Instabilities in high density two flavor quark matter

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Outline

1. Instabilities in gapless quark matter
   - Two flavor color superconductivity
   - The pattern of the symmetry breaking
   - The one loop effective action of the $U(1)_A$ Goldstone mode
   - Instabilities in the gluon sector
2. Toward a genuine ground state
   - Goldstone current and 1PW state
   - Goldstone mode in the multiple-plane-wave state
   - Meissner masses in the multiple-plane-wave state

Outline

The model

We consider neutral two flavor superconductive quark matter with action

$$S = \int d^4x \left[\bar{\psi}_{i\alpha} \left(i\gamma^\mu \partial_\mu + \mu_{ij}^{\alpha\beta} \gamma_0 \right) \psi_{j\beta} + (L \rightarrow R) + \mathcal{L}_\Delta \right] .$$

Condensation in the quark-quark channel is described by the lagrangian $\mathcal{L}_\Delta$ which is given by

$$\mathcal{L}_\Delta = -\frac{\Delta}{2} \psi_{i\alpha}^T C \psi_{j\beta} \epsilon^{\alpha\beta 3} \epsilon_{ij} + H.c. - (L \rightarrow R) \ ;$$

it can be obtained in the mean field approximation from a local four-fermion interaction. We assume that in the ground state

$$\langle \psi_{i\alpha}^L C \psi_{j\beta}^L \rangle = -\langle \psi_{i\alpha}^R C \psi_{j\beta}^R \rangle \propto \Delta \epsilon^{\alpha\beta 3} \epsilon_{ij} \neq 0 \ ,$$

where the superscripts L, R denote left-handed and right-handed quarks respectively.

The choice of the chemical potentials

The quark chemical potential matrix is given by

$$\boldsymbol{\mu} = (\mu \mathbf{1}_F - \mathbf{Q}\mu_e) \otimes \mathbf{1}_C + \mathbf{1}_F \otimes (\mu_3 \mathbf{T}_3 + \mu_8 \mathbf{T}_8) \ .$$

The symmetry of the ground state implies $\mu_3 = 0$; moreover color neutrality results in $\mu_8 = \mathcal{O}(\Delta^2/\mu) \ll \Delta$. Finally, $\mu_e = \mathcal{O}(0.1 \times \mu) \gg \mu_8$. As a consequence we assume that the ground state is neutral with

$$\mu_3 = \mu_8 = 0 \ , \qquad \mu_e \neq 0 \ .$$

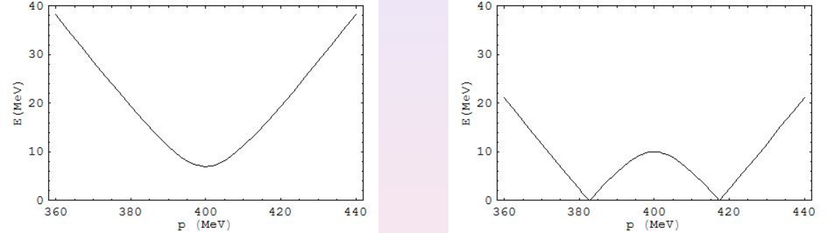
The quark chemical potentials can be written as

$$\begin{aligned} \mu_u &= \bar{\mu} - \delta\mu \ , \\ \mu_d &= \bar{\mu} + \delta\mu \ , \end{aligned}$$

with $\delta\mu = \mu_e/2$.

The 2SC and g2SC states

The dispersion law of the paired quarks are as follows



$\delta\mu/\Delta < 1$ gapped 2SC

$\delta\mu/\Delta > 1$ gapless 2SC

(*Huang and Shovkovy, 2003*)

Transformation properties of the condensate

Neglecting electromagnetism, the symmetry group of two massless flavor QCD,

$$G_{QCD} = SU(3)_c \otimes U(2)_V \otimes U(2)_A \ ,$$

is broken down by the quark condensate $\langle\psi\psi\rangle$ to

$$G_{2SC} = SU(2)_c \otimes U(2)_V \otimes SU(2)_A \ .$$

In particular, there is a broken $U(1)_A$ (*Pisarski and Rischke, 1999*).

Goldstone theorem: 1 scalar, corresponding to the breaking of $U(1)_A$.

The color group is broken too: $5/8$ gluons become massive (Meissner effect, familiar from ordinary superconductivity).

The coupling of the scalar to the quarks

The Goldstone field ϕ is a quantum excitation of the ground state, as it describes phase fluctuations of the condensate. It can be introduced in the model by the replacement, in the quark lagrangian,

$$\langle\psi\psi\rangle \rightarrow e^{2i\phi/f}\langle\psi\psi\rangle$$

This gives rise, at the leading order in the field, to interaction vertices of ϕ with the quarks,

$$\mathcal{L}_3 = \left(\frac{2i\phi}{f}\right)\frac{\mu^2}{\pi}\int\frac{d\mathbf{n}}{8\pi}\int\frac{d\ell_{\parallel}d\ell_0}{(2\pi)^2}\,\chi^\dagger\begin{pmatrix}0 & \mathbf{\Delta}\\ -\mathbf{\Delta} & 0\end{pmatrix}\chi + (L\rightarrow R)\ ,$$

$$\mathcal{L}_4 = -\left(\frac{2\phi^2}{f^2}\right)\frac{\mu^2}{\pi}\int\frac{d\mathbf{n}}{8\pi}\int\frac{d\ell_{\parallel}d\ell_0}{(2\pi)^2}\,\chi^\dagger\begin{pmatrix}0 & \mathbf{\Delta}\\ \mathbf{\Delta} & 0\end{pmatrix}\chi + (L\rightarrow R)\ .$$

The relevant diagrams

Integration over the fermion fields in the partition function gives rise to the one loop effective action of ϕ, $\mathcal{L}(p) = \mathcal{L}_{s.e.}(p) + \mathcal{L}_{tad}$, with

$$\mathcal{L}_{s.e.}(p) = \frac{i}{2}\left(\frac{2i\phi}{f}\right)^2 \mathrm{Tr}\left[D(\ell + p)\begin{pmatrix} 0 & \mathbf{\Delta} \\ -\mathbf{\Delta} & 0 \end{pmatrix} D(\ell)\begin{pmatrix} 0 & \mathbf{\Delta} \\ -\mathbf{\Delta} & 0 \end{pmatrix}\right] ,$$

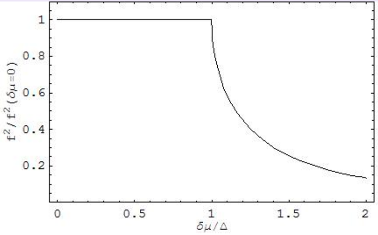
$$\mathcal{L}_{tad} = -i\left(-\frac{2\phi^2}{f^2}\right)\mathrm{Tr}\left[D(\ell)\begin{pmatrix} 0 & \mathbf{\Delta} \\ \mathbf{\Delta} & 0 \end{pmatrix}\right] .$$

In the low energy regime $p \ll \Delta$ one has

$$\mathcal{L}(p) = \frac{1}{2}\left[p_0^2\phi^2 - v^2(\mathbf{p}\phi)\cdot(\mathbf{p}\phi)\right] .$$

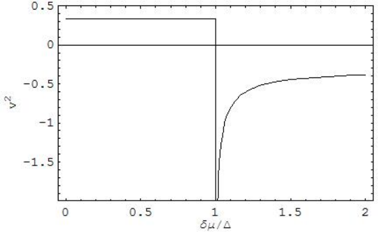
Low energy parameters of ϕ

Evaluating the loop integrals we get



f^2, obtained by requiring canonical normalization of $\mathcal{L}(p)$,

$$f^2 = \frac{4\mu^2}{\pi^2}\left(1 - \theta(\delta\mu - \Delta)\frac{\sqrt{\delta\mu^2 - \Delta^2}}{\delta\mu}\right)$$



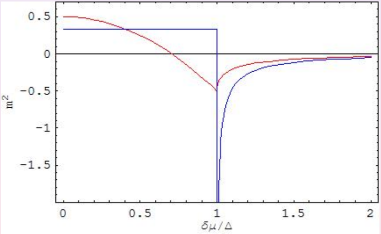
$$v^2 = \frac{1}{3}\theta(\Delta - \delta\mu) - \frac{1}{3}\theta(\delta\mu - \Delta)\frac{\delta\mu}{\sqrt{\delta\mu^2 - \Delta^2}}$$

Gluon sector I: Meissner masses

The condensate breaks $SU(3)_c$ down to $SU(2)_c$: as a consequence, 5/8 gluons are massive (Higgs mechanism). In particular, one can evaluate the Meissner masses, defined by $m_M^2 = -\Pi(p_0 = 0, \mathbf{p} \to 0)$. One finds

$$m_{M,4}^2 = \frac{4\alpha_s \mu^2}{3\pi} H\left(\frac{\delta\mu}{\Delta}\right)$$

$$m_{M,8}^2 = \frac{4\alpha_s \mu^2}{3\pi} G\left(\frac{\delta\mu}{\Delta}\right)$$



(Shovkovy and Huang, 2004). One notices the relation

$$f^2 v^2 \propto m_{M,8}^2 \ ,$$

which links the instabilities of the gluon and the Goldstone sector.

Gluon sector II: dispersion laws

Beside Meissner instability, the dispersion laws of dynamical gluons have been studied by *Shovkovy et al., 2006*. They have found that

- Gluons with $a = 4, \ldots, 7$ have a negative squared plasmon mass for $\delta\mu \leq \Delta/\sqrt{2}$.
- Gluon $a = 8$ is massless and has a negative squared velocity.

It is interesting to notice that these kind of instabilities are found both for the electric and for the magnetic gluon modes, while the Meissner instability develops only for the magnetic gluons.

Outline

The ϕ current

The low energy lagrangian of ϕ reads

$$\mathcal{L} = \frac{1}{2}\left(\partial_0\phi\partial_0\phi - v^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\phi\right) \ .$$

In the regime $\delta\mu > \Delta$ we find $v^2 < 0$. We interpret this result as $\langle\boldsymbol{\nabla}\phi\rangle \neq 0$ in the ground state.
Ansatz (the simplest one):

$$\phi(t,\mathbf{x}) = \boldsymbol{\Phi}\cdot\mathbf{x} + h(t,\mathbf{x}) \ ,$$

which implies, assuming $\langle\boldsymbol{\nabla}h\rangle = 0$,

$$\langle\boldsymbol{\nabla}\phi\rangle = \boldsymbol{\Phi} \ ,$$

and interpret $\boldsymbol{\Phi}$ as a Goldstone current.

The action of h and the value of Φ

In the quark lagrangian

$$\langle\psi\psi\rangle \to e^{2i\mathbf{\Phi}\cdot\mathbf{x}/f}\langle\psi\psi\rangle e^{2ih/f} \ .$$

The value Φ_0 of $|\mathbf{\Phi}|$ in the ground state is evaluated by minimizing the thermodynamic potential Ω. Expanding the quark propagator in powers of Δ/q with $q \equiv |\mathbf{\Phi}|/f$ we get

$$\Phi_0^2 \approx 1.2 \times f^2\delta\mu^2 \ .$$

Moreover, expanding in the small field h/f one has

$$\mathcal{L}[h] = \frac{1}{2}\left((\partial_0 h)^2 - v_i v_j \partial_i h \partial_j h\right) \ ,$$

and the low energy parameters are $(\Phi_x = \Phi_y = 0, \Phi_z = \Phi_0)$

$$f^2 \approx 0.46\mu^2\Delta^2/\delta\mu^2 \ , \qquad \mathbf{v} = (0,0,1) \ .$$

Therefore the tensor $v_i v_j$ is semidefinite positive.

Equivalence with the 1PW LOFF state

In the quark lagrangian, after the introduction of the phase fluctuation ϕ,

$$\Delta \propto \langle\psi\psi\rangle \rightarrow \Delta e^{2i\mathbf{\Phi}\cdot\mathbf{x}/f} e^{2ih/f} \ .$$

This is equivalent to say that in the ground state

$$\langle\psi\psi\rangle \propto \Delta e^{2i\mathbf{q}\cdot\mathbf{x}} \ , \qquad \mathbf{q} \leftrightarrow \mathbf{\Phi}/f \ ,$$

and the fluctuation field h plays the role of the $U(1)_A$ Goldstone field in the state with the inhomogeneous quark condensate.

This is the 1PW LOFF state. Hence the ansatz for the current implies an equivalence between the g2SC and the 1PW LOFF state, and the equality of their free energies.

The $U(1)_A$ boson does not suffer the gradient instability in the 1PW state.

From 1PW to MPW state

Problem

The state with Goldstone current, equivalent to the 1PW state, may not have enough free energy to be the good ground state of two flavor QCD.

As a matter of fact, from the LOFF literature we know that the interval in which the 1PW state is energetically favored is, in the weak coupling limit,

$$0.707\Delta_0 \leq \delta\mu \leq 0.754\Delta_0 \ , \quad \Delta_0 \equiv \Delta(\delta\mu = 0) \ .$$

Possible solution

The interval in $\delta\mu$ where the 1PW state is favored can be enlarged by adding more plane waves, hence considering crystalline LOFF structures (Bowers and Rajagopal, 2002).

Action of the Goldstone in the MPW state

The ground state of quark matter can be determined only by evaluation of the free energy.

Before that, it is important to investigate the stability (or instability) of a given QCD phase.
The above considerations led us to consider the MPW crystalline superconductor.

Ansatz in the MPW phase

$$\langle\psi\psi\rangle \propto \Delta \sum_{a=1}^{P} e^{2i\mathbf{q}^a\cdot\mathbf{x}}$$

Action of the Goldstone in the MPW state

In the MPW state

$$\langle\psi\psi\rangle \propto \Delta \sum_{a=1}^{P} e^{2i\mathbf{q}^a\cdot\mathbf{x}}$$

We consider first the $U(1)_A$ mode:

$$\mathcal{L}[h] = \frac{1}{2}\left((\partial_0\phi)^2 - v_i v_j \partial_i\phi\partial_j\phi\right) \;,$$

where

$$v_i v_j = \sum_{a=1}^{P} (\hat{q}^a)_i (\hat{q}^a)_j / P \;;$$

$v_i v_j$ is definite positive (easily proved), hence the MPW does not suffer the gradient instability.

The gluon sector: Meissner masses

The explicit evaluation of the polarization tensor $\Pi^{ab}_{\mu\nu}$ of the 5/8 broken gluons leads to the relations (*Gatto and M. R. , 2007*)

$$\left(\mathcal{M}^{ij}_{44}\right)^2 = \frac{f^2}{16} v_i v_j \ , \qquad \left(\mathcal{M}^{ij}_{88}\right)^2 = \frac{f^2}{12} v_i v_j \ .$$

The positivity of the matrix $v_i v_j$ implies the positivity of the (Meissner) mass matrix of the gluons. Hence, the MPW state does not suffer the chromomagnetic instability. Just as an example:

1PW:

$$v_i v_j = [\mathrm{diag}(0,0,1)]_{ij}$$

(q along the z-axes)

FCC:

$$v_i v_j = \frac{1}{3}\delta_{ij}$$

(8 q pointing to the vertices of a cube)

Free energy

From the LOFF literature we know that the LOFF state in the favored phase in the following intervals (*M. R. et al., 2004*)

- BCC structure ($P = 6$, the wave vectors pointing to the faces of a cube) is stable in the interval $0.71\Delta_0 \leq \delta\mu \leq 0.95\Delta_0$;
- FCC structure is favored in the interval $0.95\Delta_0 \leq \delta\mu \leq 1.35\Delta_0$.

Therefore the LOFF state in the MPW configuration is stable in the interval

$$0.71 \leq \frac{\delta\mu}{\Delta_0} \leq 1.35 \; .$$

For larger values of $\delta\mu$ the LOFF state is not energetically favored, and cannot cure the instability problem of two flavor quark matter.

Summary

Summary

- Instability in the g2SC phase: Goldstone and gluon sectors.
- Boson current state $\Leftrightarrow$ 1PW LOFF state, with no Goldstone instability.
- Stability analysis of the MPW state.

Open Problems

- Dispersion laws of the gluons in the MPW crystalline phase.
- Smearing procedure to evaluate Goldstone properties, Meissner masses, glue dispersion laws.
- Comparison of the free energies of the MPW superconductor and of the gluonic phases.