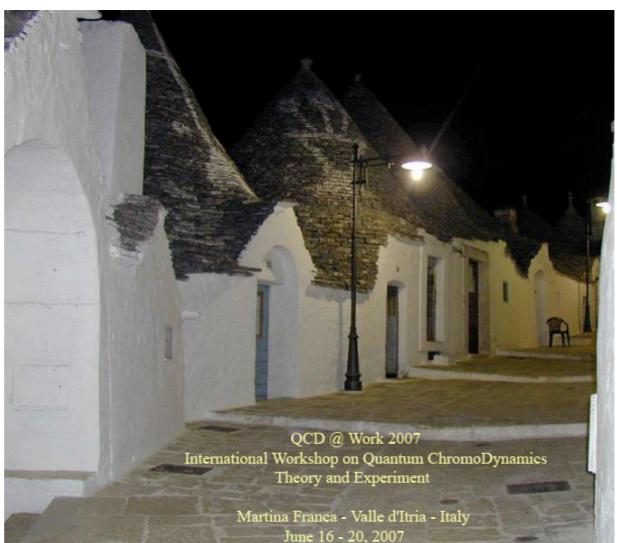


QCD Initial State Radiation: A New Approach

Federico Ceccopieri, L.T. - hep-ph



QCD@Work Martina Franca 18/06/2007

Outline

- Fracture Functions
- Semi-inclusive distributions (initial state)
 - Applications (Diffraction)
 - Scaling in diffraction (recent results)

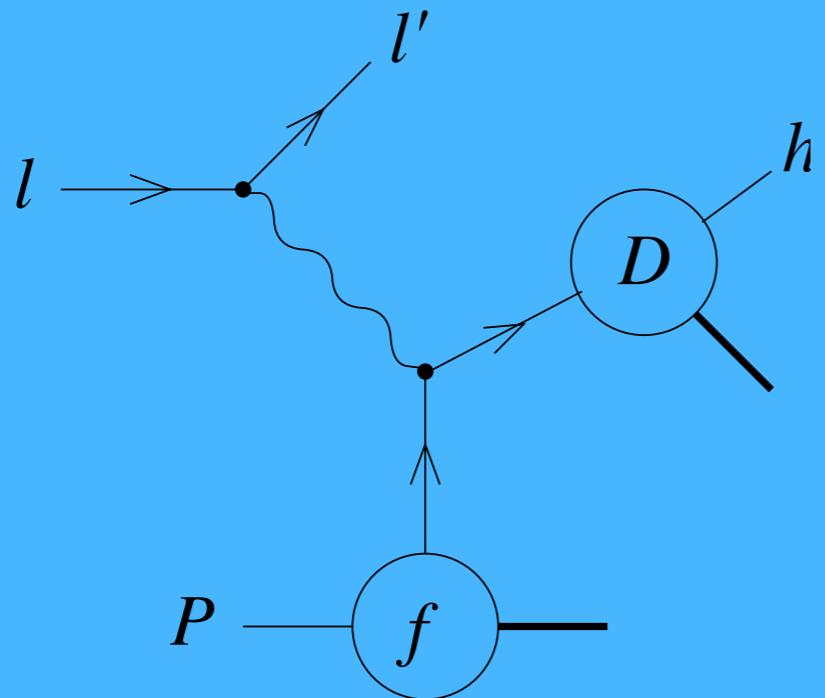
Initial states in hadronic processes:

Multiple hadrons within FF formalism

New Jet-like Fracture Functions

Semi Inclusive Deep Inelastic Scattering

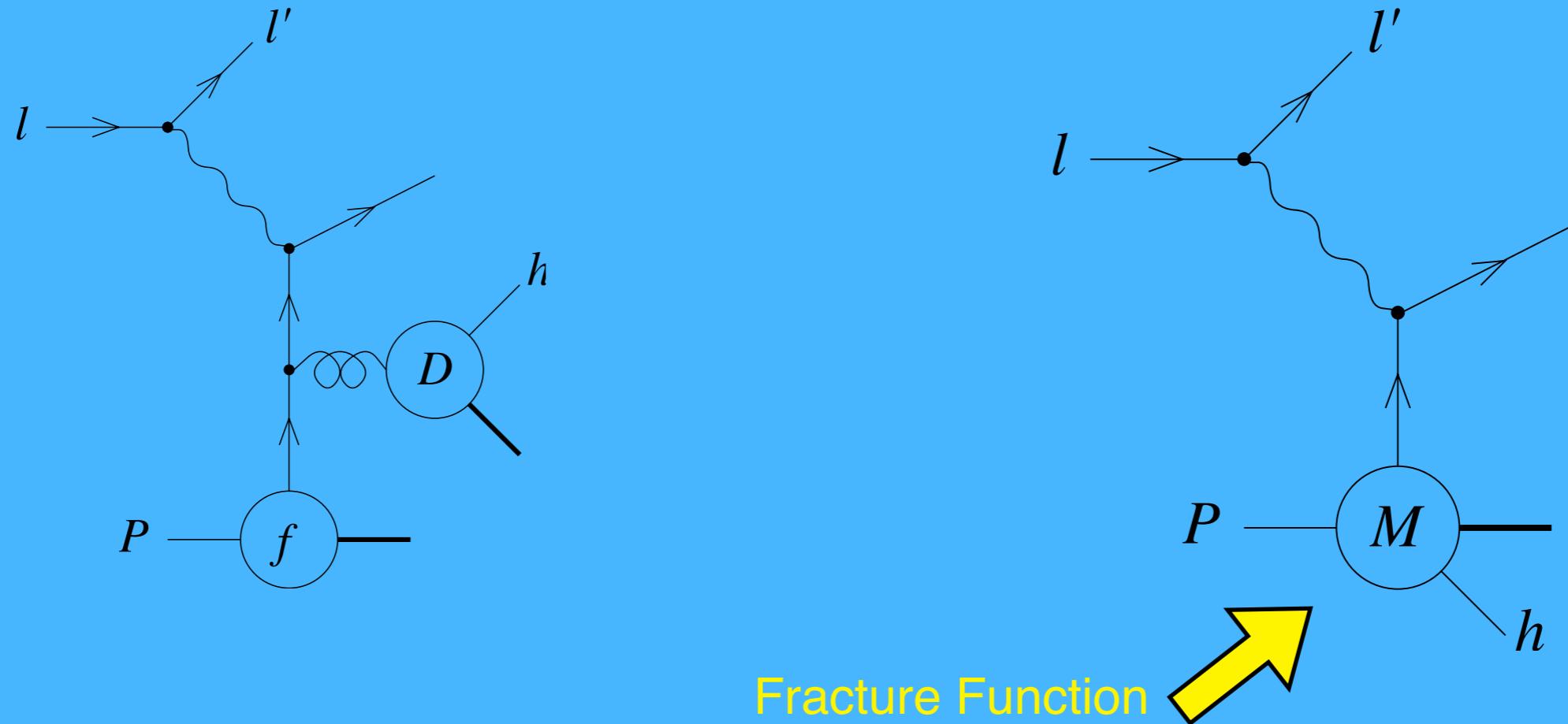
Current Fragmentation



$$\sigma_C = \int \frac{dx'}{x'} \frac{dz'}{z'} F_P^i(x', Q^2) \hat{\sigma}_{ij}(x/x', z_h/z', Q^2) D_h^j(z', Q^2)$$

Hadrons may also come from elsewhere !

Semi Inclusive Deep Inelastic Scattering Target Fragmentation



$$\sigma_T = \int \frac{dx'}{x'} M_{Ph'}^i(x', z_h, Q^2) \hat{\sigma}_i(x/x', Q^2)$$

L.Trentadue and G.Veneziano, Phys.Lett. **B323** (1994) 201

Fracture Functions = Fragmentation + structure

Properties:

- Do not depend on the arbitrary chosen scale Q_0^2 i.e.

$$\frac{\partial}{\partial Q_0^2} M_{p,h}^j(x, z, Q^2) = 0$$

- Both $D_l^h(x, Q^2)$ and $F_p^i(x, Q^2)$ satisfy the usual Altarelli Parisi evolution equations and $\sum_h \int_0^1 dz z D_l^h(x, Q^2) = 1$ and $\sum_i \int_0^1 dx x F_p^i(x, Q^2) = 1$ with

$$\sum_i \int_0^1 du u P_i^j(u) = 0$$

$M_{p,h}^j(x, z, Q^2)$ satisfies the momentum sum rule:

$$\sum_h \int_0^1 dz z M_{p,h}^j(x, z, Q^2) = (1 - x) F_p^j(x, Q^2)$$

accounting for the s-channel unitarity constraint.

The combination of the Fracture Function with the target-Fragmentation evolution gives the evolution equation:

$$\begin{aligned} \frac{\partial}{\partial \log Q^2} M_{i,h/p}(\xi, \zeta, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\xi/(1-\zeta)}^1 \frac{du}{u} P_j^i(u) M_{j,h/N}\left(\frac{\xi}{u}, \zeta, Q^2\right) \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_{\xi}^{\xi/(\xi+\zeta)} \frac{du}{\xi(1-u)} \hat{P}_j^{i,l}(u) f_{j/p}\left(\frac{\xi}{u}, Q^2\right) D_{h/l}\left(\frac{\zeta u}{\xi(1-u)}, Q^2\right) \end{aligned}$$

Homogeneous (usual Altarelli Parisi type) term + Inhomogeneous term

Several properties:

- 1) Fracture Functions satisfy unitarity

$$\sum_h \int_0^1 dz z M_{p,h}^j(x, z; \mu^2) = (1 - x) \cdot F_p^j(x, \mu^2).$$

- 2) Fracture Functions factorize

*M.Grazzini, L.Trentadue and G.Veneziano, Nucl.Phys.B519(1998)394
J.Collins, Phys.Rev.D57(1998)3051*

- 3) Extended $M(x,z,t,Q^2)$ -Fracture Functions satisfy Gribov-Lipatov-Altarelli-Parisi type evolution equations

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}_{A,A'}^j(x, z, t, Q^2) = \frac{\alpha_S(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) \mathcal{M}_{A,A'}^i(x/u, z, t, Q^2)$$

Applications:

Difraction:

$$e^-(k) + A(P) \rightarrow e^-(k') + A(P') + X$$

- According to the Ingelman-Schlein model,

[G.Ingelman and P.Schlein, Phys.Lett. **B152** (1985) 256.]

the diffractive structure function $F_2^{diff}(x, Q^2)$ is

$$F_2^{diff}(x, Q^2) = \sum_a \int d\xi \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} \hat{\sigma}\left(\frac{x}{\xi}, Q^2, \mu\right)$$

But

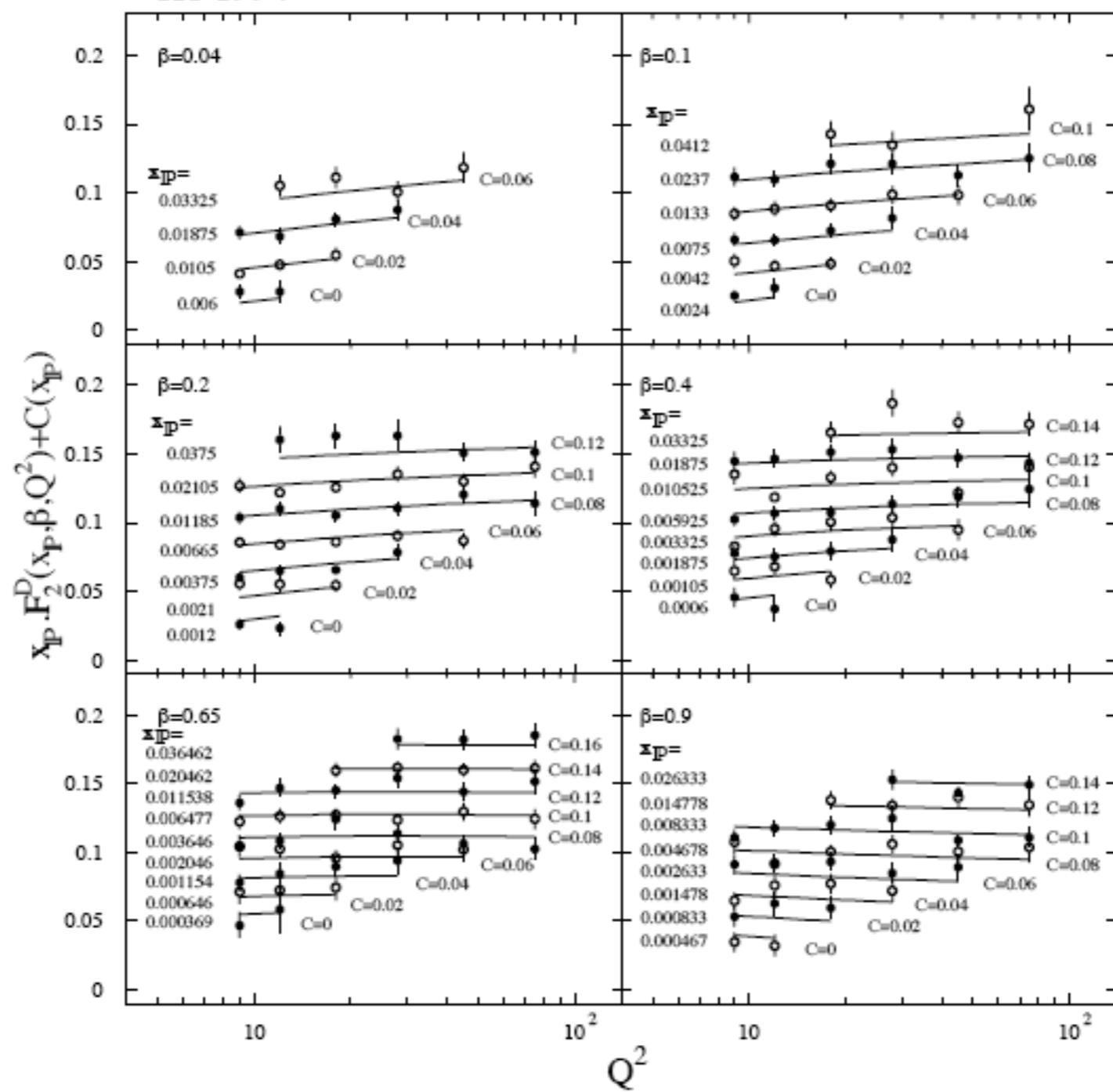
$$\int_0^\infty d|t| \frac{df_{a/P}^{diff}(\xi, \mu, x_P, t)}{dx_P dt} = M_{AA}(\xi, \mu, z = 1 - x_P)$$

- Analyzing $F_2^{D(3)}(\beta, \mu, x_P)$, H1 collaboration at HERA observed a possible Q^2 dependence of the diffractive distributions

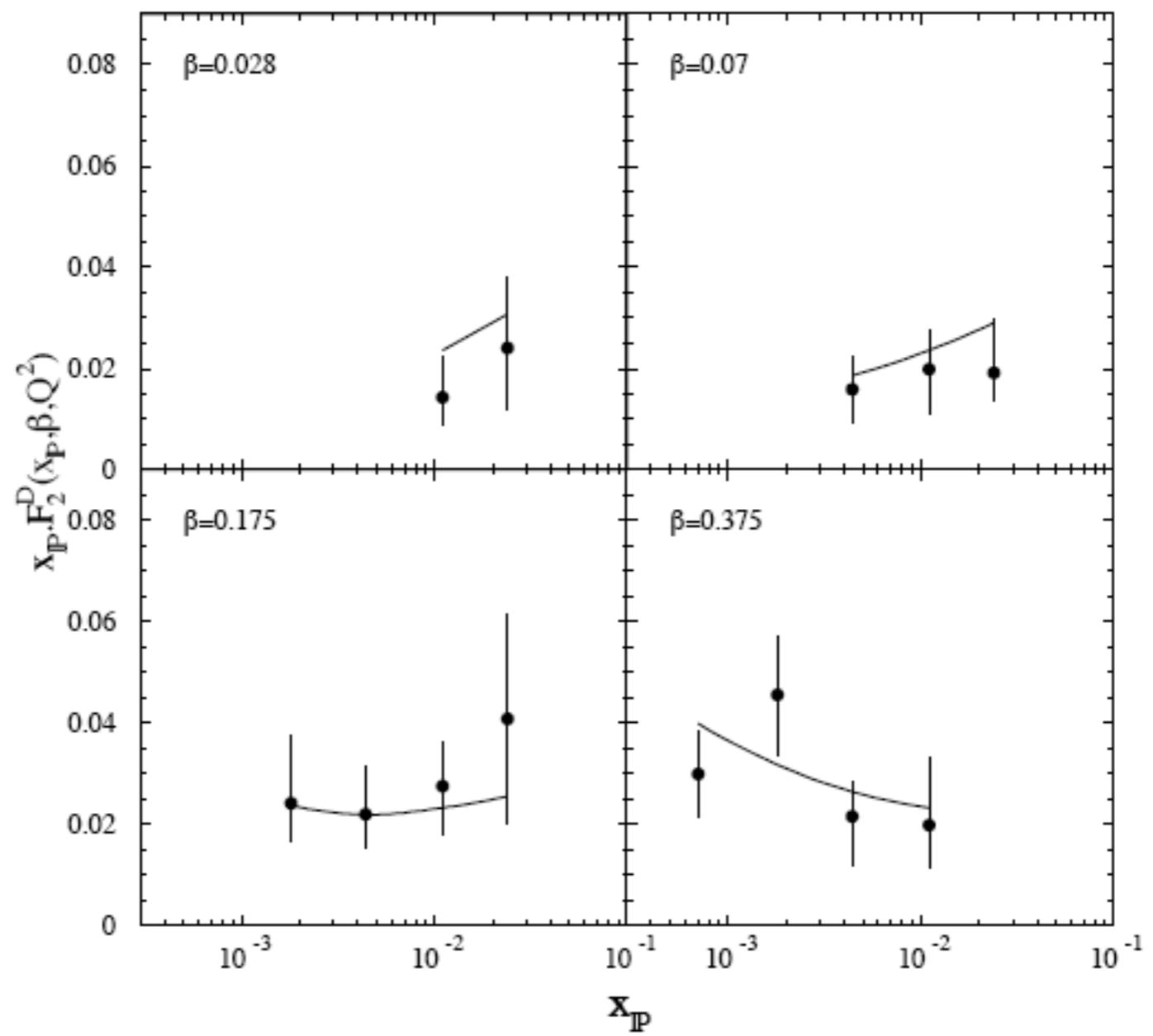
↓

A logarithmic dependence on Q^2 is implicitly contained in fracture functions!

H1 1994



ZEUS 1994

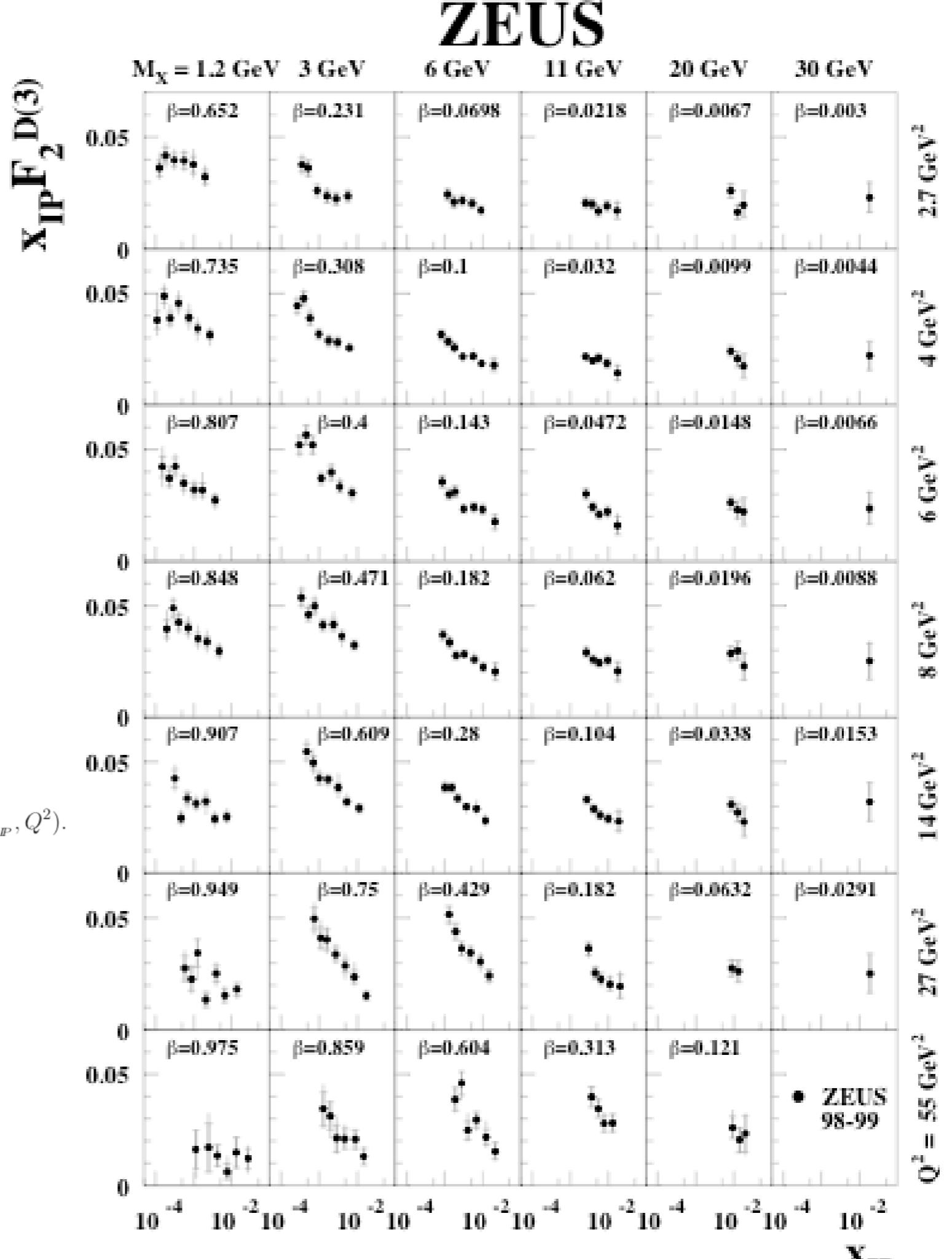


Study of deep inelastic inclusive and diffractive scattering with the ZEUS forward plug calorimeter

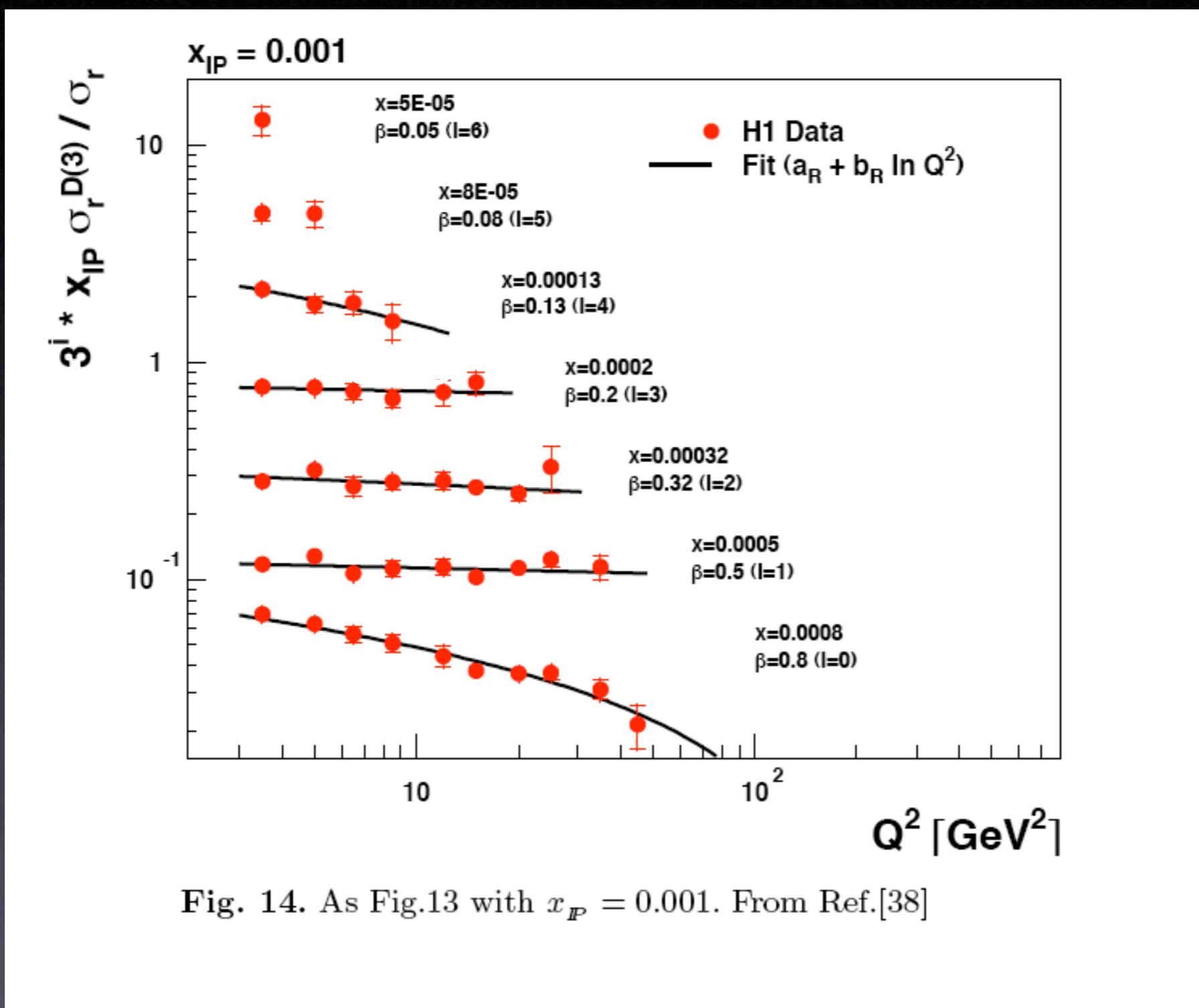
ZEUS Collaboration

section. The data are also presented in terms of the diffractive structure function, $F_2^{D(3)}(\beta, x_p, Q^2)$, of the proton. For fixed β , the Q^2 dependence of $x_p F_2^{D(3)}$ changes with x_p , in violation of Regge factorisation. For fixed x_p , $x_p F_2^{D(3)}$ rises as $\beta \rightarrow 0$, the rise accelerating with increasing Q^2 . These positive scaling violations suggest substantial contributions of perturbative effects in the diffractive DIS cross section.

$$\frac{1}{2M_X} \frac{d\sigma_{\gamma^* p \rightarrow XN}^{\text{diff}}(M_X, W, Q^2)}{dM_X} = \frac{4\pi^2 \alpha}{Q^2(Q^2 + M_X^2)} x_{_P} F_2^{D(3)}(\beta, x_{_P}, Q^2).$$



H1- 2006



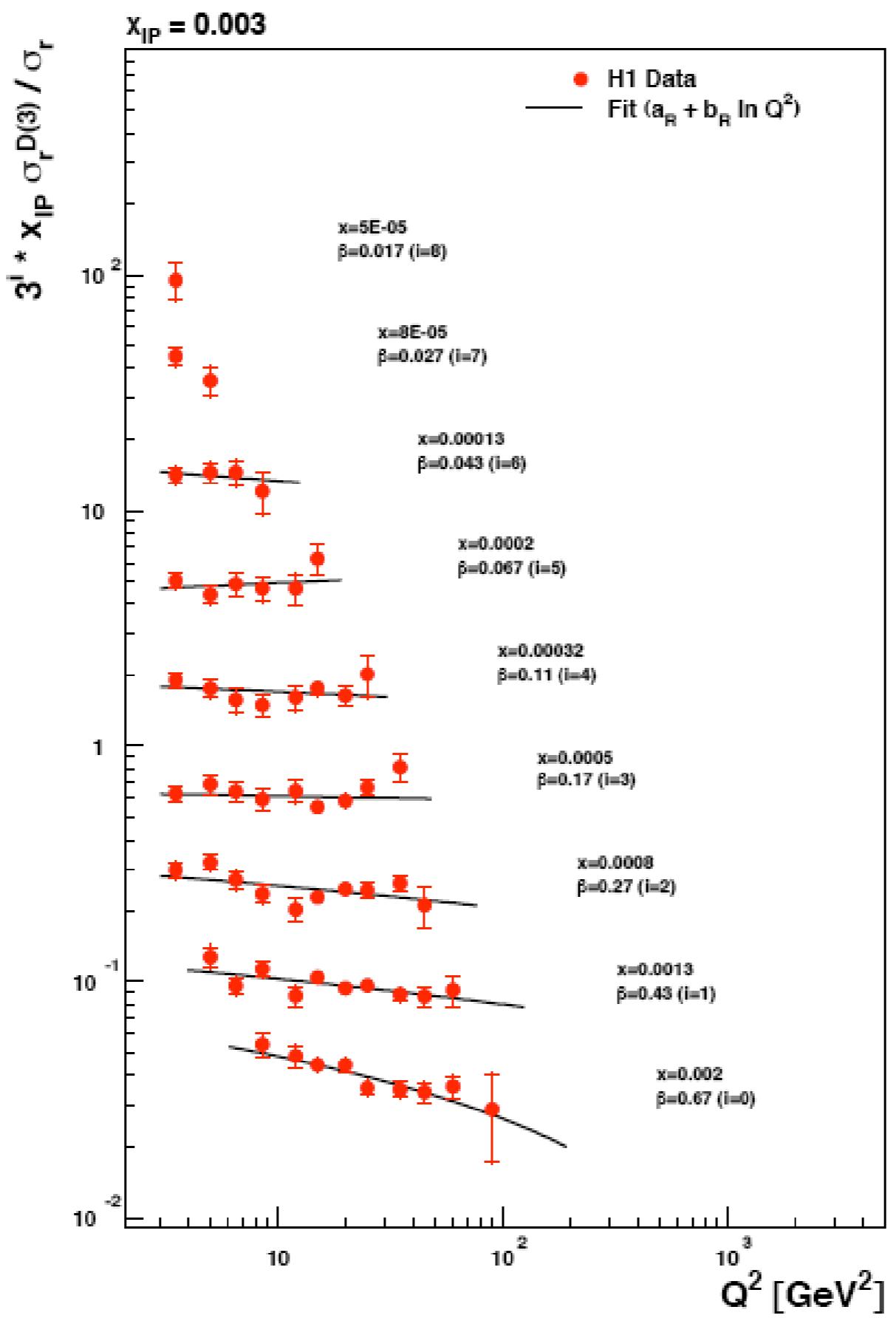
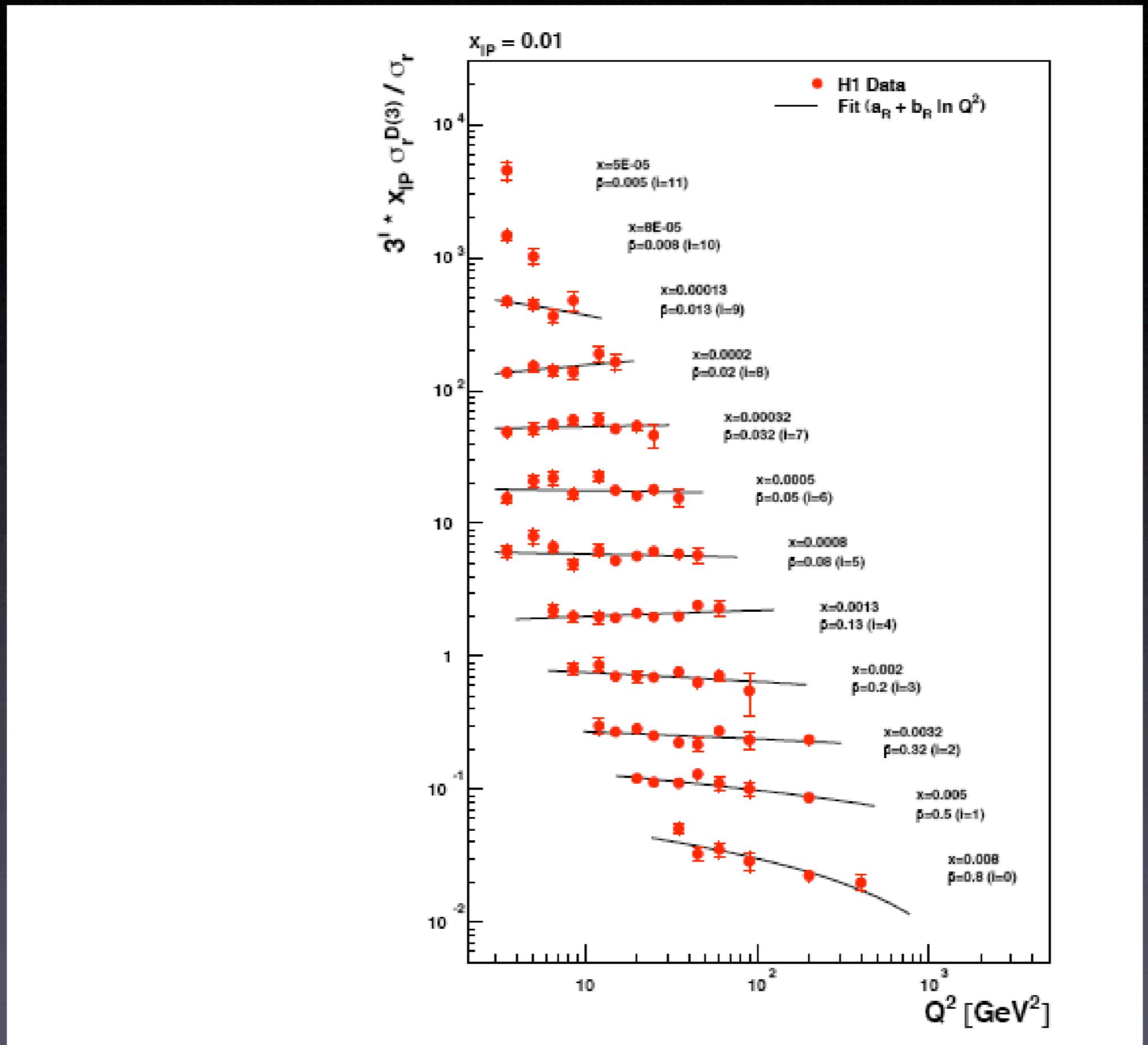


Fig. 15. As Fig.13 with $x_P = 0.003$. From Ref.[38]



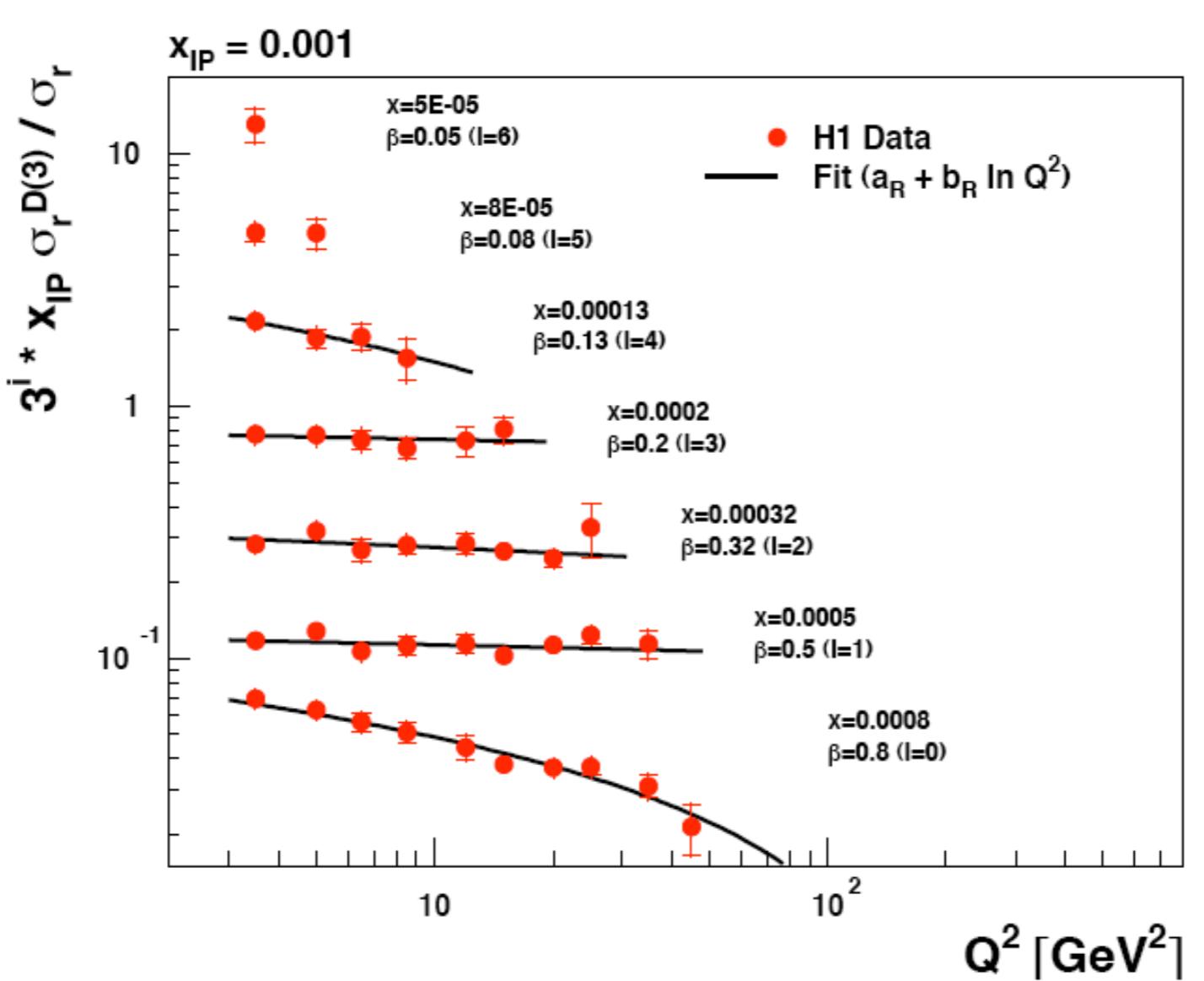


Fig. 14. As Fig.13 with $x_{IP} = 0.001$. From Ref.[38]

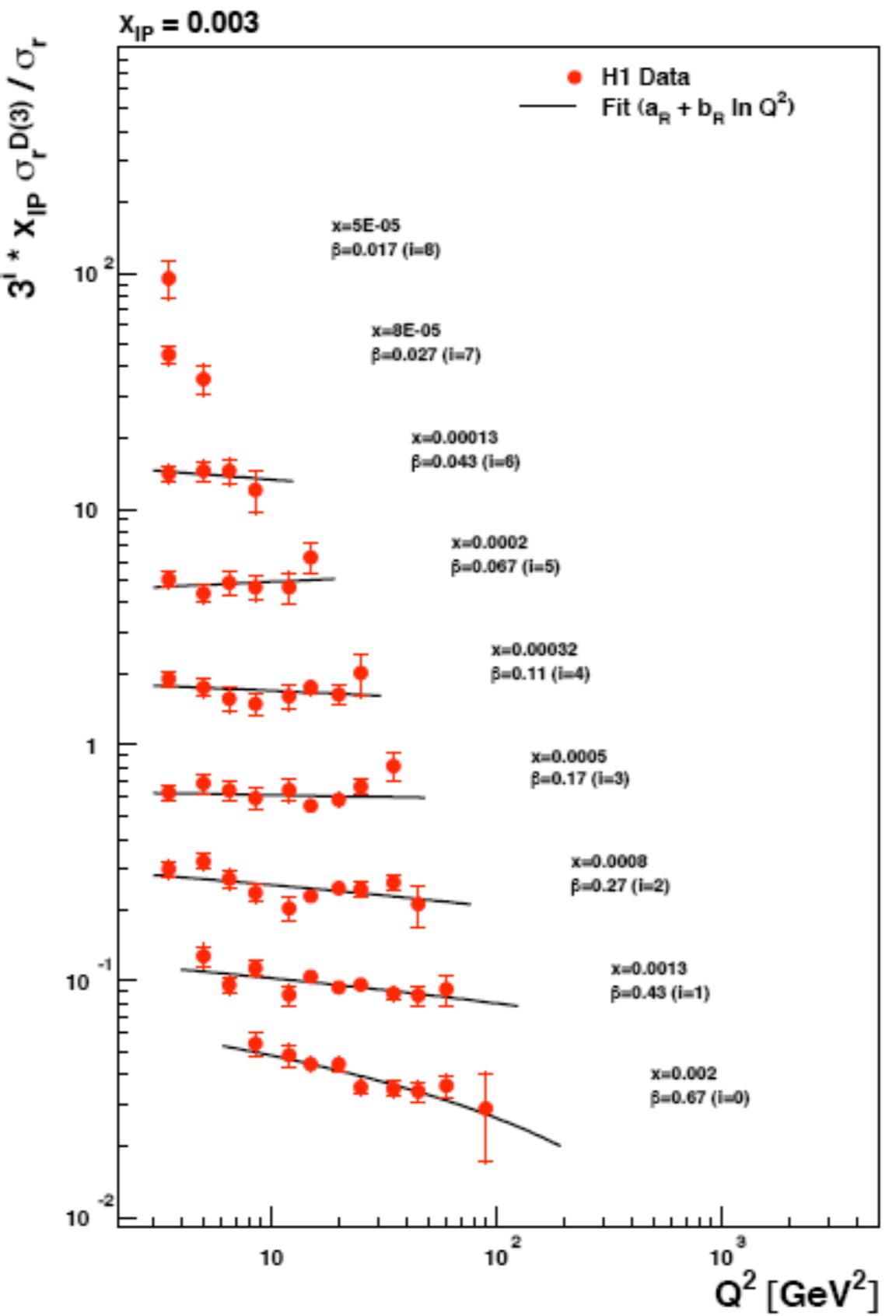
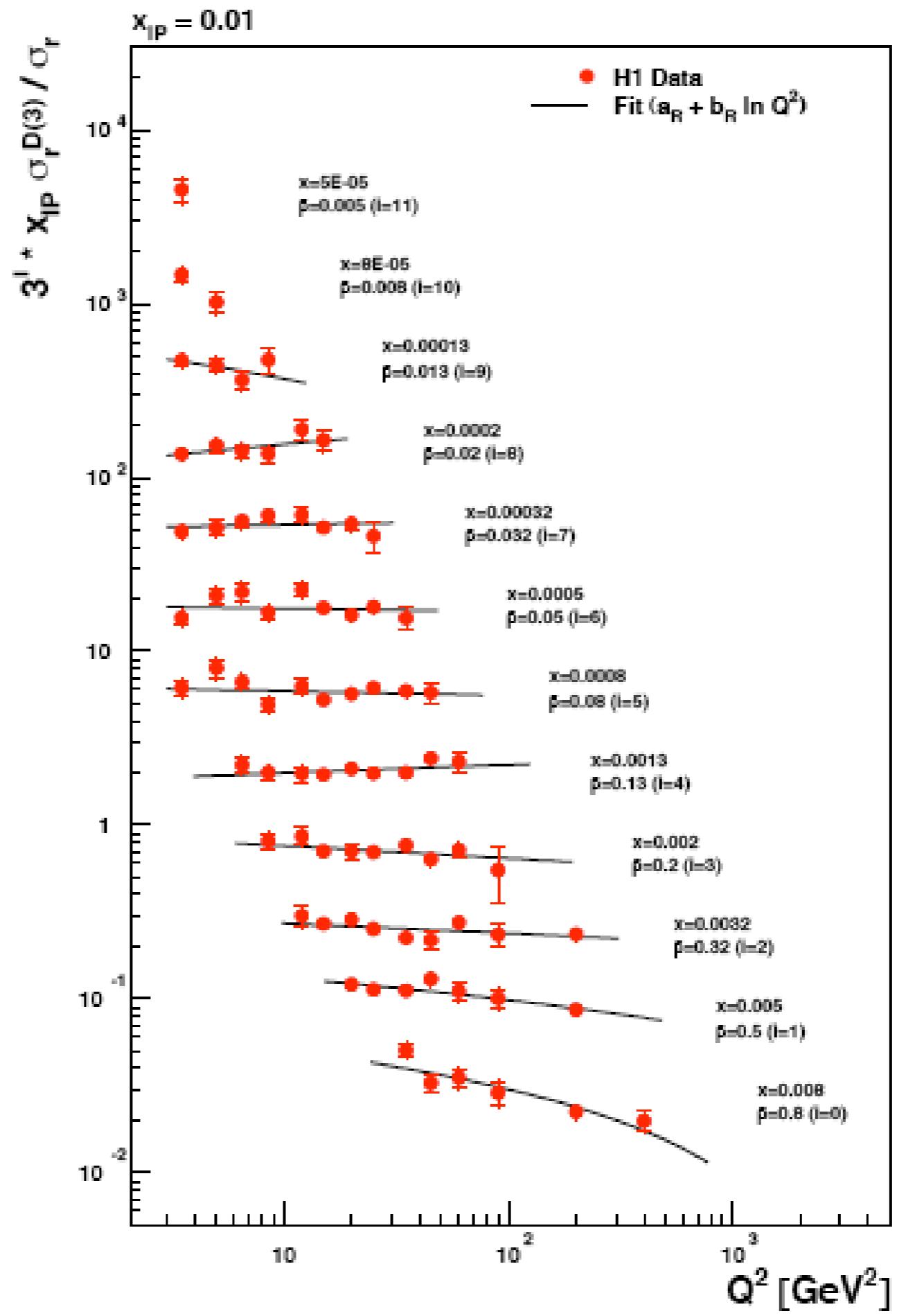


Fig. 15. As Fig.13 with $x_{IP} = 0.003$. From Ref.[38]



Next to leading order evolution of SIDIS processes in the forward region*

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We compute the order α_s^2 quark initiated corrections to semi-inclusive deep inelastic scattering extending the approach developed recently for the gluon contributions. With these corrections we complete the order α_s^2 QCD description of these processes, verifying explicitly the factorization of collinear singularities. We also obtain the corresponding NLO evolution kernels, relevant for the scale dependence of fracture functions. We compare the non-homogeneous evolution effects driven by these kernels with those obtained at leading order accuracy and discuss their phenomenological implications.

PACS numbers: 12.38.Bx, 13.85.Ni

Keywords: Semi-Inclusive DIS; perturbative QCD; Fracture functions

$$\begin{aligned} \frac{\partial M_{i,h/P}^r(\xi, \zeta, M^2)}{\partial \log M^2} &= \frac{\alpha_s(M^2)}{2\pi} \int_{\frac{\xi}{1-\zeta}}^1 \frac{du}{u} \left[P_{i \leftarrow j}^{(0)}(u) + \frac{\alpha_s(M^2)}{2\pi} P_{i \leftarrow j}^{(1)}(u) \right] M_{j,h/P}^r\left(\frac{\xi}{u}, \zeta, M^2\right) \\ &+ \frac{\alpha_s(M^2)}{2\pi} \frac{1}{\xi} \int_{\xi}^{\frac{\xi}{1-\zeta}} \frac{du}{u} \int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}} \frac{dv}{v} \left[\tilde{P}_{k i \leftarrow j}^{(0)}(u, v) + \frac{\alpha_s(M^2)}{2\pi} P_{k i \leftarrow j}^{(1)}(u, v) \right] f_{j/P}^r\left(\frac{\xi}{u}, M^2\right) D_{h/k}^r\left(\frac{\zeta}{\xi v}, M^2\right), \end{aligned}$$

Other interesting applications and developments:

LEPTO and Polarized SIDIS: the Question of Independent Fragmentation

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March 8, 2005

Abstract

Hadron production in the LEPTO event generator is modeled as a product of distribution functions and LUND hadronization functions (LHF_s) weighted by the hard scattering cross sections. The description of polarized SIDIS within this formalism includes a new nonperturbative input – polarized LHF_s. It is shown that this approach does not correspond to the commonly adopted one with the independent fragmentation functions. The purity method used by the HERMES collaboration mixes up the two approaches and ignores the contributions from polarized LHF_s. This method cannot be considered a precise tool for the extraction of polarized quark distributions from measured SIDIS asymmetries.

1 Introduction

Recently, the important issue of the extraction of polarized quark distributions was again addressed by the HERMES collaboration [1]. They have used the LO analysis of semi-inclusive deep inelastic scattering (SIDIS) based on the so called purity method.

It is evident that the theoretical description of SIDIS is much more complicated than that of DIS owing to our poor knowledge of the nonperturbative hadronization mechanism. Traditionally, one distinguishes two regions for hadron production: the current fragmentation region, $x_F > 0$ and the target fragmentation region, $x_F < 0$ ¹. The common assumption is that hadrons in the current fragmentation region with $z > 0.2$ are produced in the independent quark fragmentation. Then, in the LO approximation of perturbative QCD the SIDIS cross section for unpolarized target is given as

$$\sigma^h(x, z, Q^2) \propto (1 + (1 - y)^2) \sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2) \quad (1)$$

¹We use the standard SIDIS notations and variables like in [1].

LEPTO and Polarized SIDIS: the Question of Independent Fragmentation

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Abstract

Hadron production in the LEPTO event generator is modeled as a product of distribution functions and LUND hadronization functions (LHF_s) weighted by the hard scattering cross sections. The description of polarized SIDIS within this formalism includes a new nonperturbative input – polarized LHF_s. It is shown that this approach does not correspond to the commonly adopted one with the independent fragmentation functions. The purity method used by the HERMES collaboration mixes up the two approaches and ignores the contributions from polarized LHF_s. This method cannot be considered a precise tool for the extraction of polarized quark distributions from measured SIDIS asymmetries.

Sources of time reversal odd spin asymmetries in QCD

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Received 20 October 2002

Generation of T-odd single spin asymmetries (SSA) by the various ingredients of QCD factorization is discussed. The possible use of SSA in studies of Generalized Parton Distribution (GPD) at HERA with the polarized lepton beam is suggested. The role of GPD in the investigation of orbital angular momenta of partons is discussed. The generalization of Equivalence principle, leading to the equipartition of momenta and total angular momenta, violated in perturbation theory, but possibly restored due to confinement and chiral symmetry breaking, is proposed. The T-odd fragmentation and fracture function are considered. The T-odd distribution (Sivers) function may be only effective, due to the imaginary cuts in the SIDIS and Drell-Yan process, while the existence of such universal functions should lead, after the integration over transverse momentum, to the strong T violation in polarized DIS.

Key words: single spin asymmetry, distribution, fragmentation

1 Introduction

The single transverse spin asymmetries are most easily studied experimentally, as they require only one polarized particle (usually, the polarized target). At the same time, they are known to be one of the most subtle effects in QCD. They should be proportional to mass scale, and the only scale in "naive" perturbative QCD is that of the current quark mass.

Fracture Functions : spin dependent

/0211027 v2 11 Nov 2002

Spin-dependent, interference and T -odd fragmentation and
fracture functions

O. V. TERYAEV,

Joint Institute for Nuclear Research, Dubna, 141980 Russia

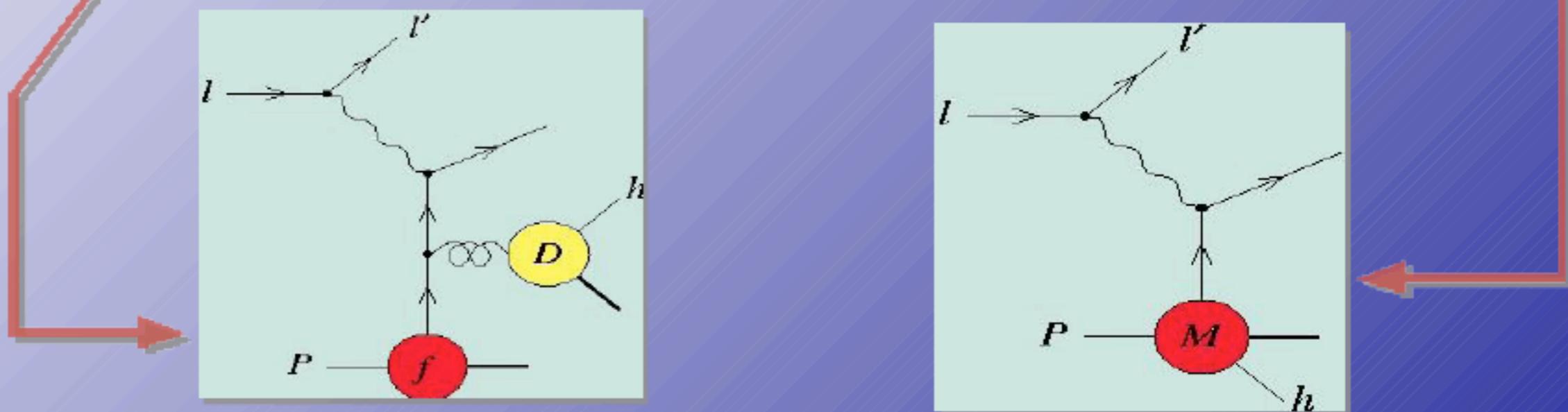
Fracture functions, originally suggested to describe the production of diffractive and leading hadrons in semi-inclusive DIS, may be also applied at fixed target energies. They may also include interference and final state interaction, providing a source for azimuthal asymmetries at HERMES and (especially) Λ polarization at NOMAD. The recent papers by Brodsky, Hwang and Schmidt, and by Gluck and Reya, may be understood in terms of fracture functions.

Transverse Momenta

F. Ceccopieri, L.T.,
Phys.Lett.B (2006)

- **QCD** predicts the scale dependence of M :

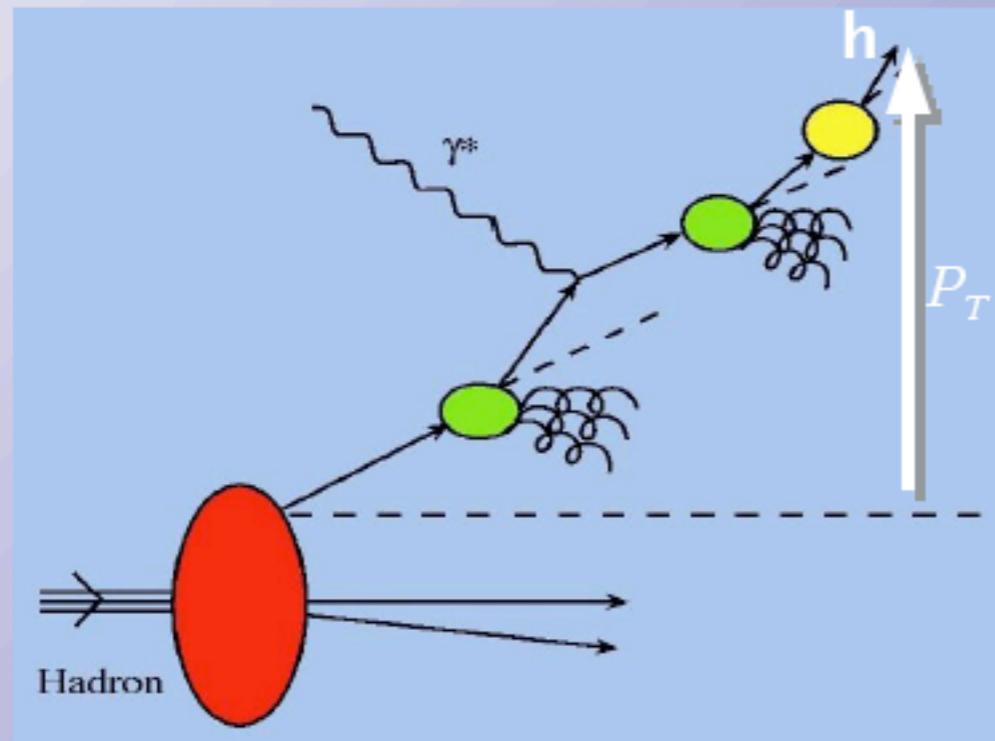
$$\frac{\partial}{\partial \log Q^2} M_{i,h/p}(x, z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{x/(1-z)}^1 \frac{du}{u} P_j^i(u) M_{j,h/N}\left(\frac{x}{u}, z, Q^2\right) + \\ + \frac{\alpha_s(Q^2)}{2\pi} \int_x^{x/(x+z)} \frac{du}{x(1-u)} \hat{P}_j^{i,I}(u) F_{j/p}\left(\frac{x}{u}, Q^2\right) D_{h/I}\left(\frac{zu}{x(1-u)}, Q^2\right)$$



→ Leading twist SIDIS cross section is thus:

$$\frac{d^3 \sigma_p^h}{dx_B dQ^2 dz_h} \propto \sum_{i=q,\bar{q}} e_i^2 \left[\underbrace{F_{i/p}(x_B, Q^2) D_{h/i}(z_h, Q^2)}_{\sigma_{current}} + \underbrace{(1-x_B) M_{i,h/p}(x_B, (1-x_B)z_h, Q^2)}_{\sigma_{target}} \right]$$

- Sources of transverse momentum in $I+P \rightarrow I+h+X$:

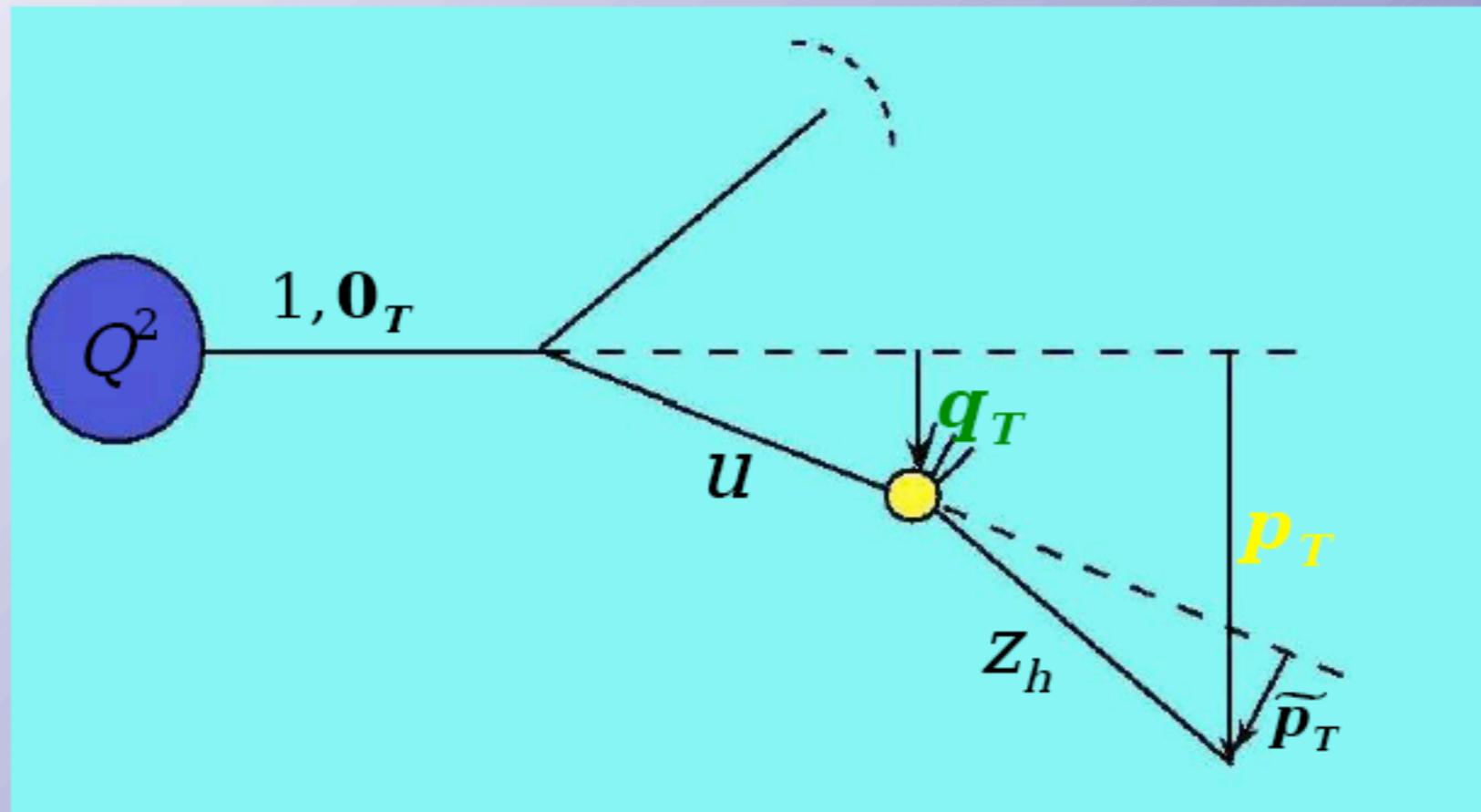


- Intrinsic distribution k_T [8]
- Radiative q_T
- Intrinsic fragmentation p_T [8]

Detected hadron transverse momentum : $\mathbf{P}_T \approx \mathbf{p}_T + z_h \mathbf{k}_T$

[8] M. Anselmino et al.

- Time-like TMD DGLAP evolution equation



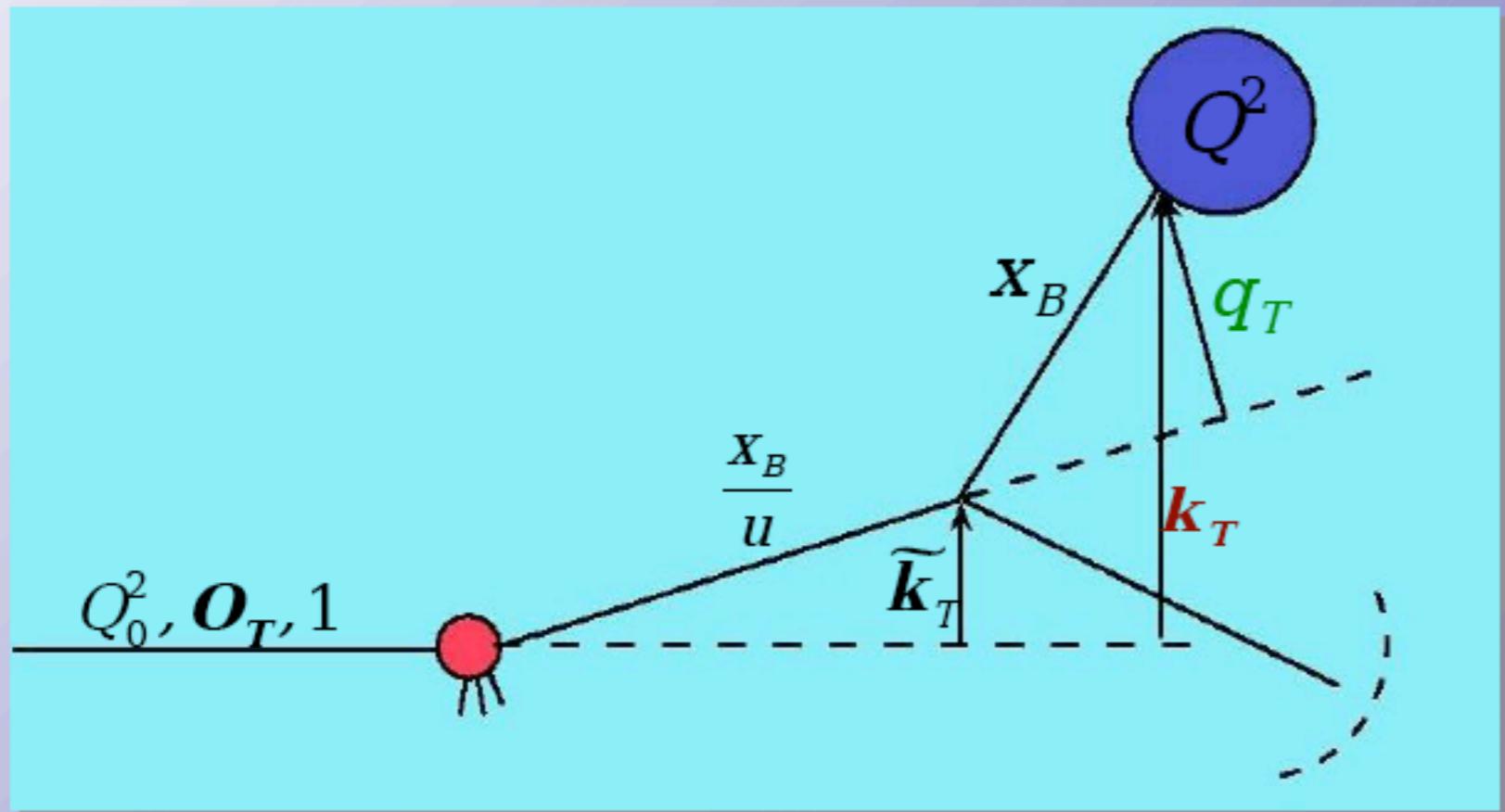
Branching kinematics:

$$\widetilde{\mathbf{p}}_T = \mathbf{P}_T - \frac{z_h}{u} \mathbf{q}_T$$

$$u(1-u) Q^2 = \mathbf{P}_T^2$$

$$Q^2 \frac{\partial D_a^b(Q^2, z_h, \mathbf{P}_T)}{\partial Q^2} = \\ = \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{du}{u} \int \frac{d^2 \mathbf{q}_T}{\pi} \delta[u(1-u)Q^2 - \mathbf{q}_T^2] P_a^c(w) D_c^b \left(Q^2, \frac{z_h}{u}, \mathbf{P}_T - \frac{z_h}{u} \mathbf{q}_T \right)$$

- Space-like TMD DGLAP evolution equation



Branching kinematics:

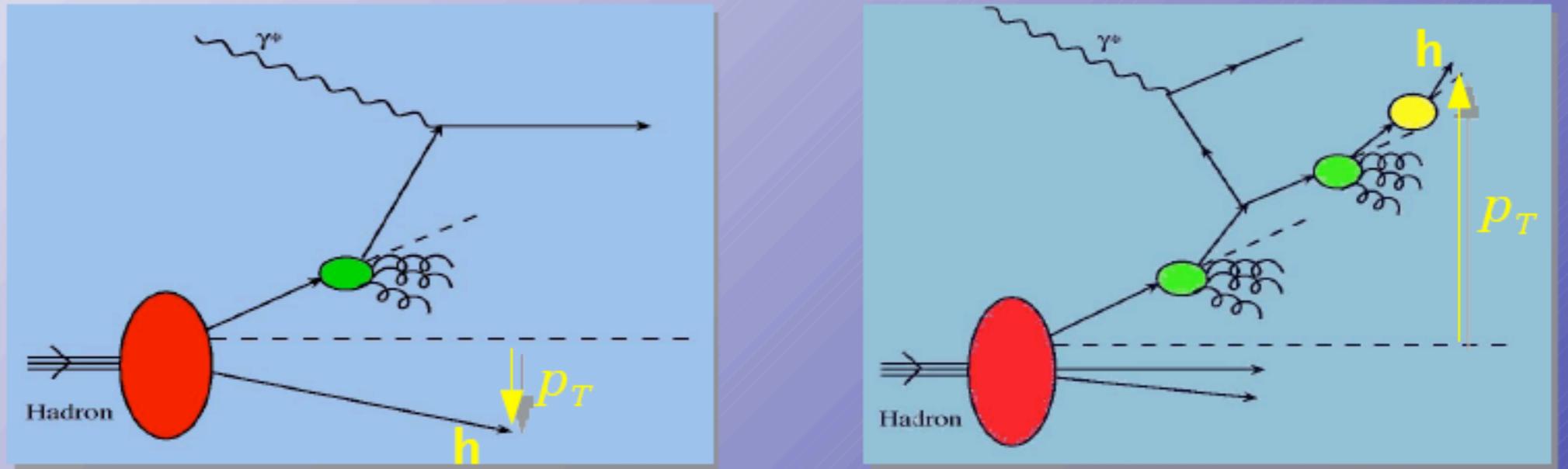
$$\tilde{\mathbf{k}}_T = \frac{\mathbf{K}_T - \mathbf{q}_T}{u}$$

$$(1-u)Q^2 = \mathbf{q}_T^2$$

$$Q^2 \frac{\partial f_a^b(Q^2, x_B, \mathbf{k}_T)}{\partial Q^2} =$$

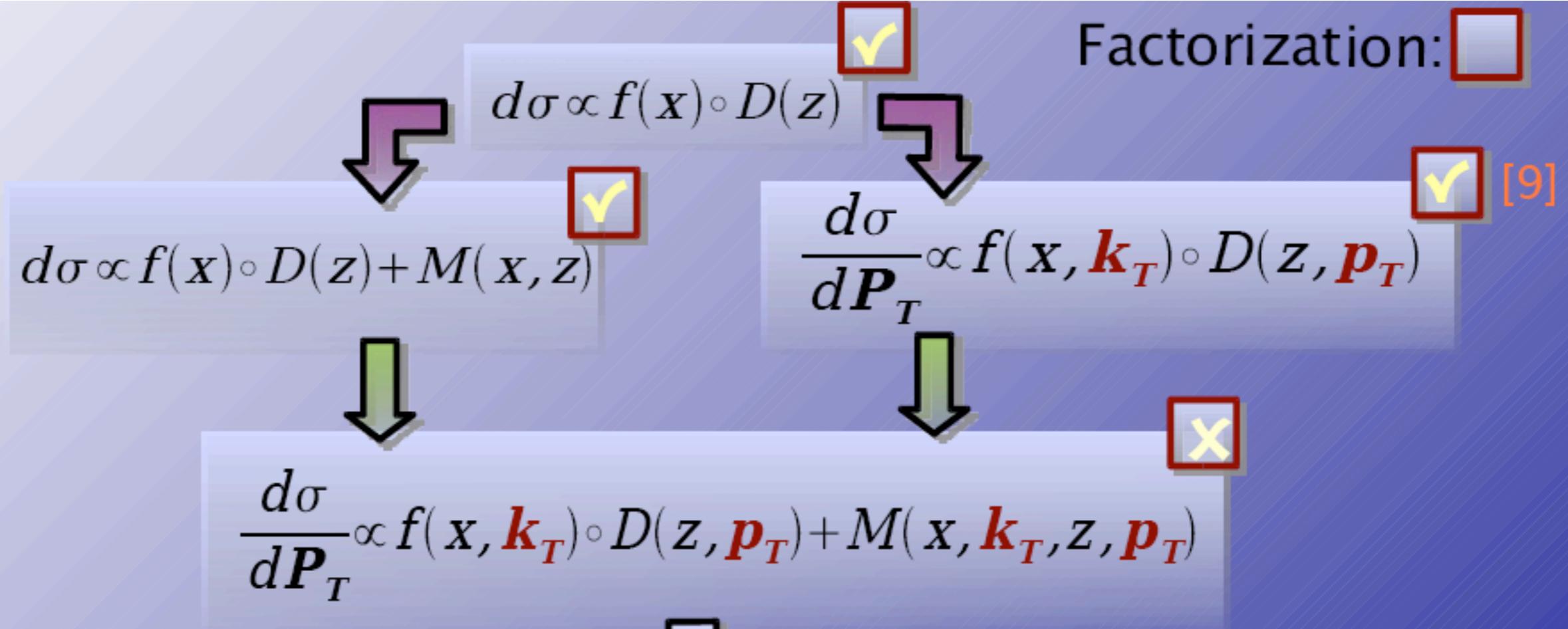
$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \int \frac{d^2 \mathbf{q}_T}{\pi} \delta[(1-u)Q^2 - \mathbf{q}_T^2] P_a^c(u) f_c^b \left(Q^2, \frac{x_B}{u}, \frac{\mathbf{k}_T - \mathbf{q}_T}{u} \right)$$

- Fracture functions TMD evolution equation

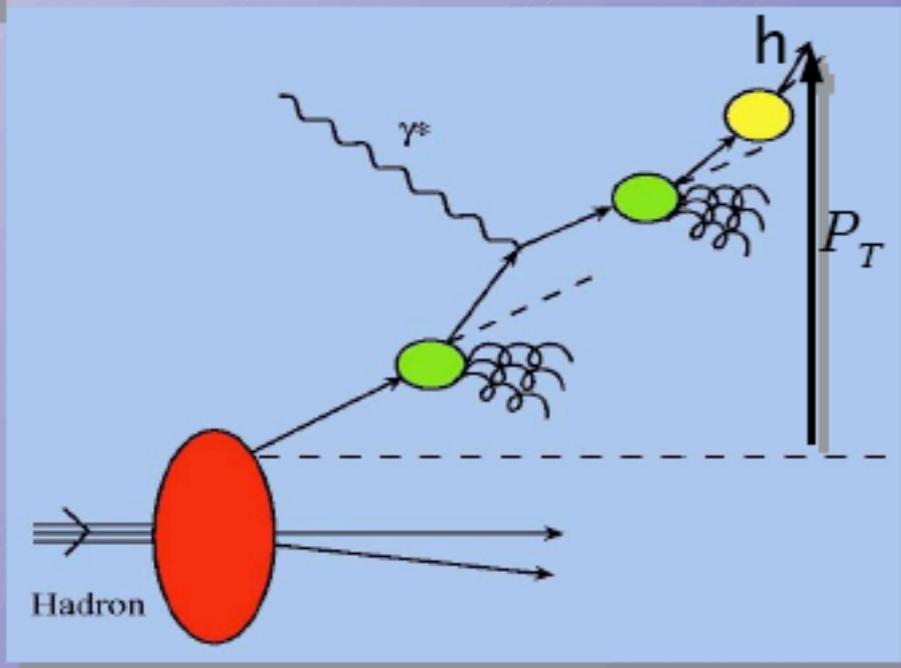


$$\begin{aligned}
 & Q^2 \frac{\partial \mathcal{M}_{\mathbf{p}, \mathbf{h}}^j(Q^2, \mathbf{x}_B, \mathbf{k}_T, z_h, \mathbf{p}_T)}{\partial Q^2} = \\
 & = \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u^3} \int \frac{d^2 \mathbf{q}_T}{\pi} \delta[(1-u)Q^2 - \mathbf{q}_T^2] P_i^j(u) \mathcal{M}_{\mathbf{p}, \mathbf{h}}^i\left(Q^2, \frac{\mathbf{x}_B}{u}, \frac{\mathbf{k}_T - \mathbf{q}_T}{u}, z_h, \mathbf{p}_T\right) + \\
 & + \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u} \int \frac{d^2 \mathbf{q}_T}{\pi} \delta[(1-u)Q^2 - \mathbf{q}_T^2] \frac{u}{\mathbf{x}_B(1-u)} \hat{P}_i^{j,I}(u) \cdot \\
 & \cdot \mathcal{F}_{\mathbf{p}}^i\left(Q^2, \frac{\mathbf{x}_B}{u}, \frac{\mathbf{k}_T - \mathbf{q}_T}{u}\right) D_I^h\left(Q^2, \frac{z_h u}{\mathbf{x}_B(1-u)}, \mathbf{p}_T - \frac{z_h u}{\mathbf{x}_B(1-u)} \mathbf{q}_T\right)
 \end{aligned}$$

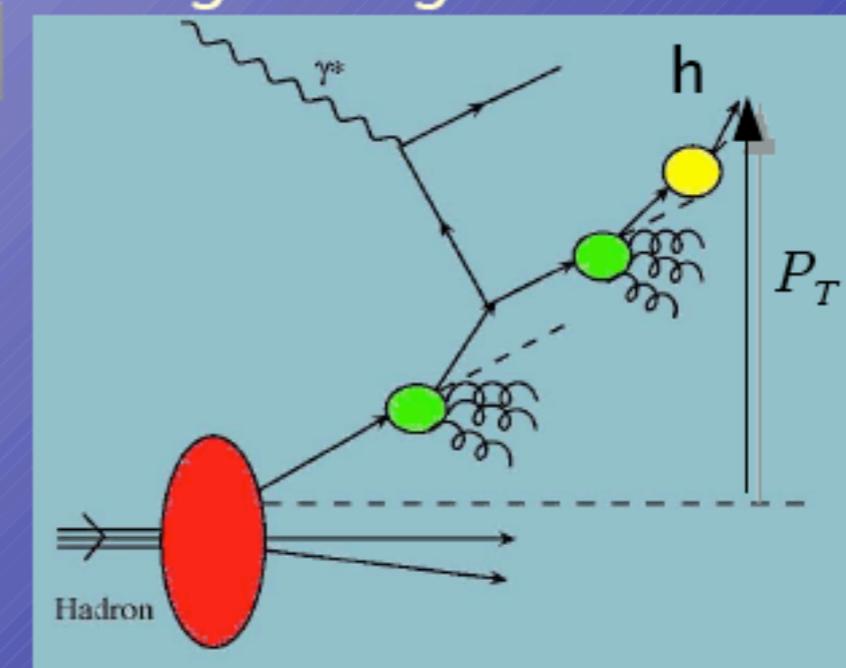
Increasing phase space

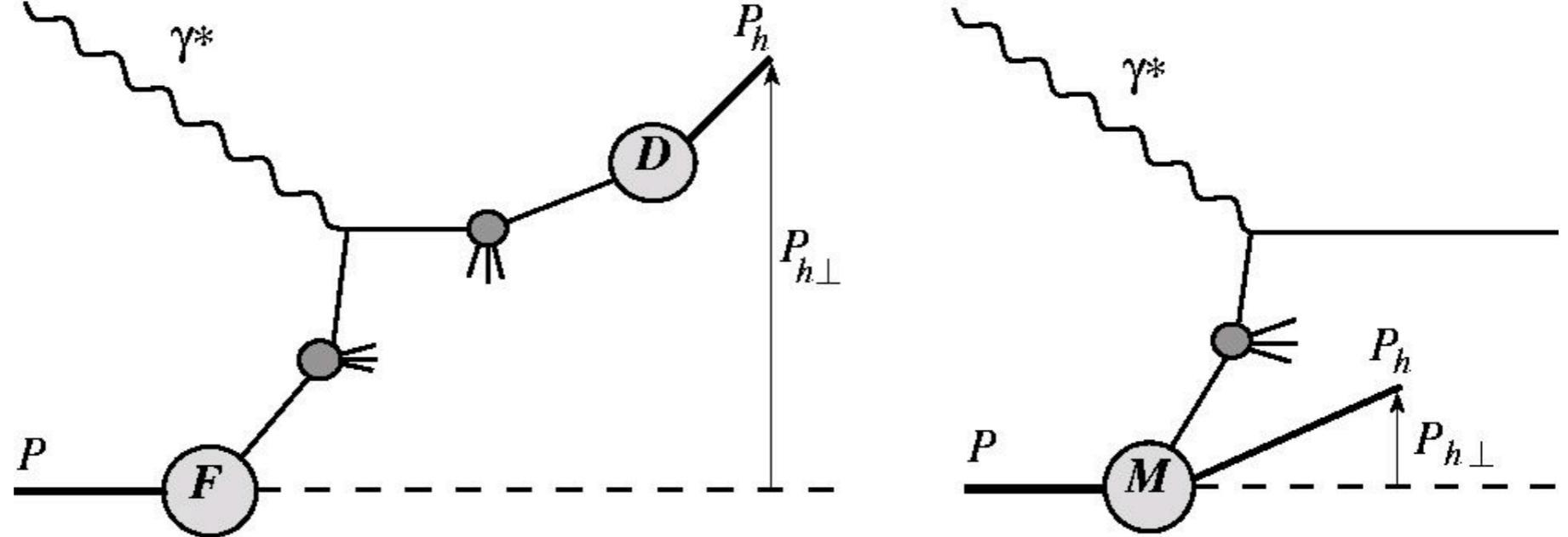


Current fragmentation



Target fragmentation





$$Q^2 \frac{\partial \mathcal{M}_{P,h}^i(x, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \left\{ \int_{\frac{x}{1-z}}^1 \frac{du}{u^3} P_j^i(u) \int \frac{d^2 \boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 - \right. \\ \cdot \mathcal{M}_{P,h}^j\left(Q^2, \frac{x}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}, z, \boldsymbol{p}_\perp\right) + \int_x^{\frac{x}{x+z}} \frac{du}{x(1-u)u^2} \hat{P}_j^{i,l}(u) \frac{d^2 \boldsymbol{q}_\perp}{\pi} \delta((1-u)Q^2 - \\ \cdot \mathcal{F}_P^j\left(\frac{x}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}, Q^2\right) \mathcal{D}_l^h\left(\frac{zu}{x(1-u)}, \boldsymbol{p}_\perp - \frac{zu}{x(1-u)} \boldsymbol{q}_\perp, Q^2\right) \right\}.$$

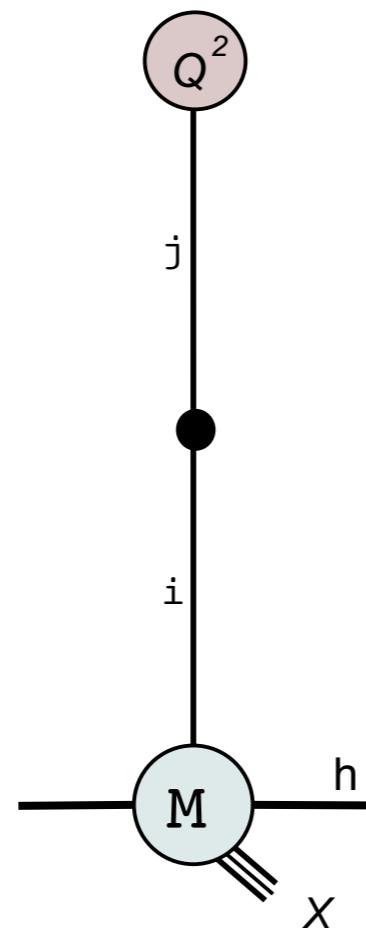
$$\int d^2 \boldsymbol{k}_\perp \int d^2 \boldsymbol{p}_\perp \mathcal{M}_{P,h}^i(x, \boldsymbol{k}_\perp, z, \boldsymbol{p}_\perp, Q^2) = \mathcal{M}_{P,h}^i(x, z, Q^2),$$

Extension to multiple hadron
distributions

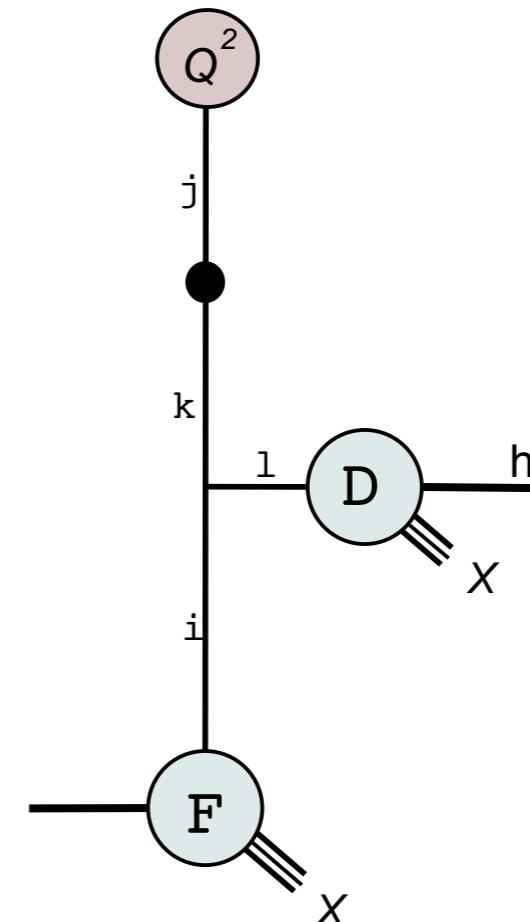
$$M_{h/P}^j(x, z, Y) = M_{A,h/P}^j(x, z, Y) + M_{B,h/P}^j(x, z, Y),$$

$$M_{A,h/P}^j(x, z, Y) = \int_x^{1-z} \frac{dw}{w} E_i^j\left(\frac{x}{w}, Y - y_0\right) M_{A,h/P}^i(w, z, y_0),$$

$$M_{B,h/P}^j(x, z, Y) = \int_{y_0}^Y dy \int_{x+z}^1 \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{z}{w}} \frac{du}{u(1-u)} \cdot \\ \cdot E_k^j\left(\frac{x}{wu}, Y - y\right) \hat{P}_i^{kl}(u) D_l^h\left(\frac{z}{w(1-u)}, y\right) F_P^i(w, y).$$



(A)



(B)

$$\begin{aligned} \frac{\partial}{\partial Y} M_{A,h/P}^j(x,z,Y) &= \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{A,h/P}^j(x/u,z,Y), \\ \frac{\partial}{\partial Y} M_{B,h/P}^j(x,z,Y) &= \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{B,h/P}^j(x/u,z,Y) + \\ &\quad \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) D_l^h \left(\frac{zu}{x(1-u)}, Y \right) F_P^i(x/u, Y). \end{aligned}$$

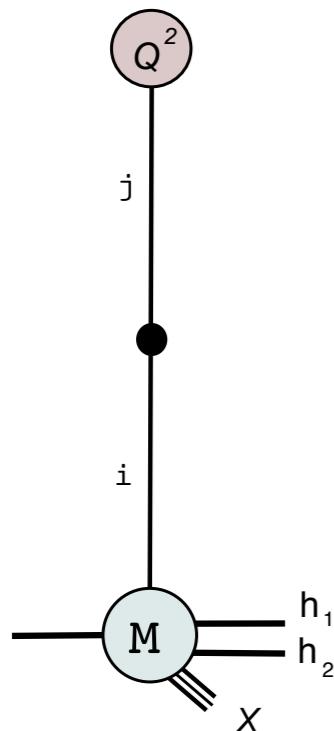
$$Y = \frac{1}{2\pi\beta_0} \ln \left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)} \right], \quad dY = \frac{\alpha_s(Q^2)}{2\pi} \frac{dQ^2}{Q^2}.$$

$$\begin{aligned} Q^2 \frac{\partial}{\partial Q^2} M_{h/P}^j(x,z,Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{h/P}^i(x/u,z,Q^2) + \\ &\quad \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{x+z}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) D_l^h \left(\frac{zu}{x(1-u)}, Q^2 \right) F_P^i(x/u, Q^2). \end{aligned}$$

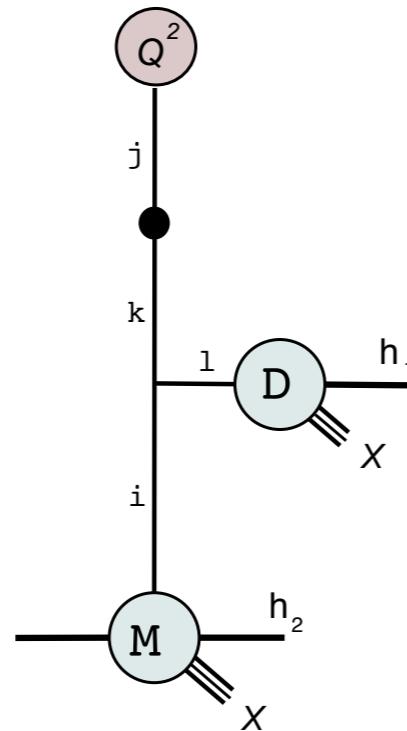
Di-hadron Fracture Functions

$$\sigma_T = \int \frac{du}{u} M_{h_1, h_2 / P}^j(u, z_1, z_2, Q^2) \hat{\sigma}_j(x/u, Q^2).$$

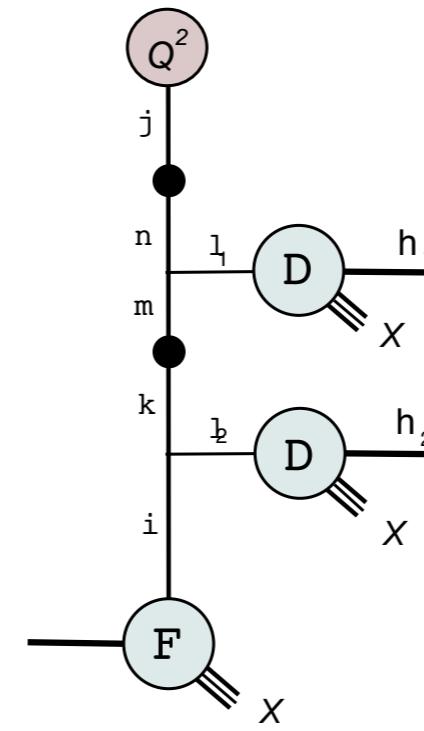
$$M_{h_1, h_2 / P}^j(x, z_1, z_2, Y) = \sum_{X=A,B,C,D} M_{X, h_1, h_2 / P}^j(x, z_1, z_2, Y).$$



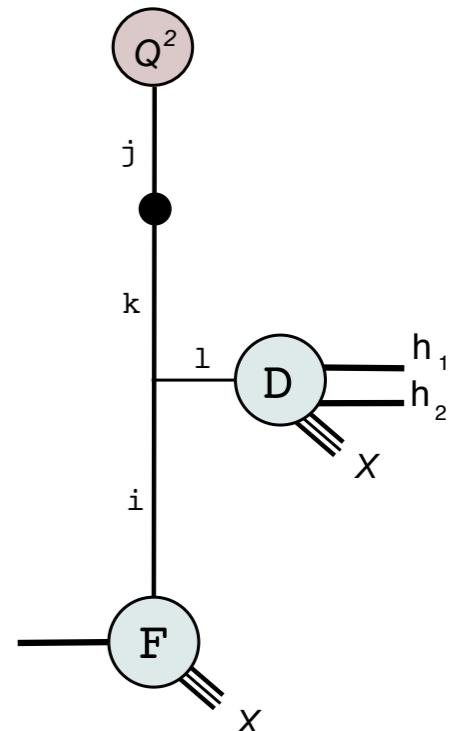
(A)



(B)



(C)



(D)

$$\begin{aligned}
M_{A,h_1,h_2/P}^j(x, z_1, z_2, Y) &= \int_x^{1-z_1-z_2} \frac{dw}{w} E_i^j\left(\frac{x}{w}, Y - y_0\right) M_{A,h_1,h_2/P}^i(w, z_1, z_2, y_0), \\
M_{B,h_1,h_2/P}^j(x, z_1, z_2, Y) &= \int_{y_0}^Y dy \int_{x+z_1}^{1-z_2} \frac{dw}{w^2} \int_{x/w}^{1-z_1/w} \frac{du}{u(1-u)} E_k^j\left(\frac{x}{wu}, Y - y\right) \hat{P}_i^{kl}(u) \cdot \\
&\quad M_{A,h_2/P}^i(w, z_2, y) D_l^{h_1}\left(\frac{z_1}{w(1-u)}, y\right) + (h_1, z_1) \leftrightarrow (h_2, z_2), \\
M_{C,h_1,h_2/P}^j(x, z_1, z_2, Y) &= \int_{y_0}^Y dy_2 \int_{y_2}^Y dy_1 \int_{x+z_1+z_2}^1 \frac{dw}{w^2} \int_{x+z_1}^{w-z_2} \frac{dx_2}{x_2^2} \int_{x/x_2}^{1-z_1/x_2} \frac{du_1}{u_1(1-u_1)} \hat{P}_m^{nl_1}(u_1) \cdot \\
&\quad \int_{x_2/w}^{1-z_2/w} \frac{du_2}{u_2(1-u_2)} \hat{P}_i^{kl_2}(u_2) E_k^m\left(\frac{x_2}{wu_2}, y_1 - y_2\right) E_n^j\left(\frac{x}{u_1 x_2}, Y - y_1\right) \cdot \\
&\quad D_{l_2}^{h_2}\left(\frac{z_2}{w(1-u_2)}, y_2\right) D_{l_1}^{h_1}\left(\frac{z_1}{x_2(1-u_1)}, y_1\right) F_P^i(w, y_1) + (h_1, z_1) \leftrightarrow (h_2, z_2), \\
M_{D,h_1,h_2/P}^j(x, z_1, z_2, Y) &= \int_{y_0}^Y dy \int_{x+z_1+z_2}^1 \frac{dw}{w} \int_{x/w}^{1-(z_1+z_2)/w} \frac{1}{w^2(1-u)^2} \frac{du}{u} E_k^j\left(\frac{x}{wu}, Y - y\right) \cdot \\
&\quad \hat{P}_i^{kl}(u) F_P^i(w, y) D_l^{h_1, h_2}\left(\frac{z_1}{w(1-u)}, \frac{z_2}{w(1-u)}, y\right).
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial Y} M_{A,h_1,h_2/P}^j &= \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{A,h_1,h_2/P}^i(x/u, z_1, z_2, Y) + , \\
\frac{\partial}{\partial Y} M_{B,h_1,h_2/P}^j &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_i^{jl}(u) M_{A,h_2/P}^i(x/u, z_2, Y) D_l^{h_1} \left(\frac{z_1 u}{x(1-u)}, Y \right) + \\
&\quad +(h_1, z_1) \leftrightarrow (h_2, z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{B,h_1,h_2/P}^i(x/u, z_1, z_2, Y) , \\
\frac{\partial}{\partial Y} M_{C,h_1,h_2/P}^j &= \int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}} \frac{du}{u} \frac{u}{x(1-u)} \hat{P}_m^{jl_1}(u) M_{B,h_2/P}^m(x/u, z_2, Y) D_{l_1}^{h_1} \left(\frac{z_1 u}{x(1-u)}, Y \right) \\
&\quad +(h_1, z_1) \leftrightarrow (h_2, z_2) + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{C,h_1,h_2/P}^i(x/u, z_1, z_2, Y) , \\
\frac{\partial}{\partial Y} M_{D,h_1,h_2/P}^j &= \int_x^{\frac{x}{x+z_1+z_2}} \frac{du}{u} \frac{u^2}{x^2(1-u)^2} \hat{P}_i^{jl}(u) F_P^i(x/u, Y) D_l^{h_1,h_2} \left(\frac{z_1 u}{x(1-u)}, \frac{z_2 u}{x(1-u)}, Y \right) \\
&\quad + \int_{\frac{x}{1-z_1-z_2}}^1 \frac{du}{u} P_i^j(u) M_{D,h_1,h_2/P}^i(x/u, z_1, z_2, Y) .
\end{aligned}$$

$$Q^2\frac{\partial}{\partial Q^2}M_{h_1,h_2/P}^j(x,z_1,z_2,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x}{1-z_1-z_2}}^1\frac{du}{u}P_i^j(u)M_{h_1,h_2/P}^i(x/u,z_1,z_2,Q^2)+\\+\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x}{1-z_2}}^{\frac{x}{x+z_1}}\frac{du}{u}\frac{u}{x(1-u)}\hat P_i^{jl}(u)M_{h_2/P}^i(x/u,z_2,Q^2)D_l^{h_1}\Big(\frac{z_1u}{x(1-u)},Q^2\Big)+(h_1,z_1)\leftrightarrow(h_2,z_2)-\\+\frac{\alpha_s(Q^2)}{2\pi}\int_x^{\frac{x}{x+z_1+z_2}}\frac{du}{u}\frac{u^2}{x^2(1-u)^2}\hat P_i^{jl}(u)F_P^i(x/u,Q^2)D_l^{h_1,h_2}\Big(\frac{z_1u}{x(1-u)},\frac{z_2u}{x(1-u)},Q^2\Big)\,.$$

$$\sum_{h_2}\int dz_2\, z_2\, M_{h_1,h_2/P}^i(x,z_1,z_2,Q^2)=(1-x-z_1)M_{h_1/P}^i(x,z_1,Q^2)\,,\\ \sum_{h_1,h_2}\int dz_1\, z_1\int dz_2\, z_2\, M_{2,h_1h_2/P}^i(x,z_1,z_2,Q^2)=(1-x)F_P^i(x,Q^2)\,.$$

n-hadrons

$$\begin{aligned} Q^2 \frac{\partial M_{n/P}^j}{\partial Q^2}(x, z_1, \dots, z_n, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-\sum_{k=1}^n z_k}}^1 \frac{du}{u} P_i^j(u) M_{n/P}^i(x/u, z_1, \dots, z_n, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \sum_{q=1}^{n-1} \mathcal{P}_n\{h, z\} \\ &\cdot \int_{\frac{x}{1-\sum_{k=q+1}^n z_k}}^{\frac{x}{1-\sum_{k=1}^q z_k}} \frac{du}{u} \left(\frac{u}{x(1-u)} \right)^q \hat{P}_i^{jl}(u) M_{n-q}^i(x/u, z_{q+1}, \dots, z_n, Q^2) D_l^q \left(\frac{z_1 u}{x(1-u)}, \dots, \frac{z_q u}{x(1-u)}, Q^2 \right) + \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{\frac{x}{1-\sum_{k=1}^n z_k}} \frac{du}{u} \left(\frac{u}{x(1-u)} \right)^n \hat{P}_i^{jl}(u) F_P^i(x/u, Q^2) D_l^n \left(\frac{z_1 u}{x(1-u)}, \dots, \frac{z_n u}{x(1-u)}, Q^2 \right). \end{aligned}$$

Jet-like Fracture Functions

for

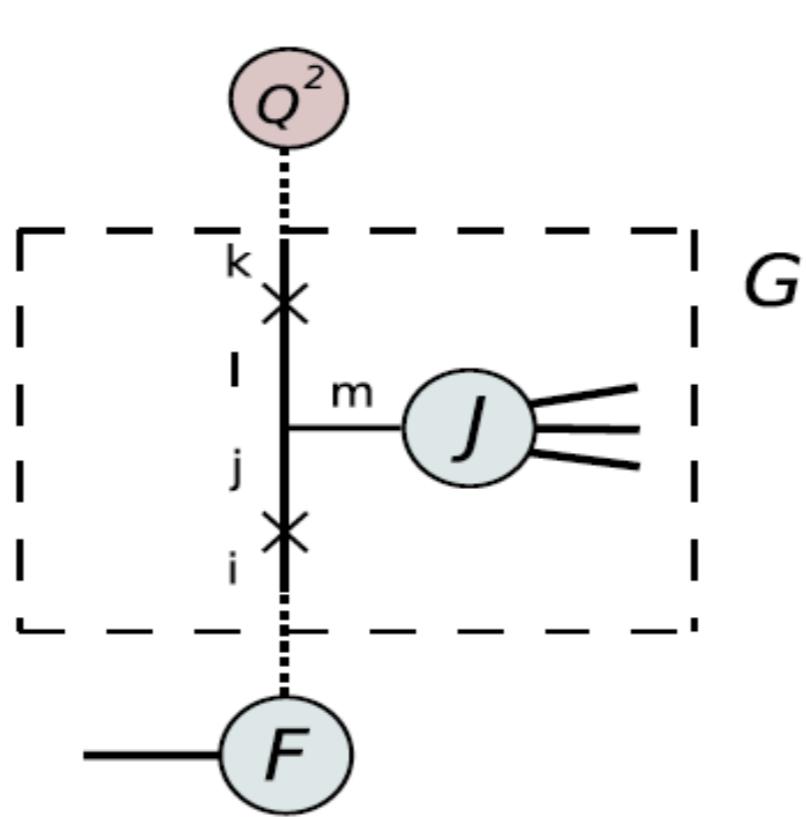
inclusive initial state radiation

$$F_2(x,Q^2)=\sum_{n=1}^\infty F_2^{(n)}(x,Q^2)\,.$$

$$F_2^{(n)}(x,Q^2;E_t^2,y_{\rm cut})=\sum_{i=q,\bar q}\int_x^1\frac{dz}{z}F_P^i(x/z,\mu_F^2)R_{2,i}^{(n)}\Big(z,\alpha_s,\frac{Q^2}{E_t^2},y_{\rm cut}\Big)\,,$$

$$\Lambda^2 \, \ll \, E_t^2 \, \leq \, Q^2 \, .$$

$$G_i^k(u, Q_i^2, Q_j^2) \equiv \Delta_i^j(Q_i^2, Q_j^2) \hat{P}_j^{lm}(u) J_m(Q_j^2, Q_0^2) \Delta_l^k(Q_l^2, Q_k^2)$$



$$J(Q^2, k^2) = \int_0^1 dz \, d(z, Q^2, k^2).$$

$$\int^{Q^2} dk^2 \, d(z, Q^2, k^2) \equiv D(z, Q^2).$$

$$\Delta_i^j(Q_i^2, Q_j^2) \equiv \exp \left[- \sum_k \int_{Q_i^2}^{Q_j^2} \frac{dt}{t} \int_{\frac{Q_i^2}{Q_j^2}}^{1 - \frac{Q_i^2}{Q_j^2}} dz \frac{\alpha_s(t)}{2\pi} \hat{P}_i^{jk}(z) \right],$$

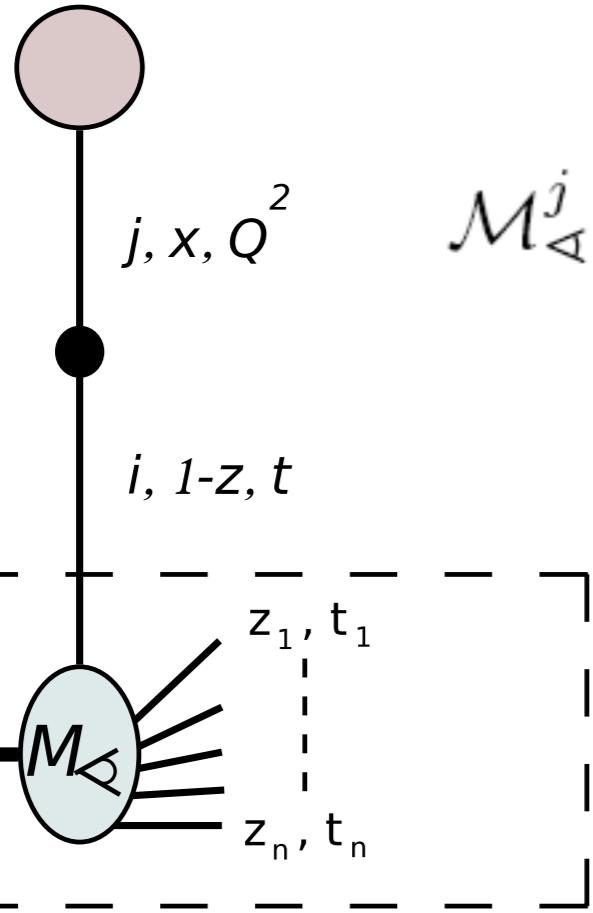
Catani Dokshitzer Webber

$$\boxed{\frac{1}{\sigma_{tot}} \frac{d\sigma}{dxdQ^2dzdt} \propto x \sum_{i=q,\bar q} e_i^2 \; {\cal M}_{\scriptscriptstyle \triangleleft}^i(x,Q^2,z,t) \,.}$$

$${\mathcal R}:~~t_i=-(P-h_i)^2< t, ~~~~t_0\leq t\leq Q^2~.~~~~~z=\sum_iz_i,~~h_i\in {\mathcal R}\,.$$

$$\Sigma^{(n)}_{excl} \equiv \frac{1}{n!} \frac{d^{2n+2} \sigma^{(n)}}{dxdQ^2 \prod_{m=1}^n dz_m dt_m}$$

$$\boxed{\frac{1}{\sigma_{tot}} \frac{d\sigma}{dxdQ^2dzdt} \equiv \frac{1}{\sigma_{tot}} \sum_{k=1}^\infty \Big\{ \prod_{m=1}^k \int_{t_0}^t dt_m \int_0^1 dz_m \Big\} \, \Sigma^{(k)}_{excl} \, \delta\Big(z-\sum_{k=1}^n z_k\Big)}$$



$$\mathcal{M}_{\triangleleft}^j(x,Q^2,z,t) = \int_x^{1-z} \frac{dw}{w} \mathcal{M}_{\triangleleft}^i(w,t,z,t) \, E_i^j(x/w,t,Q^2).$$

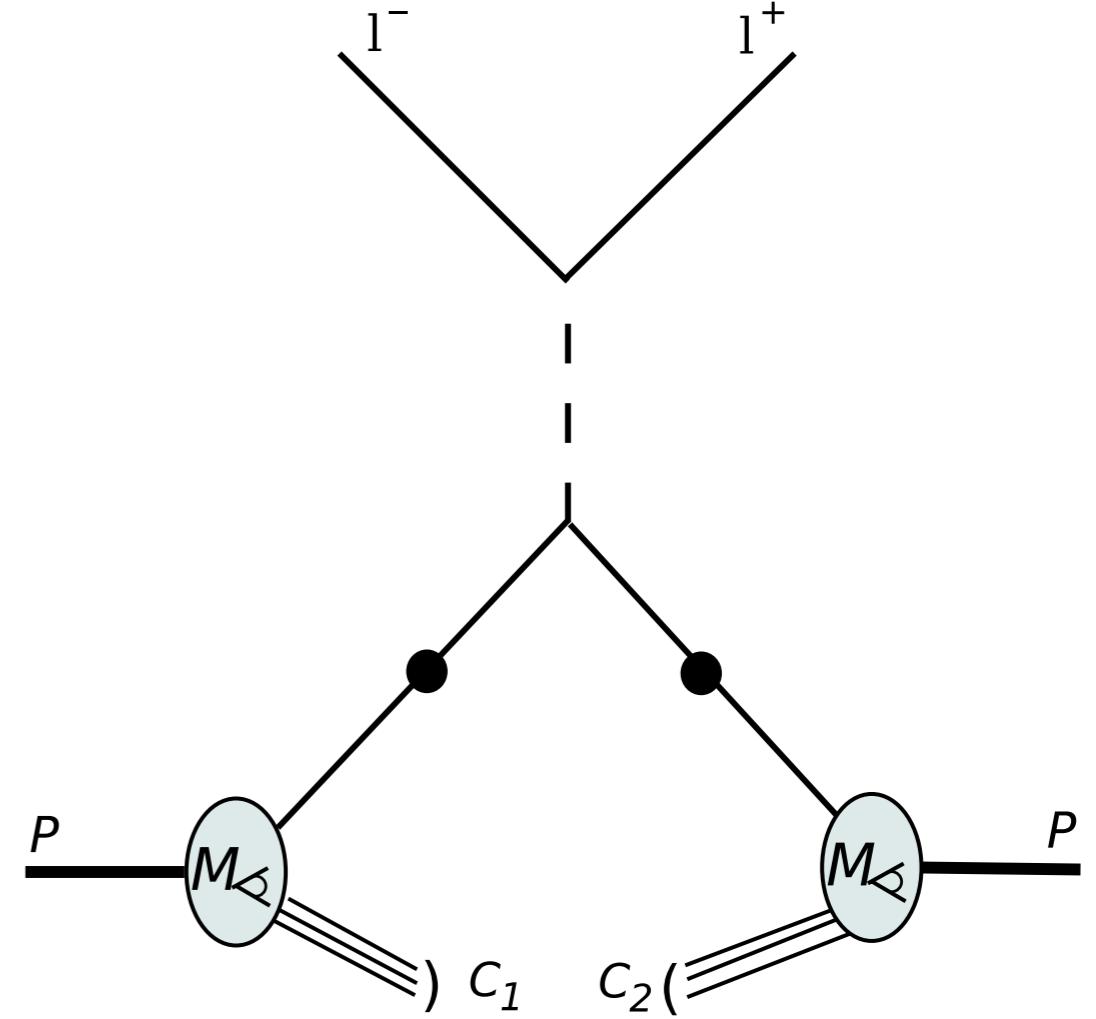
$$P_k^j(u)=P_k^{j\,(0)}(u)+\frac{\alpha_s}{2\pi}P_k^{j\,(1)}(u)+\dots$$

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}_{\triangleleft}^j(x,Q^2,z,t) = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_k^j(u) \mathcal{M}_{\triangleleft}^k(x/u,Q^2,z,t).$$

$$F_2^{(n+1)}(x,Q^2;E_t^2,y_{\rm cut})=\sum_{i=q,\bar q}\int_0^1dz\int_{\frac{x}{1-z}}^1\frac{du}{u}\mathcal{M}_{\triangleleft}^i(x/u,Q^2,z,y_{\rm cut}E_t^2)\,R_{2,i}^{(n)}\Big(u,\alpha_s,\frac{Q^2}{E_t^2},y_{\rm cut}\Big)$$

Drell-Yan

$$p + p \rightarrow C_1 + C_2 + \gamma^* \rightarrow l^+ l^- + X .$$



$$\frac{d\sigma^{DY}}{dt_1 dt_2 dQ^2 dz_1 dz_2} = \sum_{i,j=q,\bar{q}} \int \int dx_1 dx_2 \left(\mathcal{M}_{\triangleleft}^i(x_1, Q^2, z_1, t_1) \mathcal{M}_{\triangleright}^j(x_2, Q^2, z_2, t_2) + i \leftrightarrow j \right) \delta(s - x_1 x_2 Q^2),$$

Conclusions

Alternative QCD based approach to initial state radiation including target remnants

QCD evolution known

NLO and NNLO improvement + coherence

Useful framework to investigate factorization

A potentially useful tool for minimum bias and underlying event

