(x) = \varepsilon^{ijk} [u^i(x)(C\gamma_\nu) u^j(x)] (\gamma_5 \gamma^\nu) d_\delta^k(x) \Rightarrow B = \text{Proton}$

■ Current  $j_\mu$

e.g.  $j_\mu^{\text{em}}(x) = e_u \bar{u}(x) \gamma_\mu u(x) + e_d \bar{d}(x) \gamma_\mu d(x) \Rightarrow \text{em form factors}$

e.g.  $j_\mu^{\text{weak}}(x) = \bar{u}(x) \gamma_\mu (1 - \gamma_5) d(x) \Rightarrow \text{weak decay}$

Now: Express  $T_\mu$  in two different ways



# Example: EM form factors of the Nucleon

- Rosenbluth-formula (1955) for elastic  $e^-$ -N scattering (1 photon exchange)

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

$$Q^2 = -q^2, \quad \tau = \frac{Q^2}{4M^2c^2}, \quad \theta_e = \text{scattering angle of } e^-$$

- Electric  $G_E(Q^2)$  and magnetic  $G_M(Q^2)$  **Sachs form factors**
- Interpretation in **Breit frame**:
  - $G_E(Q^2)$  Fourier transform of electric charge distribution
  - $G_M(Q^2)$  Fourier transform of magnetization density
- Relation of Dirac and Pauli form factors to Sachs form factors

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2).$$

- $G_M^p(0) = \mu_p = 2.79$ ,  $G_M^n(0) = \mu_n = -1.91$ .
- Def.:  $\langle P', s | j_\mu^{\text{em}} | P \rangle = \bar{N}(P') \left[ \gamma_\mu F_1 - i \frac{\sigma_{\mu\nu} q^\nu}{2m_p} F_2 \right] N(P)$



# L.H.S. of the sum rule

Inserting a full set of one-particle states in the correlation function we obtain

$$T_\mu(P, q) = \frac{1}{m_p^2 - (P')^2} \sum_s \langle 0 | \eta_p | P', s \rangle \langle P', s | j_\mu^{\text{em}} | P \rangle + \dots$$

where the dots represent contributions of higher resonances.

The arising matrix elements read

- $\langle 0 | \eta_{\text{CZ}} | P \rangle = f_N P z \not{z} N(P)$       or       $\langle 0 | \eta_{\text{Ioffe}} | P \rangle = \lambda_1 m_p N(P)$   
with the non-perturbative constants  $f_N, \lambda_1$
- $\langle P', s | j_\mu^{\text{em}} | P \rangle = \bar{N}(P') \left[ \gamma_\mu F_1 - i \frac{\sigma_{\mu\nu} q^\nu}{2m_p} F_2 \right] N(P)$

$$\Rightarrow T_\mu = T_\mu(F_1, F_2)$$

Which  $\eta$  to use?

Determine  $f_N, \lambda_1, \dots$



# R.H.S. of the sum rule

Do all possible Wick-contractions in  $T_\mu$ :

$$T_\mu(P, q) \propto \int d^4x e^{-ipx} \dots \Gamma_1^{\alpha\beta} \Gamma_2^{\delta\gamma} 4\langle 0 | \epsilon_{ijk} u_\alpha^i u_\beta^j d_\gamma^k | N(P) \rangle$$

and insert the Baryon DA

$$4\langle 0 | \epsilon_{ijk} u_\alpha^i u_\beta^j d_\gamma^k | N(P) \rangle = \sum \Gamma_3^{\alpha\beta} \Gamma_4^\gamma F$$

with Dirac structures  $\Gamma_i$  and 24 distribution amplitudes  $F$

In the end all DAs can be reduced to eight non-perturbative parameters

$$\Rightarrow T_\mu = T_\mu(f_N, \lambda_1, \lambda_2, V_1^d, A_1^u, f_1^d, f_1^u, f_2^d)$$

Determine  $f_N, \lambda_1, \dots$



# Combining L.H.S. and R.H.S.

In order to combine the two expressions for  $T_\mu$  we perform first a projection

$$\Rightarrow \Lambda_+ T_z = pz (m_p \mathcal{A}_1^{\text{em}} + q_\perp \mathcal{B}_1^{\text{em}}) N^+(P)$$

with  $z^2 = 0 = qz; p_\mu = P_\mu - z_\mu m_p^2/(2Pz); \Lambda^+ = p^z/(2pz); \Lambda^+ N = N^+$

$$\mathcal{B}_1^{\text{em}} = \frac{\lambda_1 F_2^{\text{em}}}{m_p^2 - P'^2} = -2e_d \int_0^1 \frac{dx_3}{(q - x_3 P)^2} \int_0^{1-x_3} dx_1 120 x_1 x_2 x_3 \delta(1-x_1-x_2-x_3) f_N + \dots$$

To suppress the contributions of higher resonances we perform a Borel trafo

$$F_2^{\text{em}} = 2e_d \frac{1}{\lambda_1} \int_{x_0}^1 \frac{dx}{x} \int_0^{1-x} dx_1 120 x_1 x_2 x \delta(1-x_1-x_2-x) f_N + \dots$$



# Current activities - including higher twist DAs

- The nucleon DA up to twist 6  
Braun, Fries, Mahnke, Stein, 2000
- LCSR: em form factors of the nucleon  
use  $\eta_{CZ}$ ;  $x^2$ -corrections to  $V_1$   
Braun, A.L., Mahnke, Stein, 2001
- LCSR: em form factors of the nucleon  
use isospin conserving  $\eta_{Im.}$ .  
A.L., Wittmann, Stein, 2003
- LCSR: form factors of the nucleon,  $n \rightarrow p$   
use  $\eta_{CZ}, \eta_{Im.}, \eta_{Ioffe}$ ;  $x^2$ -correct. to  $A_1, T_1$   
estimate of non-perturbative parameters  
Braun, A.L., Wittmann, 2006
- LCSR:  $N \rightarrow \Delta$   
Braun, A.L., Peters, Radyushkin, 2005
- DA + LCSR:  $N \rightarrow N + \pi$   
Braun, Ivanov, A.L., Peters, 2006
- LCSR:  $\Lambda_b \rightarrow p l \bar{\nu}$   $x^2$ -corrections to  $V_1, A_1, T_1$   
Huang, Wang, 2004
- LCSR: Scalar form-factor of the nucleon  
use  $\eta_{CZ}$   
Z. Wang, Wan, Yang, 2006
- LCSR: axial and induced pseudoscalar  
form-factor of the nucleon  
use  $\eta_{CZ}$  Z. Wang, Wan, Yang, 2006
- LCSR:  $\Lambda_c \rightarrow \Lambda l \bar{\nu}$   
Axial  $\Lambda$ -DAs of leading conformal spin  
Huang, Wang, 2006
- LCSR:  $\Sigma \rightarrow N$   
use  $\eta_{Ioffe}$  Z. Wang, 2006
- LCSR: Scalar form-factor of the nucleon  
use general  $\eta$  Aliev, Savci, 2006



# Determining the Nucleon DA upto twist 6 I

$$4\langle 0 | \varepsilon^{ijk} u_\alpha^i(a_1 x) u_\beta^j(a_2 x) d_\gamma^k(a_3 x) | P \rangle =$$

$$\begin{aligned}
& \mathcal{S}_1 M C_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta} (\not{x}\gamma_5 N)_\gamma \mathcal{P}_1 M (\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{P}_2 M^2 (\gamma_5 C)_{\alpha\beta} (\not{x}N)_\gamma \\
& + \left( \mathcal{V}_1 + \frac{x^2 m_N^2}{4} \mathcal{V}_1^M \right) (\not{P}C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{V}_2 M (\not{P}C)_{\alpha\beta} (\not{x}\gamma_5 N)_\gamma + \mathcal{V}_3 M (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma \\
& + \mathcal{V}_4 M^2 (\not{x}C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{V}_5 M^2 (\gamma_\mu C)_{\alpha\beta} (i\sigma^{\mu\nu} x_\nu \gamma_5 N)_\gamma + \mathcal{V}_6 M^3 (\not{x}C)_{\alpha\beta} (\not{x}\gamma_5 N)_\gamma \\
& + \left( \mathcal{A}_1 + \frac{x^2 m_N^2}{4} \mathcal{A}_1^M \right) (\not{P}\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_2 M (\not{P}\gamma_5 C)_{\alpha\beta} (\not{x}N)_\gamma + \mathcal{A}_3 M (\gamma_\mu \gamma_5 C)_{\alpha\beta} (\gamma^\mu N)_\gamma \\
& + \mathcal{A}_4 M^2 (\not{x}\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_5 M^2 (\gamma_\mu \gamma_5 C)_{\alpha\beta} (i\sigma^{\mu\nu} x_\nu N)_\gamma + \mathcal{A}_6 M^3 (\not{x}\gamma_5 C)_{\alpha\beta} (\not{x}N)_\gamma \\
& + \left( \mathcal{T}_1 + \frac{x^2 m_N^2}{4} \mathcal{T}_1^M \right) (i\sigma_{\mu P} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_2 M (i\sigma_{x P} C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{T}_3 M (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 N)_\gamma \\
& + \mathcal{T}_4 M (P^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma + \mathcal{T}_5 M^2 (x^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_6 M^2 (i\sigma_{x P} C)_{\alpha\beta} (\not{x}\gamma_5 N)_\gamma \\
& + \mathcal{T}_7 M^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \not{x}\gamma_5 N)_\gamma + \mathcal{T}_8 M^3 (x^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\rho} x_\rho \gamma_5 N)_\gamma \quad \text{BFMS 2000}
\end{aligned}$$

- \* The 24 functions  $\mathcal{F}^{(i)} = \mathcal{S}_i, \mathcal{P}_i, \mathcal{A}_i, \mathcal{V}_i, \mathcal{T}_i$  can be related to 8 LCDAs of twist-3 to twist-6.
- \* In leading conformal spin we have 3 parameters:  $\lambda_1, \lambda_2, f_N$
- \* in NL conformal spin we have 5 parameters:  $V_1^d, A_1^u, f_1^d, f_1^u, f_2^d$
- \*  $\mathcal{V}_1^M$ : BLMS 2001
- \*  $\mathcal{A}_1^M, \mathcal{T}_1^M$ : Huang, Wang 2004; BLW 2006.



# Determining the Nucleon DA upto twist 6 II

Definition of the non-perturbative parameters

$$\begin{aligned} \langle 0 | \varepsilon^{ijk} [u^i(0)(C\gamma) u^j(0)] (\gamma_5 \gamma) d_\delta^k(0) | P \rangle &= f_N p z \gamma N(P) \\ \langle 0 | \varepsilon^{ijk} [u^i(0)(C\gamma_\mu) u^j(0)] (\gamma_5 \gamma^\mu) d_\delta^k(0) | P \rangle &= \lambda_1 m_N N(P) \\ \langle 0 | \varepsilon^{ijk} [u^i(0)(C\sigma_{\mu\nu}) u^j(0)] (\gamma_5 \sigma^{\mu\nu}) d_\delta^k(0) | P \rangle &= \lambda_2 m_N N(P) \\ \\ \langle 0 | \varepsilon^{ijk} [u^i(0)(C\gamma) u^j(0)] (\gamma_5 \gamma) (iz \vec{D} d_\delta^k)(0) | P \rangle &= f_N V_1^d (pz)^2 \gamma N(P) \\ \langle 0 | \varepsilon^{ijk} [u^i(0)(C\gamma) \gamma_5 iz \stackrel{\leftrightarrow}{D} u^j(0)] (\gamma \gamma_5 d_\delta^k)(0) | P \rangle &= -f_N A_1^u (pz)^2 \gamma N(P) \\ \langle 0 | \varepsilon^{ijk} [u^i(0) C \gamma_\mu u^j(0)] \gamma \gamma_5 \gamma^\mu (iz \vec{D} d^k)(0) | P \rangle &= \lambda_1 f_1^d (pz) M \gamma N(P), \\ \langle 0 | \varepsilon^{ijk} [u^i(0) C \sigma_{\mu\nu} u^j(0)] \gamma \gamma_5 \sigma^{\mu\nu} (iz \vec{D} d^k)(0) | P \rangle &= \lambda_2 f_2^d (pz) M \gamma N(P), \\ \langle 0 | \varepsilon^{ijk} [u^i(0) C \gamma_\mu \gamma_5 iz \stackrel{\leftrightarrow}{D} u^j(0)] \gamma \gamma^\mu d^k(0) | P \rangle &= \lambda_1 f_1^u (pz) M \gamma N(P), \end{aligned}$$



# Determining the Nucleon DA upto twist 6 III

## ■ Leading twist: $f_N, V_1^d, A_1^u$

- ◆ QCD SR: Chernyak, Zhitnitsky 1984; King, Sachrajda 1987; Gari, Stefanis 1987; Chernyak, Ogloblin, Zhitnitsky 1988, 1989; Quark model: Bolz, Kroll 1996.

$$f_N = (5.0 \pm 0.5) \cdot 10^{-3} \text{GeV}^2 \quad A_1^u = 0.38 \pm 0.15 \quad V_1^d = 0.23 \pm 0.03$$

- ◆ Lattice: Martinelli, Sachrajda 1989; Aoki et al. 2006., QCD-SF in progress
- ◆ Asymptotic:  $A_1^u = 0$   $V_1^d = 1/3$
- ◆ LCSR: BLW 2006  $A_1^u = 0.13$   $V_1^d = 0.30$

## ■ Higher twist

- ◆ leading conformal spin:  $\lambda_1, \lambda_2$  - QCD SR: BFMS 2000, BLW 2006

$$\lambda_1 = -(2.7 \pm 0.9) \cdot 10^{-2} \text{GeV}^2 \quad \lambda_2 = (5.4 \pm 1.9) \cdot 10^{-2} \text{GeV}^2$$

- ◆ next-to-leading conformal spin:  $f_1^d, f_1^u, f_2^d$

Method	$f_1^d$	$f_1^u$	$f_2^d$	authors
QCD SR	$0.40 \pm 0.05$	$0.07 \pm 0.05$	$0.22 \pm 0.05$	BFMS 2000, BLW 2006
LCSR	0.33	0.09	0.25	BFMS 2000, BLW 2006
asymptotic	0.30	0.10	4/15	



# LCSR for the nucleon form factors

- EM - form factors using  $\eta_{CZ}$ : BLMS 2001
  - ◆ Higher twist is important
- EM - form factors using isospin conserving  $\eta_{IM}$ : LWS 2004
  - ◆  $\eta_{CZ}$  leads to unphysical isospin violating effects
- EM and weak form factors, compare different  $\eta$ s: BLW 2006
  - ◆ surprisingly good description of data

⇒  $\eta_{\text{Ioffe}}$  seems to be the best choice

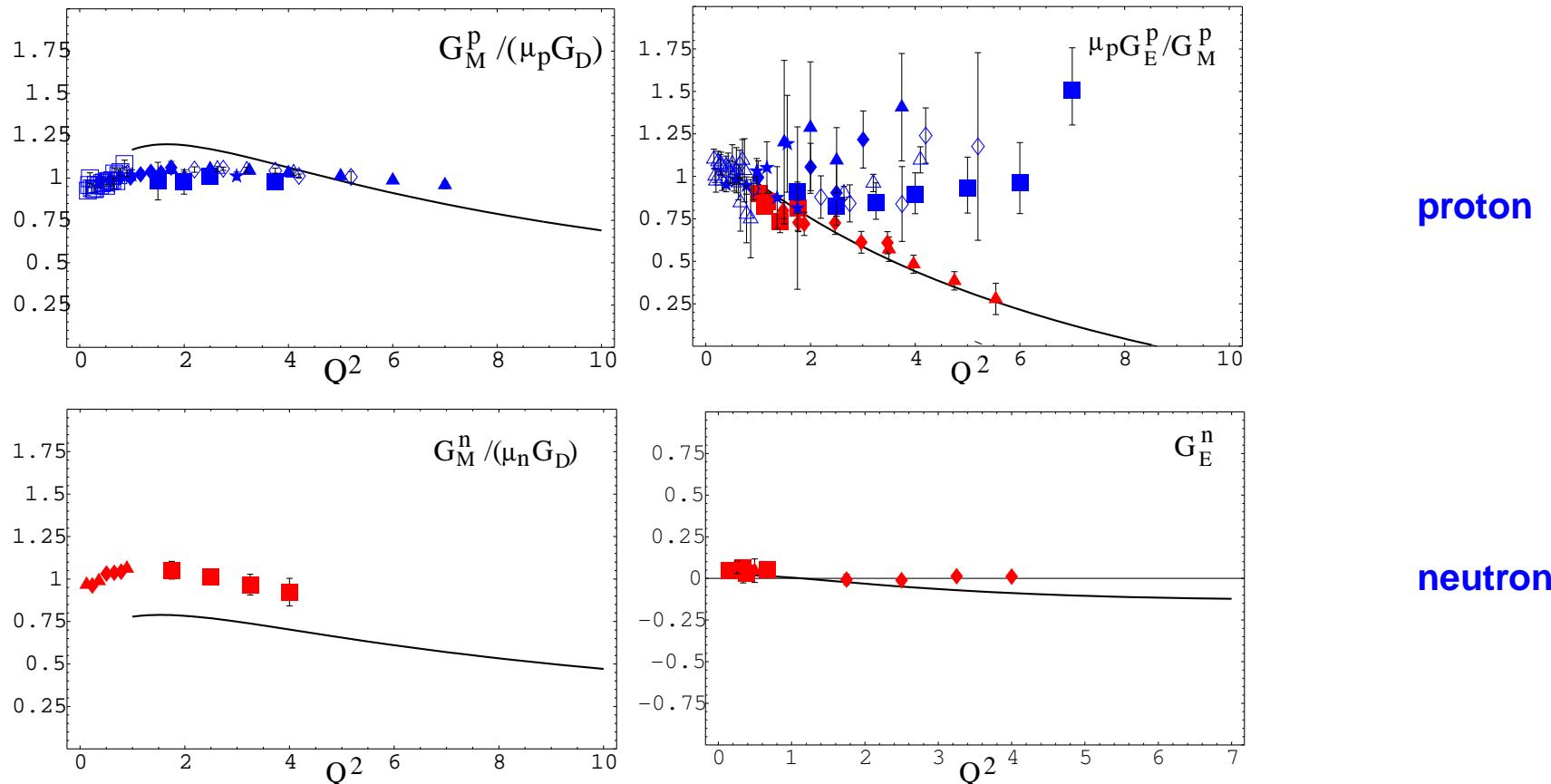
- Scalar form factor of the nucleon using  $\eta_{CZ}$ : Wang, Wan 2006
- Scalar form factor of the nucleon using general  $\eta$ : Aliev, Savci 2006
- Axial and ps form factor of the nucleon using  $\eta_{CZ}$ : Wang, Wan 2006

- \* ⇒ use  $\eta_{\text{Ioffe}}$  to compare experiment and LCSR
- \* FF depend on 5 Parameters  $\lambda_1/f_N, A_1^u, V_1^d, f_1^d, f_1^u$



# Nucleon electromagnetic form factors

$$\langle N(P') | j_\mu^{\text{em}}(0) | N(P) \rangle = \bar{N}(P') \left[ \gamma_\mu F_1(Q^2) - i \frac{\sigma_{\mu\nu} q^\nu}{2m_N} F_2(Q^2) \right] N(P)$$



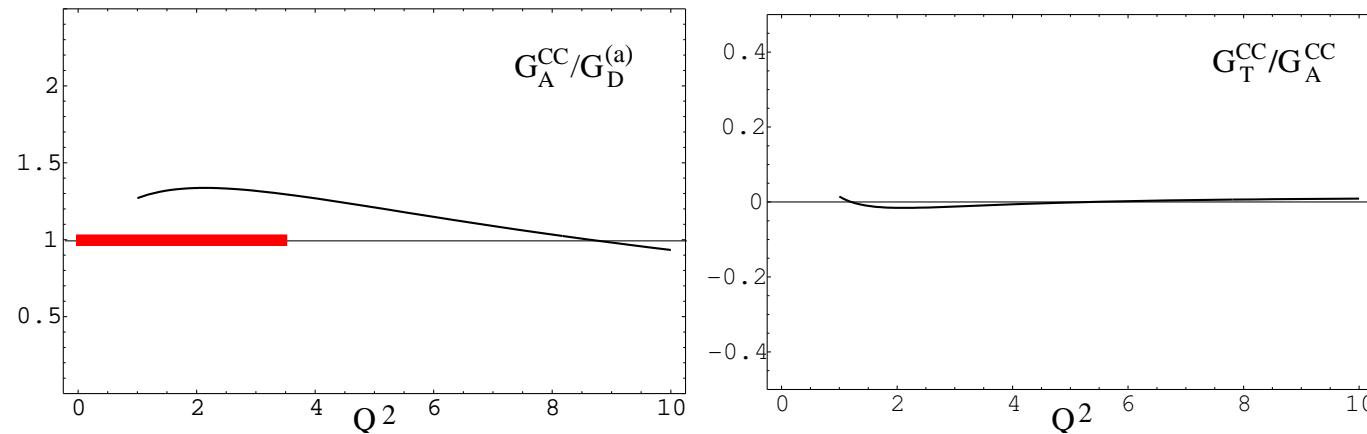
- Leading order LCSR, BLW distribution amplitudes

Braun, Lenz, Wittmann; PRD73 (2006) 094019



# Nucleon axial vector form factors

$$\langle N(P') | A_\mu(0) | N(P) \rangle = \bar{N}(P') \left[ \gamma_\mu G_A(Q^2) - \frac{q_\mu}{2m_N} G_P(Q^2) - i \frac{\sigma_{\mu\nu} q^\nu}{2m_N} G_T(Q^2) \right] \gamma_5 N(P)$$



charged  
current

- Leading order LCSR, BLW distribution amplitudes

Braun, Lenz, Wittmann; PRD73 (2006) 094019



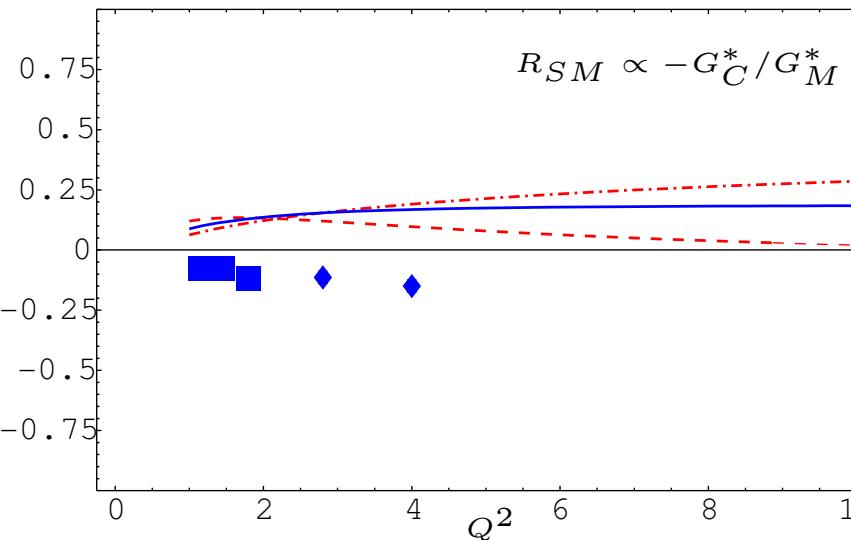
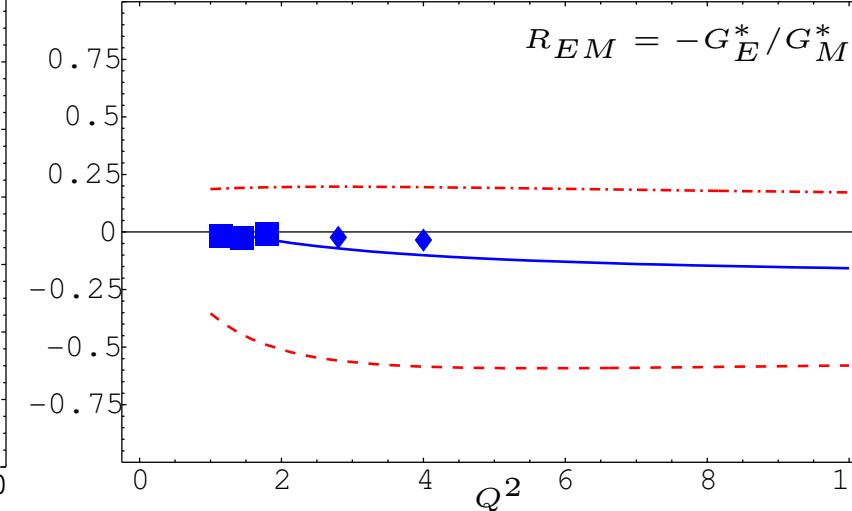
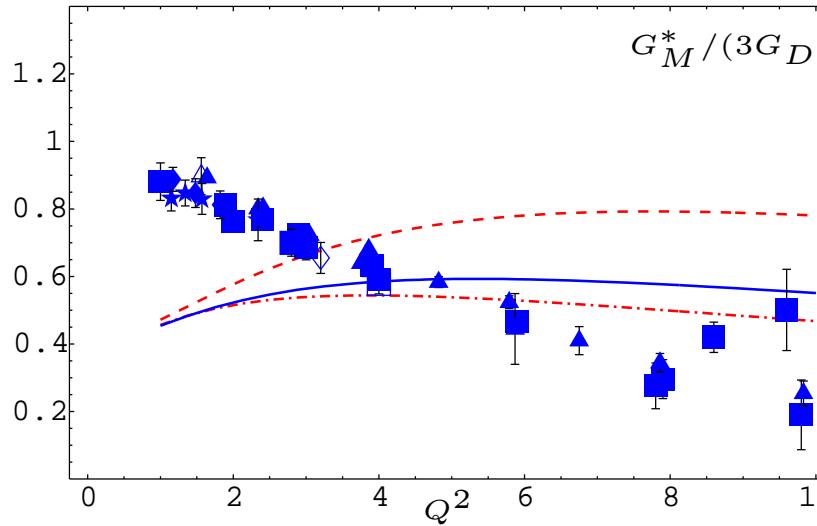
# LCSR for transitions

- Huang, Wang, 2004:  $\Lambda_b \rightarrow pl\nu$ 
  - ◆ Interpolating field:  $\eta_{\Lambda_b} = \epsilon_{ijk} u^i C \not{d}^j \cdot \gamma_5 \not{b}^k$  vs.  $\eta_{\Lambda_b} = \epsilon_{ijk} u^i C \not{d}^j \cdot \gamma_5 \not{h}_v^k$
  - ◆ Determine  $f_\Lambda$  from QCD-SR
  - ◆ HQET  $\approx 1/10$  QCD!
- BLPR 2005:  $N \rightarrow \Delta$ 
  - ◆ Disentangle  $N \rightarrow N^*$  (spin 1/2)
  - ◆ Interpolating field:  $\eta_\Delta = \epsilon_{ijk} (2u^i C \gamma_\mu d^j \cdot \not{u}^k + u^i C \gamma_\mu u^j \cdot \not{d}^k)$
- BLW 2006:  $n \rightarrow p$
- Huang, Wang, 2006:  $\Lambda_c \rightarrow \Lambda l\nu$ 
  - ◆ Determine  $\Lambda$ -DA up to twist-6 and leading conformal spin
  - ◆ Interpolating field:  $\eta_{\Lambda_c} = \epsilon_{ijk} u^i C \gamma_5 \not{d}^j \cdot \not{c}^k$
  - ◆ only tw-3 agrees with experiment
- Wang, 2006:  $\Sigma \rightarrow N$ 
  - ◆ Interpolating field:  $\eta_\Sigma = \epsilon_{ijk} d^i C \gamma_\mu d^j \cdot \gamma_5 \gamma^\mu s^k$
  - ◆ Wang compared results for  $Q^2 = 0$  with the data!



# $N\Delta\gamma$ transition form factors

- magnetic, electric and quadrupole form factors exist:



asymptotic  
CZ model  
BLW model

Braun, Lenz, Radyushkin, Peters; PRD73 (2006) 034020



# Outlook

## ■ Nucleon DA

- ◆  $\alpha_s$  corrections to LCSR for form factors of the nucleon,  
Compare LCSR to experiment and fit the non-perturbative parameters  
[Braun, A.L., Paszek-Kumericki, Peters, in progress](#)
- ◆ Lattice determination of the non-perturbative parameters

## ■ Decay of heavy baryons

- ◆ LCSR for  $\Lambda_b \rightarrow p l \bar{\nu}$   
[A.L., Wankerl, in progress](#)
- ◆ LCSR for  $\Lambda_b \rightarrow \Lambda_c l + l$   
[A.L., Rohrwild, planned](#)

## ■ LCSR for pion electro-production

[Braun, Ivanov, A.L., Peters, in progress](#)