Chiral Perturbation theory with tensor sources

and its phenomenological applications

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Outline

1. χPT in the presence of an external tensor source
   - Tensor currents in QCD
   - Higher order terms
   - Reduction of the basis
2. Two point correlators (Preliminary)
   - Definitions
   - OPE vs Large-N_C
3. Radiative pion decay
   - Beyond SM interactions

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- Useful to define $U(x) = u(x)^2$ transforming as $U(x) \rightarrow g_R\, U(x)\, g_L^\dagger$

External field method [Leutwyler (1993)]

QCD currents

$$V^a_\mu(x) = \bar{q}(x)\,\frac{\lambda^a}{2}\,\gamma_\mu q(x)\,,\quad A^a_\mu(x) = \bar{q}(x)\,\frac{\lambda^a}{2}\,\gamma_\mu\gamma_5 q(x)\,,$$

$$S^a(x) = \bar{q}(x)\,\lambda^a\,q(x)\,,\quad P^a(x) = i\,\bar{q}(x)\,\lambda^a\,\gamma_5 q(x),$$

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Extended QCD Lagrangian

$$\mathcal{L}_{QCD} \;=\; \mathcal{L}^0_{QCD} + \mathcal{L}_{ext}\,,$$

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$$\begin{aligned}
\mathcal{L}_{QCD} &= \mathcal{L}^0_{QCD} + \mathcal{L}_{ext}\,,\\
\mathcal{L}_{ext} &= \bar{q}\,\gamma_\mu(v^\mu + \gamma_5\,a^\mu)q - \bar{q}(s - i\,\gamma_5\,p)\,q\\
&= \bar{q}_R\,\gamma_\mu\,r^\mu\,q_L + \bar{q}_L\,\gamma_\mu\,\ell^\mu\,q_R - \bar{q}_R(s + i\,p)q_L - \bar{q}_L(s + i\,p)q_R\,,
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\end{aligned}$$

building blocks $X \to h(g,\Phi)X\,h\,(g,\Phi)^\dagger$ [Ecker, Gasser, Pich & de Rafael'89]

$$\begin{aligned}
u_\mu &= i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right\} \equiv iu^\dagger\,D_\mu U\,u^\dagger\,,\\
h_{\mu\nu} &= \nabla_\mu u_\nu + \nabla_\nu u_\mu\,,\\
f_\pm^{\mu\nu} &= u\,F_L^{\mu\nu}\,u^\dagger \pm u^\dagger\,F_R^{\mu\nu}\,u\,,\\
\chi_\pm &= u^\dagger\,\chi\,u^\dagger \pm u\,\chi^\dagger\,u\,.
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Resonances chiral theory (RχT) basics

RχT lagrangian [Ecker et al'89]

$$\mathcal{L}_R = \sum_{R=V,A,S,P} \left\{ \mathcal{L}^R_{\text{kin}} + \mathcal{L}^1_{\text{int}}(s=1) + \mathcal{L}^1_{\text{int}}(s=0) + \mathcal{L}^2_{\text{int}} \right\}$$

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The role of RχT: the missing link

$$\text{QCD}\left\{\begin{array}{l} E << M_\rho \Longrightarrow \text{chiral symmetry } \chi\text{PT} \\ \\ \end{array}\right.$$

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$$\mathrm{QCD}\begin{cases} E << M_\rho \Longrightarrow \text{chiral symmetry } \chi\text{PT} \\ \\ E >> M_\rho \Longrightarrow \text{perturbative QCD, OPE, asymptotic behaviours} \end{cases}$$

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$\rightsquigarrow$ Matching OPE $\Longleftrightarrow$ RχT (saturation) $\rightarrow$ determination of χ LEC's

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Transformation properties [O. Catà & V.M. '07]

External tensor source : chiral partner splitting

We define $\bar{t}^{\mu\nu} = \sum_{a=0}^{8} \dfrac{\lambda^a}{2}\,\bar{t}_a^{\mu\nu}$. Due to the identity $\sigma^{\mu\nu}\,\gamma_5 = \frac{i}{2}\,\varepsilon^{\mu\nu\lambda\rho}\,\sigma_{\lambda\rho}\,,$

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t^{\mu\nu} &= P_L^{\mu\nu\lambda\rho}\,\bar{t}_{\lambda\rho} \sim \mathcal{O}(p^2)\,,
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$P_{L\,R}$ are chirality projectors $P_R^{\mu\nu\lambda\rho} = \frac{1}{4}(g^{\mu\lambda}\,g^{\nu\rho} - g^{\nu\lambda}\,g^{\mu\rho} + i\,\varepsilon^{\mu\nu\lambda\rho})$,

Transformation properties and new building block

$$\begin{aligned}
t_{\mu\nu} &= \begin{cases} g_R t_{\mu\nu} g_L^{\dagger} & G(x)\\ t^{\dagger}_{\mu\nu} & \mathcal{P}\\ -t_{\mu\nu} & \mathcal{Q}\end{cases}\,,\\
t_{\pm}^{\mu\nu} &= u^{\dagger}\,t^{\mu\nu}\,u^{\dagger} \pm u\,(t^{\dagger})^{\mu\nu}\,u\,,
\end{aligned}$$

Tricky relation among tensors :

$$t^{\mu\nu}t^{\dagger}_{\mu\nu} = 0\,,\quad t_+^{\mu\nu}t^{+}_{\mu\nu} = t_-^{\mu\nu}t^{-}_{\mu\nu}\,,\quad t_+^{\mu\nu}t^{-}_{\mu\nu} = t_-^{\mu\nu}t^{+}_{\mu\nu}\,.$$

Lowest order Lagrangians of χPT and RχT

Lagrangians [O. Catà & V.M.'07; Ecker & Zauner'07]

$$\mathcal{L}_{(4)} \doteq \Lambda_1 \langle t_+^{\mu\nu} f_{+\mu\nu} \rangle - i\,\Lambda_2 \langle t_+^{\mu\nu} u_\mu u_\nu \rangle + \Lambda_3 \langle t_+^{\mu\nu} t^+_{\mu\nu} \rangle + \Lambda_4 \langle t_+^{\mu\nu} \rangle^2 \,,$$

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4. Since there is no tensor term in $\mathcal{L}_{(2)}$ no universal definition for b_0

Outline

Terms appearing at $\mathcal{O}(p^6)$

General form of the terms

$$t_{\mu\nu}t^{\mu\nu}u_\alpha u^\alpha; \quad t_{\mu\nu}f^{\mu\nu}\chi; \quad t_{\mu\nu}t^{\mu\nu}\chi; \quad t_{\mu\nu}\chi u^\mu u^\nu; \quad t_{\mu\nu}f^{\mu\rho}f^\nu{}_\rho; \quad t_{\mu\nu}t^{\nu\rho}h^\mu{}_\rho;$$

$$t_{\mu\nu}h^{\nu\rho}u_\rho u^\mu; \quad t_{\mu\nu}h^{\mu\alpha}h^\nu{}_\alpha; \quad t_{\mu\nu}t^{\nu\rho}f^\mu{}_\rho; \quad t_{\mu\nu}f^{\mu\nu}u_\alpha u^\alpha; \quad t^{\mu\nu}\chi_\mu u_\nu; \quad t_{\mu\nu}t^{\mu\rho}t^\nu{}_\rho;$$

$$\nabla_\rho t_{\mu\nu}\nabla^\rho t^{\mu\nu}; \quad t_{\mu\nu}h^{\mu\alpha}f^\nu{}_\alpha; \quad \nabla_\mu t^{\mu\nu}\nabla^\alpha f_{\alpha\nu}; \quad \nabla_\rho t_{\mu\nu}h^{\mu\rho}u^\nu; \quad \nabla^\mu t_{\mu\nu}f^{\nu\rho}u_\rho;$$

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So there is no genuine odd-intrinsic parity sector up to the $\mathcal{O}(p^8)$ order.

Outline

Reduction techniques [Bijnens et al '99]

1. Partial integration $(\nabla_\mu A)\, B = -A\, (\nabla_\mu B)$ leave the action invariant

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$$\boxed{\text{independent operators} \left\{ \begin{array}{cc} n_f & 120 \\ n_f = 3 & 113 \\ n_f = 2 & 78 \end{array} \right.}$$

Outline

Definitions [O. Catà & V.M. work in prep.]

Two point Green functions :

$$(\Pi_{VV})^{ab}_{\mu\nu}(q) = i\int \mathrm{d}^D x\, e^{iq\cdot x}\left\langle 0 \left| T\left\{V^a_\mu(x) V^b_\nu(0)\right\}\right| 0\right\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_{VV}(q^2)\,,$$

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FlaviA net

Outline

OPE vs Large-N_C

$$\Pi_{VV}^{OPE}(q^2) = \frac{N_C}{24\pi^2}\left[\frac{1}{\hat{\epsilon}} - \log\left(\frac{-q^2}{\mu^2}\right) + \frac{5}{3}\right] + \frac{1}{24\pi}\frac{\langle \alpha_s G^{\mu\nu} G_{\mu\nu}\rangle}{q^4}\,,$$

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Saturation with an infinite spectrum in the large-N_C

$$\Pi_{TT}^{Q}(q^2) = \sum_n^{\infty}\xi_n^2\frac{F_{Vn}^2}{-q^2+m_{Vn}^2}, \qquad \Pi_{VV}(t) = \sum_n^{\infty}\frac{F_{Vn}^2}{-q^2+m_{Vn}^2},$$

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Main results

- Matching for $\langle VV\rangle$ and $\langle TT\rangle$ the parton log, fixes for highly excited resonances the following quantities :

$$\lim_{n\to\infty}\left(\frac{F_{Vn}}{m_{Vn}}\right)^2,\ \lim_{n\to\infty}\left(\frac{F^T_{Vn}}{m_{Vn}}\right)^2,\ \lim_{n\to\infty}\left(\frac{F_{Bn}}{m_{Bn}}\right)^2\sim\frac{1}{n},\ \lim_{n\to\infty}\xi_n^2=\frac{1}{2}\simeq 0.71^2$$

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- Comparison with lattice suggests that ξ_n does not depend on n as happens with Regge models.

Outline

1. χPT in the presence of an external tensor source
   - Tensor currents in QCD
   - Higher order terms
   - Reduction of the basis
2. Two point correlators (Preliminary)
   - Definitions
   - OPE vs Large-N_C
3. Radiative pion decay
   - Beyond SM interactions

Putative tensor interaction

- Amplitude $\pi^+(r) \to e^+\nu_e\,\gamma(p)$ split into $M(\pi^+ \to \ell^+\nu_\ell\,\gamma) = M_{IB} + M_{SD}$

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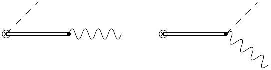
Hadronic determination

- Two types of contributions. We need the odd-intrinsic sector *VVP* for the second one [Ruiz-Femenía, Portolés & Pich'03]



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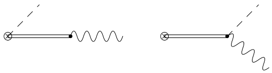

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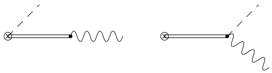

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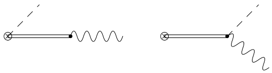

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- Determination of New Physics coupling $\boxed{F_T=(1\pm 14)\times 10^{-4}}$
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- The chiral dimension of the tensor source assigned by consistency arguments
- Tensor currents encode valuable information concerning vector mesons and b_1 mesons
- Together with large-N_C we find a sign alternation in the tensor decay constant
- Formalism has proven to be very useful for disentangling queries about the SM