Has a coupling of right-handed quarks to W been observed?

Micaela Oertel

LUTH, Meudon

Collaborators: V. Bernard (Strasbourg), E. Passemar & J. Stern (Orsay)

V. Bernard, M.O., E. Passemar, J.Stern, Phys. Lett. B 638 (2006) 480
V. Bernard, M.O., E. Passemar, J.Stern, in preparation

Outline of the talk

- Theoretical framework
- Structure of couplings to W
- Experimental test: $K \to \pi l \nu_l$ decays
- Structure of couplings to Z
- Z-pole data
- Summary and outlook

Theoretical framework

- High energies: larger symmetry $\longrightarrow$ observable effects at low energies?
- Decoupling scenario: new physics operators suppressed by energy scale Λ (80 operators at NLO)
- Non-decoupling effective theory
  $\longrightarrow$ Classification scheme of non-standard operators necessary: counting in momenta and parameters of explicit symmetry breaking
  $\longrightarrow$ Renormalisation order by order in this expansion scheme
- Larger symmetry constrains the interactions at low energy
- Minimal solution to get at lowest order the higgsless vertices of the SM: symmetry $S_{nat} = [SU(2)]^4 \times U(1)$ (no light elementary Higgs) (Hirn&Stern '06)
- Non-standard effects already before loop order (two operators at NLO)
- NLO: non-standard couplings of fermions to W and Z
  oblique corrections at NNLO (together with loop effects)

Structure of couplings to W at NLO

- Lepton sector:
  - Universal modification of left-handed couplings $\longrightarrow$ redefinition of G_F
  - Additional Z_2 symmetry ($\nu_R \to -\nu_R$)
    $\longrightarrow$ suppresses Dirac mass for neutrinos
    $\longrightarrow$ no right-handed charged lepton current
- Effective quark charged current interaction (universal non-standard effects)

$$\boxed{W^+_\mu \left((1+\delta)\, \bar{U}_L\, \gamma^\mu\, V^L\, D_L \,+\, \epsilon\, \bar{U}_R\, \gamma^\mu\, V^R\, D_R \right) + h.c.}$$

- V^L and V^R: two a priori independent unitary mixing matrices
- Effective couplings ($\mathcal{V}, \mathcal{A}$):

$$\bar{U}\, \underbrace{\left((1+\delta)V^L + \epsilon\, V^R\right)}_{\mathcal{V}_{\text{eff}}} \gamma_\mu D - \bar{U}\, \underbrace{\left((1+\delta)V^L - \epsilon\, V^R\right)}_{\mathcal{A}_{\text{eff}}} \gamma_\mu \gamma_5 D$$

- Right-handed charged currents also in LR extensions of SM/SUSY, extra dimensions

Non-standard parameters in the light quark sector

- Focus on the light-quark sector (u, d, s):
  - RHCs in the (non)-strange sector: $\epsilon_{ns} = \epsilon \,\mathrm{Re}\left(\frac{V^R_{ud}}{V^L_{ud}}\right) \qquad \epsilon_s = \epsilon \,\mathrm{Re}\left(\frac{V^R_{us}}{V^L_{us}}\right)$
  - Unitarity (suppose V^L_{ub} negligible) $\longrightarrow$ $|V^L_{ud}|^2 + |V^L_{us}|^2 = 1$
  - Modification of the left-handed couplings: δ
- Determination of EW couplings $\leftrightarrow$ knowledge of QCD parameters!
- Example: Neutron β-decay
- Hadronic matrix elements $\longrightarrow$ form factors of the nucleon

$$\langle p|\bar{u}\gamma_\mu d|n\rangle = g_V\, \bar{u}_p\gamma_\mu u_n \qquad \langle p|\bar{u}\gamma_\mu\gamma_5 d|n\rangle = g_A\, \bar{u}_p\gamma_\mu\gamma_5 u_n$$

- What is measured in neutron β-decay? Two independent observables

$$|\mathcal{V}^{ud}_{eff}|g_V \quad \text{and} \quad \lambda = \frac{g_A}{g_V}\frac{|\mathcal{A}^{ud}_{eff}|}{|\mathcal{V}^{ud}_{eff}|} \approx \frac{g_A}{g_V}(1 - 2\,\epsilon_{ns})$$

- Conservation of vector current (CVC) $g_V = 1$, but g_A?

What do we know about the QCD parameters?

- Assuming SM weak interactions: form factors and decay constants precisely measured in semileptonic decays
  Denote quantities extracted assuming SM couplings of quarks by a hat:

$$\hat{F}_\pi = 92.42(26)\ \mathrm{MeV}, \quad \hat{F}_K/\hat{F}_\pi = 1.182(7), \quad \hat{f}_+^{K^0\pi^-}(0) = 0.951(5)$$

- Relation between QCD and EW parameters, e.g. ($\cos\hat{\theta} = |\mathcal{V}^{ud}_{eff}|$)

$$F_\pi = \hat{F}_\pi\,(1 + 2\,\epsilon_{ns})$$

$$\left(\frac{F_{K^+}}{F_{\pi^+}}\right)^2 = \left(\frac{\hat{F}_{K^+}}{\hat{F}_{\pi^+}}\right)^2 \frac{1+2\,(\epsilon_s-\epsilon_{ns})}{1+\frac{2}{\sin^2\hat{\theta}}(\delta+\epsilon_{ns})}$$

$$|f_+^{K^0\pi^-}(0)|^2 = (\hat{f}_+^{K^0\pi^-}(0))^2\,\frac{1-2(\epsilon_s-\epsilon_{ns})}{1+\frac{2}{\sin^2\hat{\theta}}(\delta+\epsilon_{ns})}$$

- Need independent information on EW or QCD parameters!
- Here: branching ratios + EW couplings and shape of scalar $K\pi$ form factor
  $\longrightarrow$ two ways to determine $F_K/(F_\pi f_+(0))$

Scalar $K\pi$ form factor and Callan-Treiman theorem

- K_{l_3}-decays ($K \to \pi\, l\, \nu_l$): hadronic matrix elements $\longrightarrow$ two form factors

$$\langle \pi^-(p)|\bar{s}\gamma_\mu u|K^0(k)\rangle \quad = \quad f_+^{K^0\pi^-}(q^2)(k+p)_\mu + f_-^{K^0\pi^-}(q^2)(k-p)_\mu$$

- Define normalised scalar K-π form factor

$$f(t) =: \frac{f_0(t)}{f_+(0)} = \frac{1}{f_+(0)}\left(f_+(t) + \frac{t}{\Delta_{K\pi}} f_-(t)\right) \qquad (\Delta_{K\pi} = m_K^2 - m_\pi^2)$$

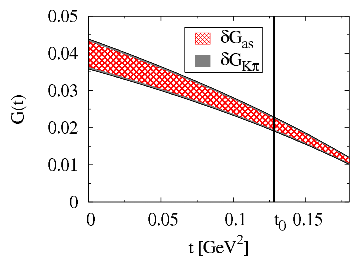
- Callan-Treiman theorem: predicts $f(m_K^2)$ in the $SU(2)\times SU(2)$ chiral limit
- For physical pion masses: $C \equiv f(\Delta_{k\pi}) = \dfrac{F_{K^+}}{F_{\pi^+} f_+^{K^0\pi^\pm}(0)} + \Delta_{\mathrm{CT}}$
- Neutral mode: $\Delta_{CT} \sim 10^{-3}$ (effect of order $m_u, m_d/\Lambda$)
  NLO χPT result (for $m_u = m_d$) (Gasser & Leutwyler '84): $\Delta_{\mathrm{CT}} = -3.5 \times 10^{-3}$
- Charged kaon mode: $\Delta_{\mathrm{CT}} \sim 10^{-2}$ (π^0-η mixing)
  $\longrightarrow$ focus on neutral mode
- Measuring C in two different ways $\longrightarrow$ test of the SM

Measurement of $\ln C$ from form factor shape

- Use twice subtracted dispersion relation:

$$\boxed{\ln f(t) = \frac{t}{\Delta_{K\pi}}\Big(\ln C - \underbrace{\frac{\Delta_{k\pi}(\Delta_{K\pi}-t)}{\pi}\int_{t_{K\pi}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}}_{G(t)}\Big)}$$

- Phase ϕ of $f(t)$ for $s \lesssim (1.5\mathrm{GeV})^2$ $\leftrightarrow K\pi$ scattering phase (Watson's theorem) (Buettiker et al. '02)
- Conservative estimate: $\phi(s > 2.77\mathrm{GeV}^2) = \pi \pm \pi \longrightarrow \ \delta G_{as}(t)$
- $G(t)$ not sensitive to ϕ at high energy



- In the whole physical region: $G(t) < 0.2 \ln C$
  $\longrightarrow$ moderate precision on $G(t)$ sufficient to measure $\ln C$ precisely

Value of $\ln C$ from measured branching ratios

- Independent determination of $\ln C$:

$$\mathrm{C} = \underbrace{\left|\frac{F_{K^+}\mathcal{A}^{us}_{eff}}{F_{\pi^+}\mathcal{A}^{ud}_{eff}}\right| \frac{1}{|f_+^{K^0\pi^-}(0)\mathcal{V}^{us}_{eff}|}|\mathcal{V}^{ud}_{eff}|}_{B_{exp}\ \text{(measured)}} \quad \times \quad \underbrace{\left|\frac{\mathcal{A}^{ud}_{eff}\mathcal{V}^{us}_{eff}}{\mathcal{V}^{ud}_{eff}\mathcal{A}^{us}_{eff}}\right|}_{1+2(\epsilon_s-\epsilon_{ns})+\mathcal{O}(\epsilon^2)} \quad + \quad \Delta_{\mathrm{CT}}$$

- Combination of the two determinations $\longrightarrow$ measurement of

$$\Delta\epsilon = \Delta_{\mathrm{CT}}/B_{exp} + 2\left(\epsilon_s - \epsilon_{ns}\right)$$

- $\Delta_{\mathrm{CT}}/B_{exp} \sim 10^{-3}$, what is the order of $2\left(\epsilon_s - \epsilon_{ns}\right)$?
- Unitarity of the right-handed mixing matrix implies

$$|\epsilon_{ns}| \lesssim \epsilon \quad \text{and} \quad |\epsilon_s| \lesssim 4.5\epsilon$$

$\longrightarrow$ ϵ_s can be enhanced if inverted mixing hierarchy in right-handed sector
$\longrightarrow$ $2\left(\epsilon_s - \epsilon_{ns}\right)$ can be of the order of several percent

Comparison of the two determinations of $\ln C$

- First direct measurement of $\ln C$ using dispersive representation (NA48, PLB '07)

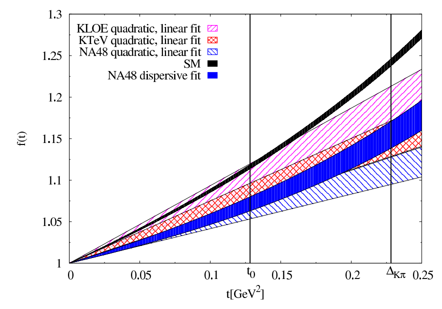
  $\ln C_{exp} = 0.1438(138)$

- Assuming SM:
  $\ln C$ from CT + B_{exp}

  $\ln C_{\mathrm{SM}} = 0.2180(35) + \Delta_{\mathrm{CT}}/B_{exp}$

  $\longrightarrow$ 5σ discrepancy?

- Hint on new physics?



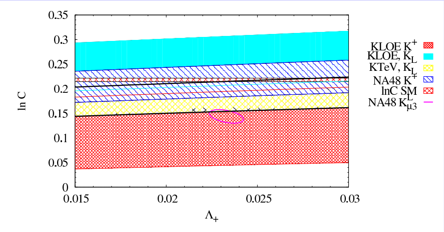
- Interpreted in terms of RHCs: $\Delta\epsilon = 2\,(\epsilon_s - \epsilon_{ns}) + \Delta_{\mathrm{CT}}/B_{exp} = -0.074(14)$

- Experimental situation: measured slope parameter (linear parametrisation of the form factor) differs for NA48 ($\lambda_0 = 0.0095(14)$), KTeV ($\lambda_0 = 0.0128(18)$), and KLOE (preliminary $\lambda_0 = 0.0156(26)$)

Independent check via branching ratios for K_{l3}

- Phase space integral for decay rate $K_{\mu 3}$ sensitive to $\ln C$
- Phase space integral for decay rate K_{e3} not sensitive to $\ln C$

  $\longrightarrow$ $\boxed{\dfrac{\Gamma(K_{\mu 3})}{\Gamma(K_{e3})} = f(\ln C)}$
- Isospin corrections small for the shape of the form factor $\longrightarrow$ compare with data from neutral and charged kaons



- Values available from NA48 (hep-ex/0702015) and KLOE (KAON '07) for charged kaons and KTeV (PRD '04), KLOE (PLB '06) for neutral kaons
- Experimental situation has to be clarified!

Structure of couplings to Z at NLO

- Effective neutral current interaction (universal non-standard effects)

$$\boxed{Z_\mu \left(\sum_f g_L^f \bar{\psi}^f{}_L \gamma^\mu \ \psi_L^f + \sum_f g_R^f \bar{\psi}^f{}_R \gamma^\mu \psi_R^f \right)}$$

- Effective couplings of left-handed fermions to Z ($\tilde{s}_w^2 = s^2/(1 - \xi^2 \rho_L)$)

$$g_L^u = \frac{1+\delta}{2} - \frac{2}{3}\tilde{s}_w^2 \qquad g_L^d = -\frac{1+\delta}{2} + \frac{1}{3}\tilde{s}_w^2 \qquad g_L^e = -\frac{1}{2} + \tilde{s}_w^2 \qquad g_L^\nu = \frac{1}{2}$$

- Effective couplings of right-handed fermions to Z:

$$g_R^u = -\frac{2}{3}\tilde{s}_w^2 + \frac{1}{2}\epsilon^u \qquad g_R^d = \frac{1}{3}\tilde{s}_w^2 - \frac{1}{2}\epsilon^d \qquad g_R^e = \tilde{s}_w^2 - \frac{1}{2}\epsilon^e \qquad g_R^\nu = \frac{1}{2}\epsilon^\nu$$

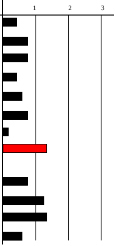
- Coupling of right-handed neutrinos enters only quadratically the Z width $\longrightarrow$ no NLO effect
- Remind redefinition of G_F : $\dfrac{G_F}{\sqrt{2}} = \dfrac{4\pi\alpha(m_Z)}{8m_W^2 s^2}(1 - \xi^2\rho_L)^2$

Experimental information: couplings to Z

1. Data near the Z resonance: (LEP/SLD compilation Phys. Rep. '06)

- Perform a fit to Z-pole data
- Take EM/QCD radiative corrections into account
- Good quality of the fit ($\chi^2/dof = 8.5/8$)
- Simultaneous determination of α_s and spurion parameters difficult
- $\alpha_s(m_Z) = 0.12$ for the presented result
- "A_{FB}^b puzzle" can be solved with universal modification of couplings important point: non-standard couplings of right-handed quarks

	Measurement	Fit	$\frac{(\lvert O^{meas} - O^{fit}\rvert)}{\sigma^{meas}}$
Γ_Z [GeV]	2.4952(23)	2.4943	
σ_{had} [nb]	41.540(37)	41.569	
R_e	20.767(25)	20.785	
A_{FB}^l	0.0171(10)	0.0165	
$\mathcal{A}_l(P_\tau)$	0.1465(32)	0.1485	
R_b	0.21629(66)	0.21685	
R_c	0.1721(30)	0.1725	
A_{FB}^b	0.0992(16)	0.1012	
A_{FB}^c	0.0707(35)	0.0707	
$\mathcal{A}_b$	0.923(20)	0.910	
$\mathcal{A}_c$	0.670(27)	0.636	
$\mathcal{A}_l$(SLD)	0.1513(21)	0.1485	
$Br(W \to l\nu)$	0.1084(9)	0.1089	



- Values for the parameters coherent with the LEET:

$$(\epsilon^e)_{\mathrm{NLO}} = -0.0024(5) \quad (\epsilon^u)_{\mathrm{NLO}} = -0.02(1) \quad (\epsilon^d)_{\mathrm{NLO}} = -0.03(1)$$
$$(\tilde{s}_w^2)_{\mathrm{NLO}} = 0.2307(2) \quad (\delta)_{\mathrm{NLO}} = -0.004(2)$$

Experimental information: couplings to Z

2. Data at low momentum transfer:

- Atomic parity violation experiments : test the weak charge

$$
\begin{aligned}
Q_W &= 4g_A^e\Big(Z\,(2g_V^u + g_V^d) + N\,(g_V^u + 2g_V^d)\Big) \\
Q_W(NLO) &= (1-\epsilon^e)\Big(Z(1-4\tilde{s}_w^2 + \delta - \epsilon^d + 2\,\epsilon^u) - N(1+\delta + 2\,\epsilon^d - \epsilon^u)\Big)
\end{aligned}
$$

- Take the value from the fit to Z pole data: $(Q_W(^{133}\mathrm{Cs}))_{\mathrm{NLO}} = -70.72 \pm 4.19$ in agreement with the experimental value, $Q_W(^{133}\mathrm{Cs}) = -72.71(49)$ (Guena et al '05)
- QWEAK experiment will measure Q_W of the proton
  NLO result very small ($Q_W^p = 0.062(22)$) because $1 - 4\tilde{s}_w^2 \approx 0$
  $\longrightarrow$ enhanced sensitivity to higher orders
- The same for weak charge of electrons (Møller scattering): higher order corrections probably important

Summary and outlook

- Summary:
  - Minimal (not quite decoupling) effective theory: first effects beyond the SM are non-standard couplings of fermions to W and Z
  - Determination of EW and QCD parameters correlated
  - $K^L_{\mu 3}$ decays: Two ways to determine $F_K/F_\pi f_+(0) \longrightarrow$ stringent test of couplings of quarks to W
  - NA48 result (dispersive representation)
    $\longrightarrow$ 5σ deviation with SM?
  - Interpretation? Direct coupling of right-handed quarks to W?
  - Other experimental results (KLOE, KTeV): need dispersive parametrisation
  - Hadronic tau decays/ Couplings to Z: no inconsistencies with the systematics of the effective theory
- Outlook:
  - Heavy quark sector
  - Loop effects (CP violation, FCNC,....)

Theoretical framework to classify non-standard EW effects

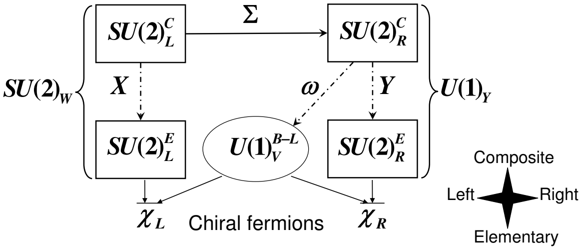
- High energies: larger symmetry $\longrightarrow$ observable effects at low energies?
- Decoupling scenario: new physics operators suppressed by energy scale Λ

$$\mathcal{L}_{eff} = \mathcal{L}_{\mathrm{SM}} + \sum_{D>4} \frac{\mathcal{O}_D}{\Lambda^{D-4}}$$

  (80 operators at NLO)
- "Not quite decoupling" effective theory (valid for $p \ll \Lambda \sim 3TeV$)
  $\longrightarrow$ Classification scheme of non-standard operators necessary:
  counting in momenta and parameters of explicit symmetry breaking
  $\longrightarrow$ Renormalisation order by order in this expansion scheme
- Larger symmetry $S_{nat} \supset SU(2) \times U(1)$ survives at low energy and becomes non-linearly realised:
  S_{nat} constrains the interactions at low energy

Structure of the minimal effective theory

- Minimal solution to get at lowest order the higgsless vertices of the SM: symmetry $S_{nat} = [SU(2)]^4 \times U(1)_{B-L}$ (Hirn&Stern '04,'06)



- GB Σ : link $L \longleftrightarrow R$
- Spurions X, Y, ω : link $C \longleftrightarrow E$
- Covariant constraints reducing the symmetry and the physical dof to

$$SU(2) \otimes U(1)_Y \Leftrightarrow \boxed{D_\mu X = D_\mu Y = D_\mu \omega = 0}$$

- $X \sim \xi \quad Y \sim \eta \quad \omega \sim \varsigma$ small expansion parameters.

- Counting $\xi, \eta \sim \mathcal{O}(p^{1/2})$ $\quad (m_{\mathrm{Dirac}} \sim \mathcal{O}(\xi\eta))$: $\kappa = \frac{k+l}{2}$ for $\mathcal{O}(\xi^k \eta^l)$

Next-to leading order

- Momentum and spurion (κ) power counting :

$$\mathcal{L}_{eff} = \sum_{d^*\geq 2} \mathcal{L}_{d^*} \quad d^* = d + \kappa = n_p + n_g + \frac{n_\psi}{2} + \kappa = 2 + 2L + \sum_v (d_v^* - 2)$$

- Leading order ($d^* = 2$): SM without Higgs

$$\begin{array}{ll} \mathcal{O}(p^2\kappa^0) : & \frac{f^2}{4}\langle D_\mu \Sigma^\dagger D^\mu \Sigma\rangle + \ldots \\ \mathcal{O}(p^1\kappa^1): & \text{fermion mass terms} \end{array}$$

- Non-standard effects before loop order, two operators at NLO ($\mathcal{O}(p^2\kappa^1)$):

$$\mathcal{O}_L = \bar{\Psi}_L \mathcal{X}^\dagger \Sigma \gamma^\mu D_\mu \Sigma^\dagger \mathcal{X} \Psi_L \qquad \mathcal{O}_R^{a,b} = \bar{\Psi}_R \mathcal{Y}_a^\dagger \Sigma^\dagger \gamma^\mu D_\mu \Sigma \mathcal{Y}_b \Psi_R$$

  ($\{a,b\} \in \{U,D\}$)
  $\mathcal{O}_L, \mathcal{O}_R$: non-standard couplings of fermions to W and Z

- Oblique corrections and loop corrections at NNLO ($d^* \geq 4$)
- What are the constraints on these non-standard couplings?

Hadronic τ-decays

- Look at inclusive hadronic τ-decay rate: $$R_\tau = \frac{\Gamma(\tau^- \to \nu \,\mathrm{hadrons})}{\Gamma(\tau^- \to \nu e^- \bar{\nu})}$$
- Separation of vector and axial, strange channel possible ($R_{\tau,V}, R_{\tau,A}, R_{\tau,S}$)
- Additional information from shape of spectral functions ($\longrightarrow$ moments):

$$R^{kl}_{\tau,V/A} = \int_0^{m_\tau^2} ds\, (1 - \frac{s}{m_\tau^2})^k\, (\frac{s}{m_\tau^2})^l\, \frac{dR_{\tau,V/A}}{ds}$$

- Theoretical description: use operator product expansion (OPE) (Braaten et al. '92, Le Diberder & Pich '92, ...):

$$R_{\tau,V} = \frac{3}{2}\, S_{EW}\, |\mathcal{V}^{ud}_{eff}|^2 \left(1 + \delta^{(0)} + \delta^{(2)}_V + \sum_{D=4,\ldots} \delta^{(D)}_V\right)$$

- Input for OPE: $\alpha_s(m_\tau), m_q(m_\tau), \langle \bar{q}q\rangle, (\langle \alpha_s G^2\rangle), \langle \bar{q}\Gamma q \bar{q}\Gamma q\rangle, \cdots$

What can we learn from the non-strange sector?

- Non-strange sector: determination of ϵ_{ns} (not sensitive to δ)
- Two possibilities:

1. V + A channel: $$R_{\tau,V+A} = 3\, S_{EW}\, |\mathcal{V}^{ud}_{eff}|^2 \left(1 + \delta^{(0)}\right)\left(1 - 2\,\epsilon_{ns} + \Delta^{+}_{ud}\right)$$
   - $|\mathcal{V}^{ud}_{eff}|$ from 0^+-0^+ superallowed β decays and CVC + small corrections
   - $\delta^{(0)}$: monotonically rising function of $\alpha_s(m_\tau) \longrightarrow$ strong correlation with ϵ_{ns}
   - Non-perturbative part $\Delta^{+}_{ud} \sim 10^{-3}$ very small

2. V and A channel:
   - Example: ratio of $R_{\tau,A}$ and $R_{\tau,V}$ $$R_{\tau,A}/R_{\tau,V} = (1 - \Delta^{-}_{ud} - 4\epsilon_{ns})$$
   - Non-perturbative part Δ^{-}_{ud} dominated by $D = 6, V - A$
   - $\Delta^{-}_{ud} \sim 10^{-2} \longrightarrow$ strong correlation with ϵ_{ns}

- Presently no quantitative determination of ϵ_{ns} possible: $-0.03 \lesssim \epsilon_{ns} \lesssim 0.03$

Strange sector

- Assuming SM (no RHCs): many studies on (ALEPH '99, Pich & Prades '99, ...)

$$\delta R_\tau = \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

$SU(3)$ symmetry breaking quantity: theoretical uncertainties reduced

- Here: proper normalisation? Look instead at

$$\frac{R_{\tau,S}}{R_{\tau,V+A}} = \frac{\sin^2\hat{\theta}}{\cos^2\hat{\theta}}\,(1 - 2\,\frac{\epsilon_{ns}+\delta}{\sin^2\hat{\theta}})\,(1+\Delta^+_{us}-\Delta^+_{ud}) \quad (\cos\hat{\theta} = |\mathcal{V}^{ud}_{eff}|)$$

- QCD part: same $SU(3)$ breaking quantity as δR_τ
  dominant term $\sim m_s^2(m_\tau)/m_\tau^2$ ($\longrightarrow$ strong dependence on m_s)
  coefficient not well known (convergence of perturbative series)?
- Very sensitive to $\delta + \epsilon_{ns}$ (values between $-0.005 \lesssim \delta + \epsilon_{ns} \lesssim 0.005$)
- Not sensitive to ϵ_s: $\Delta^-_{us}\epsilon_s \sim 10^{-4} \longrightarrow$ negligeable

Hadronic width of W

- Hadronic width of W :

$$\boxed{\Gamma_h = 6\,\Gamma_0\,(1+2\,\delta)\,\left(1+\frac{\alpha_s(m_W)}{\pi}+1.409\left(\frac{\alpha_s(m_W)}{\pi}\right)^2-12.77\left(\frac{\alpha_s(m_W)}{\pi}\right)^3\right)}$$

$\rightarrow$ modification only due to δ (left-handed couplings)

- Total width, supposing universality $\Gamma_W = 3\,\Gamma_0 + \Gamma_h$
- Combine with measured value of leptonic branching fraction
  $\Gamma(W \to l\nu)/\Gamma_W = 0.1084(9)$ (LEP EWWG, '06)
- Value of δ almost insensitive to value of $\alpha_s(m_W)$
- Taking measurement for total width: large error ($-0.03 < \delta_{\mathrm{NLO}} < 0.03$)
- δ in couplings of left-handed fermions to Z, too

W mass

- Take α, m_Z, G_F as input $\longrightarrow$ mass of W calculated
- No direct modification between LO and NLO
- Indirect modification via the definition of G_F (coupling of left-handed leptons)

$$\frac{G_F}{\sqrt{2}} = \frac{4\pi\alpha(0)}{8m_Z^2 c^2 s^2 (1-\Delta r)}(1-\xi^2\rho_L)^2$$

$$\text{at NLO } \frac{m_W^2}{m_Z^2} = \frac{h}{h+\tilde{s}^4}, \quad h = \frac{\pi\alpha(0)}{\sqrt{2}G_F m_Z^2(1-\Delta r)}$$

- At NLO no weak loops: $\Delta r = \Delta\alpha$
- LO and NLO result almost identical

$$(m_W)_{\mathrm{LO}} = 79.97 GeV, \quad (m_W)_{\mathrm{NLO}} = 79.99 GeV \quad (\xi^2\rho_L)_{\mathrm{NLO}} = 0.001(12)$$