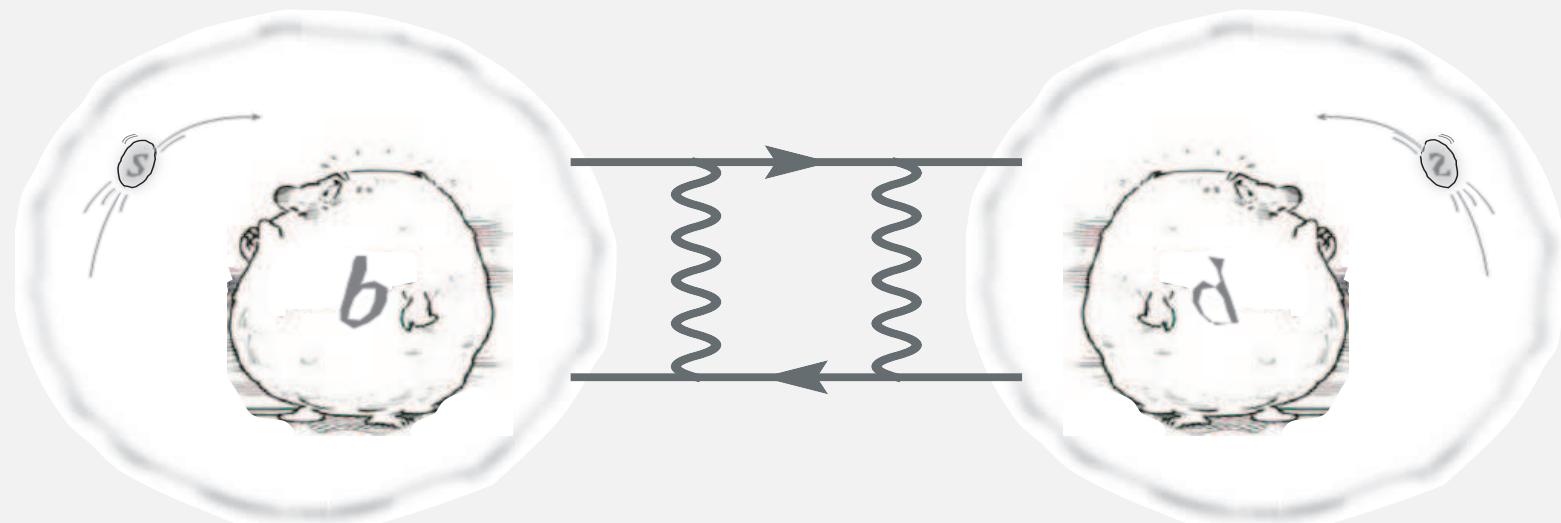


# Factorization and $B_{d,s}$ mixing

Javier Virto

QCD@Work'07, Martina Franca, June 2007



# What this talk is about

**Problem:** The phenomenology of hadronic B-decays is often obscure on the theoretical level because we don't fully understand **QCD**.

- The **Heavy Quark Limit** approach (QCDF) suffers from uncertainties due to  $1/m_b$  suppressed contributions, and other non-factorizable contributions.
- Other approaches based on **Flavor Symmetries** cannot give precise results, due to bad data and poorly estimated SU(3) breaking.

To be able to extract conclusions from experiments, theory must be more **precise** and **reliable**.

**Claim:** The situation can be improved. I show some phenomenological applications of an approach based on a QCDF-inspired quantity:  $\Delta$ .

# Express $B_q - \bar{B}_q$ Mixing

$$|B(t)\rangle = c(t) |B^{\circ}\rangle + \bar{c}(t) |\bar{B}^{\circ}\rangle + \text{decay}$$

$$i\partial_t \begin{pmatrix} c(t) \\ \bar{c}(t) \end{pmatrix} = \mathcal{H}_{\text{eff}} \begin{pmatrix} c(t) \\ \bar{c}(t) \end{pmatrix} \rightsquigarrow \text{OSCILLATIONS}.$$

$$\mathcal{H}_{\text{eff}} = M - \frac{i}{2} \Pi = \begin{pmatrix} M_0 - \frac{i}{2} \Gamma_0 & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* + \frac{i}{2} \Gamma_{12}^* & M_0 - \frac{i}{2} \Gamma_0 \end{pmatrix}$$

hermitian

$$\Rightarrow \begin{cases} |B_L\rangle = \frac{1}{\sqrt{1+|\beta/\rho|^2}} (|B^{\circ}\rangle + \frac{\beta}{\rho} |\bar{B}^{\circ}\rangle) \\ |B_H\rangle = \frac{1}{\sqrt{1+|\beta/\rho|^2}} (|B^{\circ}\rangle - \frac{\beta}{\rho} |\bar{B}^{\circ}\rangle) \end{cases}$$

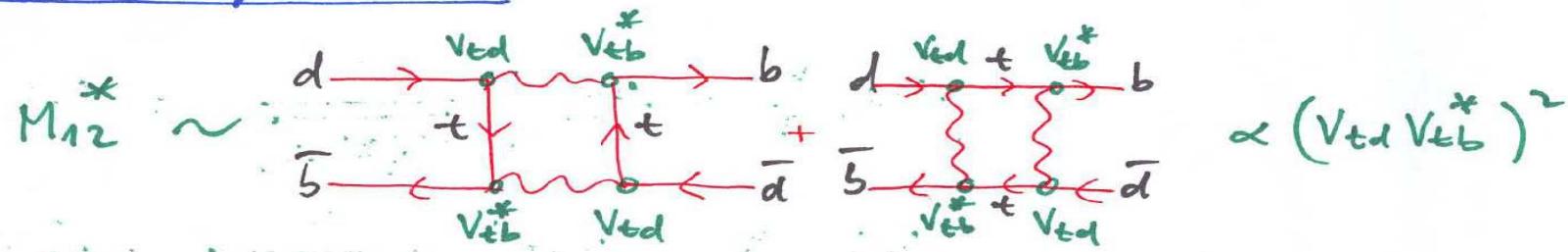
# Express $B_q - \bar{B}_q$ Mixing

Define:  $\frac{q}{p} = \left| \frac{q}{p} \right| e^{-i\phi_M}$  ← Mixing angle.

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\frac{1}{2}\Gamma_{12}^*}{M_{12} - i\frac{1}{2}\Gamma_{12}}} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \Rightarrow \underline{\phi_M = \arg M_{12}}$$

(Although unphysical phases...)

$B_d - \bar{B}_d$  Mixing in SM:



Wolf. Param  $\Rightarrow \arg M_{12}^* = 2 \arg (V_{td} V_{tb}^*) = 2 \arg (V_{td}) = -2\beta$

$$\Rightarrow \underline{\phi_d^{SM} = 2\beta}.$$

# Express $B_q - \bar{B}_q$ Mixing

Measure  $\phi_M$ : CP-asymmetries

$$A_{CP}(t) = \frac{\Gamma(B(t) \rightarrow f_{CP}) - \Gamma(\bar{B}(t) \rightarrow f_{CP})}{\Gamma(B(t) \rightarrow f_{CP}) + \Gamma(\bar{B}(t) \rightarrow f_{CP})} = \dots =$$

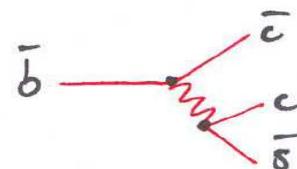
$$= \frac{A_{dir} \cos(\Delta M t) + A_{mix} \sin(\Delta M t)}{\cosh(\Delta \Gamma t/2) - A_{dir} \sinh(\Delta \Gamma t/2)}$$

$\uparrow$   
 -  $f_{CP}$  CP-eigenstate  
 $|g_f\rangle \approx 1$ .

Convention-independent quantity:  $\lambda_f \equiv \frac{q}{p} \cdot \frac{\bar{A}_f}{A_f}$

$$A_{mix} = - \frac{2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|}$$

$B_d \rightarrow J/\psi K_s$



Dominated by single amplitude

$$\Rightarrow \bar{A}_f / A_f \approx \eta_f = -1$$

$$\Rightarrow A_{mix}(B_d \rightarrow J/\psi K_s) \simeq -\sin \phi_M$$

# $\sin 2\beta$ from $b \rightarrow s$ penguins

Penguin - Mediated  $b \rightarrow s$  decays

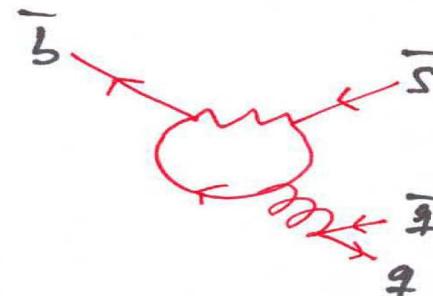
$$A = T \lambda_u^{(s)*} + P \lambda_c^{(s)*} \quad (\text{SM})$$

$$\lambda_u^{(s)*} \equiv V_{ub}^* V_{us}$$

$$\lambda_c^{(s)*} \equiv V_{cb}^* V_{cs}$$

$$\left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \approx 0.022 !!$$

CKM suppressed!



Example:  $B_d \rightarrow \phi K_s$

$= \Delta S$  (small?)

$$-A_{\text{mix}}(B_d \rightarrow \phi K_s) = \sin 2\beta + 2 \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \text{Re}\left(\frac{T}{P}\right) \sin \delta \cos 2\beta$$

$$+ \dots$$

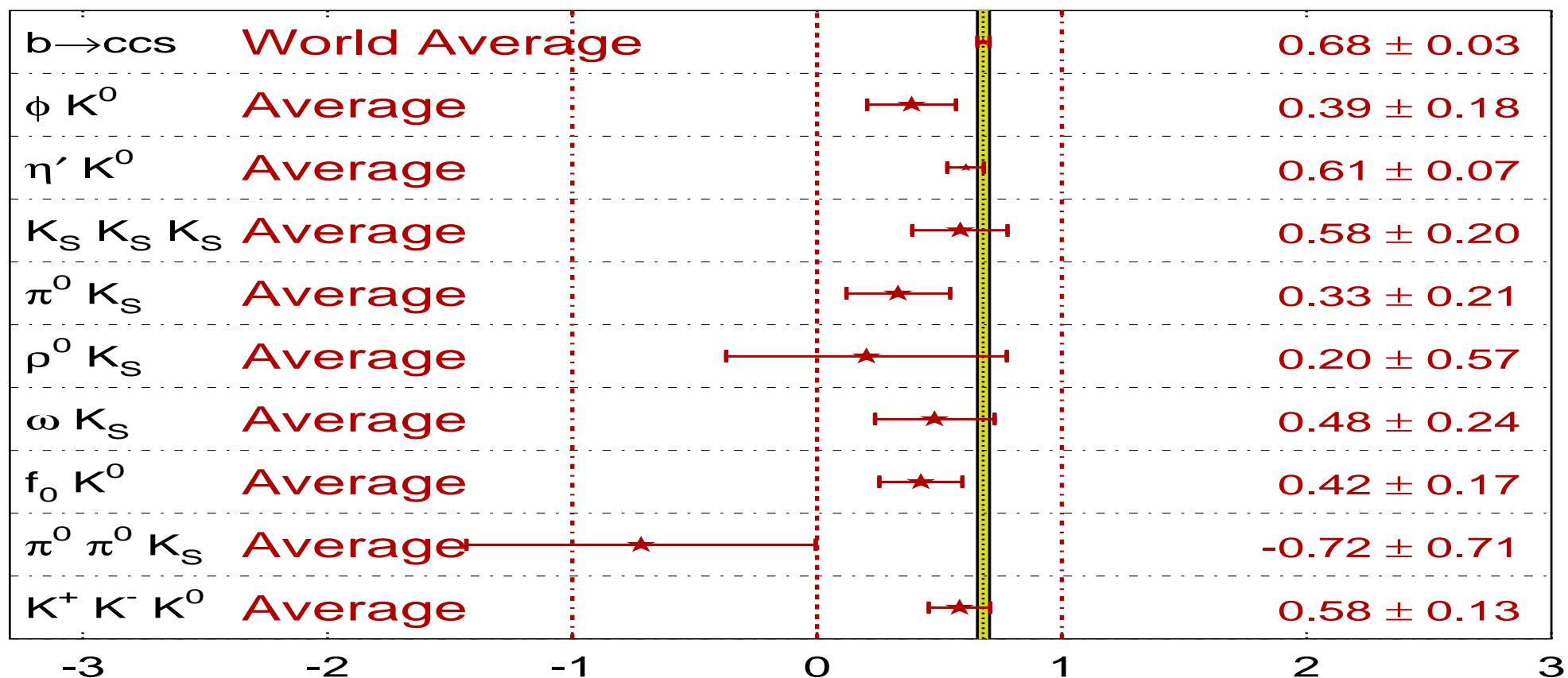
$$\approx \sin 2\beta \quad \text{as long as } T \gg P !!$$

$\Rightarrow$  MUST CONTROL HADRONIC PARS. T & P.

# $\sin 2\beta$ from $b \rightarrow s$ penguins

$$\sin(2\beta^{\text{eff}}) = \sin(2\phi_1^{\text{eff}})$$

**HFAG**  
Moriond 2007  
PRELIMINARY

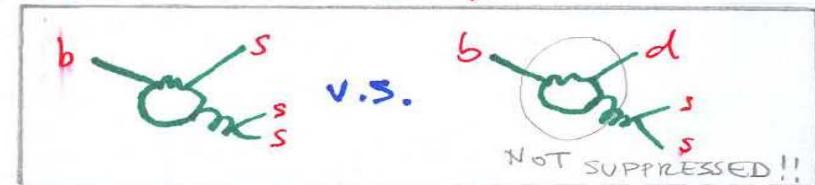


# $\sin 2\beta$ from $b \rightarrow s$ penguins

- \* Grossman, Isidori, Worah '98, Grossman, Ligeti, Nir, Quinn '03  
 $SU(3)$  + Non-cancellation assumption

$$\Delta S_{\phi k_s} < \sqrt{2} \lambda \left( \sqrt{\frac{BR(B^+ \to \phi \pi^+)}{BR(B \to \phi k_s)}} + \sqrt{\frac{BR(B^+ \to K^* K^+)}{BR(B \to \phi k_s)}} \right) + \mathcal{O}(\lambda^2)$$

$$\Rightarrow |\Delta S_{\phi k_s}| \lesssim 0.4$$



- \* Beneke '05  
QCD-Factorization  $\Rightarrow$   $0.01 < \Delta S_{\phi k_s} < 0.05$

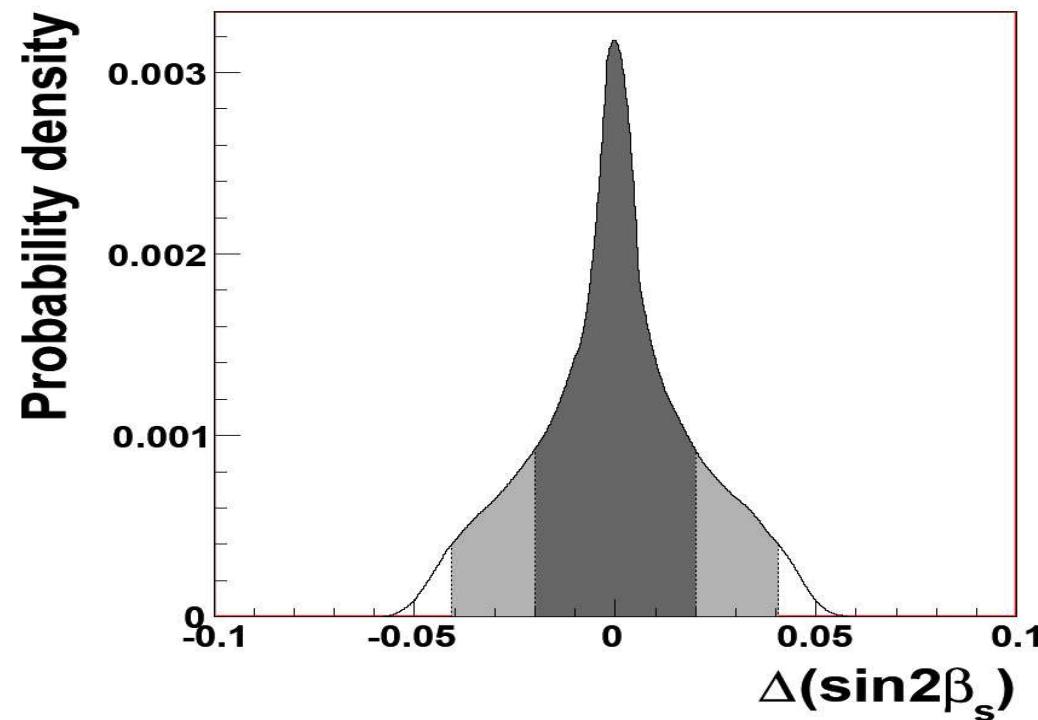
- \* EXPERIMENT :  $\underline{\Delta S_{\phi k_s}^{\text{exp}} = -0.39 \pm 0.20}$

$\Rightarrow$  THEORY IS NOT GOOD ENOUGH YET  $\rightsquigarrow$  We need alternative approaches

# $\sin 2\beta_s$ from $B_s \rightarrow K^* K^*$

- Ciuchini, Pierini, Silvestrini '07

channel	BR	S	C
$B_s \rightarrow K^{*0} \bar{K}^{*0}$	$(11.8 \pm 0.6) 10^{-6}$	$-0.07 \pm 0.02$	$0.01 \pm 0.02$
$B_d \rightarrow K^{*0} \bar{K}^{*0}$	$(5.00 \pm 0.25) 10^{-7}$	$-0.12 \pm 0.02$	$0.13 \pm 0.02$



# B decays in QCDF: A & B Operators

$$A(B_q \rightarrow M_1 M_2) = \sum_{p=u,c} \lambda_p^{(D)} \langle M_1 M_2 | \mathcal{T}_A^p + \mathcal{T}_B^p | B_q \rangle$$

$$\Rightarrow \begin{cases} T_{M_1 M_2}^q &= \langle M_1 M_2 | \mathcal{T}_A^u + \mathcal{T}_B^u | B_q \rangle \\ P_{M_1 M_2}^q &= \langle M_1 M_2 | \mathcal{T}_A^c + \mathcal{T}_B^c | B_q \rangle \end{cases}$$

$$\langle M_1 M_2 | \mathcal{T}_A^p | B_q \rangle = c_1 \delta_{pu} \alpha_1 + c_2 \delta_{pu} \alpha_2 + c_3 \alpha_3^p + c_4 \alpha_4^p + c_5 \alpha_{3,\text{EW}}^p + c_6 \alpha_{4,\text{EW}}^p$$

$$\langle M_1 M_2 | \mathcal{T}_B^p | B_q \rangle = d_1 \delta_{pu} \beta_1 + d_2 \delta_{pu} \beta_2 + d_3 \beta_3^p + d_4 \beta_4^p + d_5 \beta_{3,\text{EW}}^p + d_6 \beta_{4,\text{EW}}^p$$

# B decays in QCDF: A & B Operators

$$A(B_q \rightarrow M_1 M_2) = \sum_{p=u,c} \lambda_p^{(D)} \langle M_1 M_2 | T_A^p + T_B^p | B_q \rangle$$

$$\langle M_1 M_2 | T_A^p | B_q \rangle = c_1 \delta_{pu} \alpha_1 + c_2 \delta_{pu} \alpha_2 + c_3 \alpha_3^p + c_4 \alpha_4^p + c_5 \alpha_{3,\text{EW}}^p + c_6 \alpha_{4,\text{EW}}^p$$

$$\langle M_1 M_2 | T_B^p | B_q \rangle = d_1 \delta_{pu} \beta_1 + d_2 \delta_{pu} \beta_2 + d_3 \beta_3^p + d_4 \beta_4^p + d_5 \beta_{3,\text{EW}}^p + d_6 \beta_{4,\text{EW}}^p$$

Example:  $B_s^0 \rightarrow K^+ K^-$        $\left( A_{KK}^q = \frac{G_F}{\sqrt{2}} M_{B_q}^2 F^{B_q \rightarrow K} f_K \right)$

$$T^{s\pm} = A_{KK}^s (\bar{\alpha}_1 + \bar{\beta}_1 + \bar{\alpha}_4^u + \bar{\alpha}_{4,\text{EW}}^u + \bar{\beta}_3^u + 2\bar{\beta}_4^u - \frac{1}{2}\bar{\beta}_{3,\text{EW}}^u + \frac{1}{2}\bar{\beta}_{4,\text{EW}}^u)$$

$$P^{s\pm} = A_{KK}^s (\bar{\alpha}_4^c + \bar{\alpha}_{4,\text{EW}}^c + \bar{\beta}_3^c + 2\bar{\beta}_4^c - \frac{1}{2}\bar{\beta}_{3,\text{EW}}^c + \frac{1}{2}\bar{\beta}_{4,\text{EW}}^c)$$

# B decays in QCDF: $\alpha$ -coefficients

$$\alpha_i^p(M_1 M_2) = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8}$$
$$+ \text{Diagram 9} + \text{Diagram 10} + \text{Diagram 11} + \text{Diagram 12}$$
$$\supset \int_0^1 \frac{dy}{\bar{y}} \Phi_{m_1}(y) = \Phi_{m_1}(1) X_H^{M_1} + \text{finite}$$

$X_H^{M_1} \rightarrow$  Model dependence

# B decays in QCDF: $\beta$ -coefficients

$$\beta_i^p(M_1 M_2) = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}$$

$$\supset \int_0^1 \frac{dxdy}{\bar{x}y} \Phi_{m_2}(x) \Phi_{m_1}(y)$$

– Divergent subtractions:  $\int_0^1 \frac{dy}{y} \rightarrow X_A^{M_1}, \quad \int_0^1 dy \frac{\ln y}{y} \rightarrow -\frac{1}{2}(X_A^{M_1})^2$

$X_A^{M_1}, X_A^{M_2} \longrightarrow$  Model dependence

# $\Delta$ : A solid quantity in QCDF

- Consider the following quantity:

$$\Delta \equiv T - P$$

→ I.R. divergencies  $X_A, X_H$  CANCEL in  $\Delta$

- For  $B_q \rightarrow K^* \bar{K}^*$ :

$$\begin{cases} |\Delta_{K^*K^*}^d| &= A_{K^*K^*}^{d,0} \frac{C_F \alpha_s}{4\pi N_c} C_1 |\bar{G}_{K^*}(s_c) - \bar{G}_{K^*}(0)| \\ |\Delta_{K^*K^*}^s| &= A_{K^*K^*}^{s,0} \frac{C_F \alpha_s}{4\pi N_c} C_1 |\bar{G}_{K^*}(s_c) - \bar{G}_{K^*}(0)| \end{cases}$$

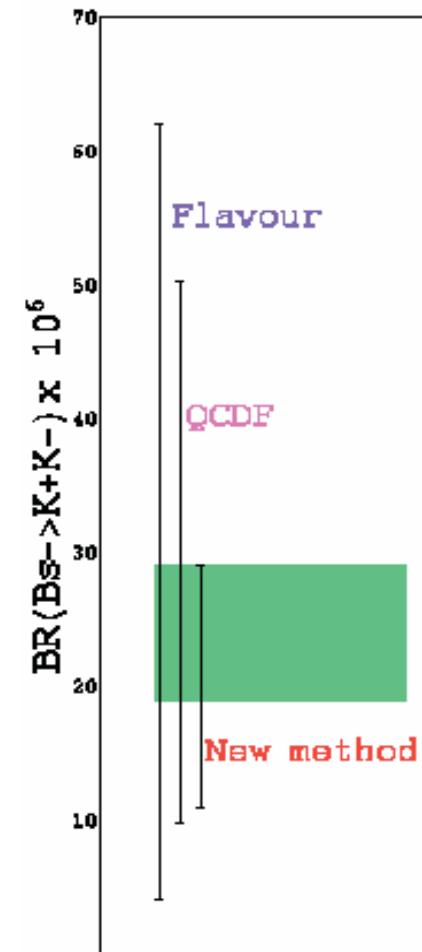
- Including QCDF input uncertainties:

$$|\Delta_{K^*K^*}^d| = (1.85 \pm 0.79) \times 10^{-7} \text{ GeV}$$

$$|\Delta_{K^*K^*}^s| = (1.62 \pm 0.69) \times 10^{-7} \text{ GeV}$$

# An application to $B \rightarrow KK$

- Our method was used to predict BR's and Asymmetries in  $B_s \rightarrow K^+K^-$  and  $B_s \rightarrow K^0\bar{K}^0$ .  
(Descotes-Genon, Matias, Virto, *Phys.Rev.Lett* **97** 061801 (2006))  
The outcome was quite promising.
- SU(3) methods suffer from large experimental uncertainties and cannot estimate SU(3)-breaking.
- QCDF has trouble with chirally enhanced  $1/m_b$  suppressed contributions, which have to be modelled and introduce huge uncertainties.



# Tree and Penguin Contributions

$$A = \lambda_u^{(D)*} \textcolor{red}{T} + \lambda_c^{(D)*} \textcolor{red}{P}, \quad \bar{A} = \lambda_u^{(D)} \textcolor{red}{T} + \lambda_c^{(D)} \textcolor{red}{P}$$

# Tree and Penguin Contributions

$$A = \lambda_u^{(D)*} \textcolor{red}{T} + \lambda_c^{(D)*} \textcolor{red}{P}, \quad \bar{A} = \lambda_u^{(D)} \textcolor{red}{T} + \lambda_c^{(D)} \textcolor{red}{P}$$



$$\textcolor{red}{T} = \textcolor{red}{P} - \Delta$$

$$|A|^2 = |\lambda_c^{(D)*} + \lambda_u^{(D)*}|^2 \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)*}}{\lambda_c^{(D)*} + \lambda_u^{(D)*}} \Delta \right|^2, \quad |\bar{A}|^2 = |\lambda_c^{(D)} + \lambda_u^{(D)}|^2 \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)}}{\lambda_c^{(D)} + \lambda_u^{(D)}} \Delta \right|^2$$

# Tree and Penguin Contributions

$$|A|^2 = |\lambda_c^{(D)*} + \lambda_u^{(D)*}|^2 \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)*}}{\lambda_c^{(D)*} + \lambda_u^{(D)*}} \Delta \right|^2 , \quad |\bar{A}|^2 = |\lambda_c^{(D)} + \lambda_u^{(D)}|^2 \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)}}{\lambda_c^{(D)} + \lambda_u^{(D)}} \Delta \right|^2$$

– But the amplitudes  $|A|^2$ ,  $|\bar{A}|^2$  are related to observables:

$$|A|^2 = BR(1 + \mathcal{A}_{\text{dir}})/g_{PS} , \quad |\bar{A}|^2 = BR(1 - \mathcal{A}_{\text{dir}})/g_{PS}$$

★  $g_{PS}$  → phase-space factor:  $g_{PS}(B_d) \simeq 8.8 \times 10^9 \text{ GeV}^{-2}$   
 $g_{PS}(B_s) \simeq 8.2 \times 10^9 \text{ GeV}^{-2}$

# Tree and Penguin Contributions

$$\frac{BR(1 + \mathcal{A}_{\text{dir}})/g_{PS}}{|\lambda_c^{(D)} + \lambda_u^{(D)}|^2} = \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)*}}{\lambda_c^{(D)*} + \lambda_u^{(D)*}} \Delta \right|^2, \quad \frac{BR(1 - \mathcal{A}_{\text{dir}})/g_{PS}}{|\lambda_c^{(D)} + \lambda_u^{(D)}|^2} = \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)}}{\lambda_c^{(D)} + \lambda_u^{(D)}} \Delta \right|^2$$

# Tree and Penguin Contributions

$$\frac{BR(1 + \mathcal{A}_{\text{dir}})/g_{PS}}{|\lambda_c^{(D)} + \lambda_u^{(D)}|^2} = \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)*}}{\lambda_c^{(D)*} + \lambda_u^{(D)*}} \Delta \right|^2, \quad \frac{BR(1 - \mathcal{A}_{\text{dir}})/g_{PS}}{|\lambda_c^{(D)} + \lambda_u^{(D)}|^2} = \left| \textcolor{red}{P} + \frac{\lambda_u^{(D)}}{\lambda_c^{(D)} + \lambda_u^{(D)}} \Delta \right|^2$$

$\downarrow$        $\downarrow$        $\downarrow$        $\downarrow$

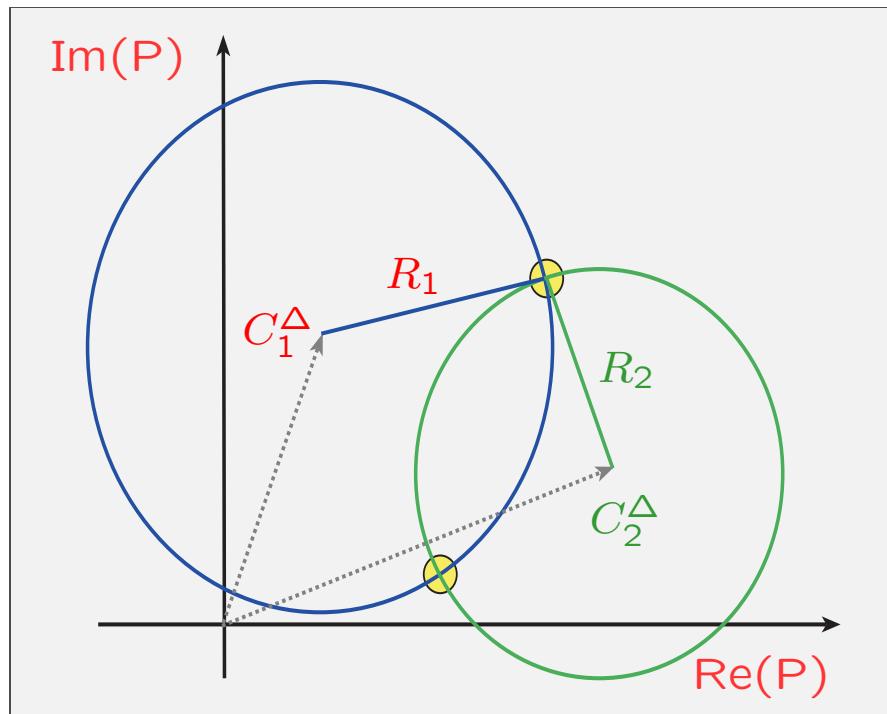
$$R_1^2 \quad -C_1^\Delta \quad R_2^2 \quad -C_2^\Delta$$

$$R_1^2 = |\textcolor{red}{P} - C_1^\Delta|^2, \quad R_2^2 = |\textcolor{red}{P} - C_2^\Delta|^2$$

- $R_i$  → depend on DATA and CKM's
- $C_i^\Delta$  → depend on  $\Delta$  and CKM's

# Tree and Penguin Contributions

$$R_1^2 = |\textcolor{red}{P} - C_1^\Delta|^2, \quad R_2^2 = |\textcolor{red}{P} - C_2^\Delta|^2$$



-Consistency Condition:

$$|R_1 - R_2| \leq |C_1^\Delta - C_2^\Delta| \leq |R_1 + R_2|$$

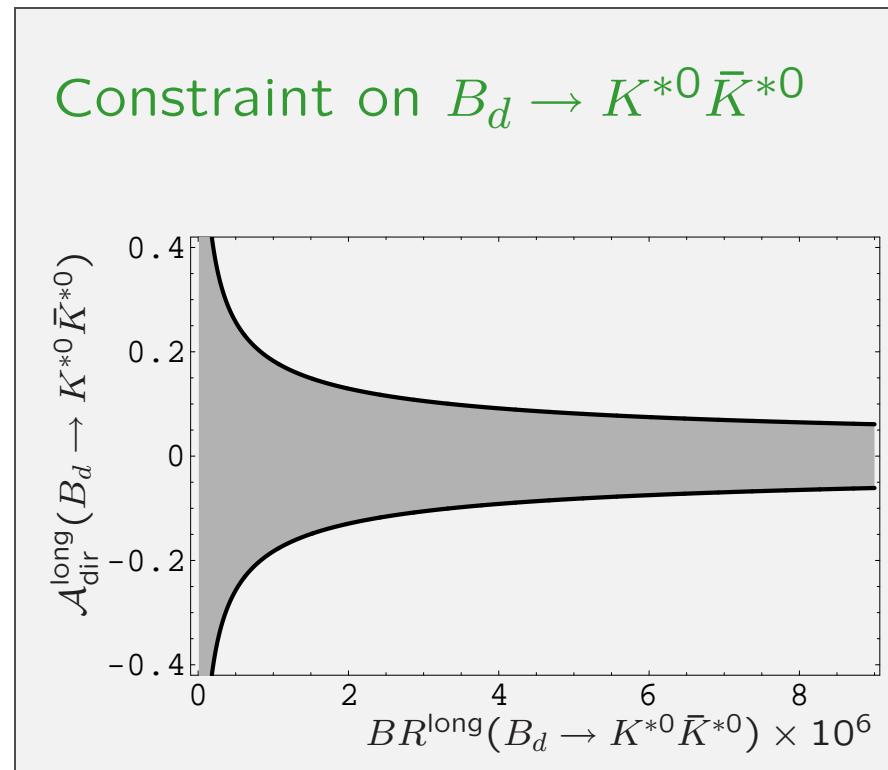
Which translates into:

$$|\mathcal{A}_{\text{dir}}| \leq \sqrt{\frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \left( 2 - \frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \right)}$$

$$\left( \widetilde{BR} \equiv BR/g_{PS}; \quad \mathcal{R}_D \sim \text{CKM's} \right)$$

# Tree and Penguin Contributions

$$R_1^2 = |\textcolor{red}{P} - C_1^\Delta|^2 , \quad R_2^2 = |\textcolor{red}{P} - C_2^\Delta|^2$$



-Consistency Condition:

$$|R_1 - R_2| \leq |C_1^\Delta - C_2^\Delta| \leq |R_1 + R_2|$$

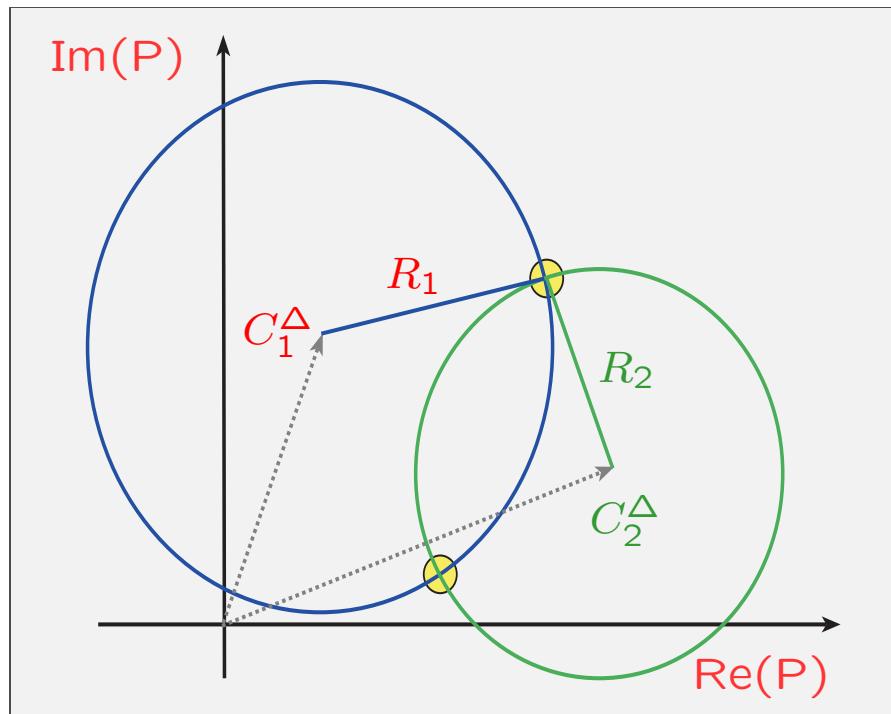
Which translates into:

$$|\mathcal{A}_{\text{dir}}| \leq \sqrt{\frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \left( 2 - \frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \right)}$$

$$\left( \widetilde{BR} \equiv BR/g_{PS} ; \quad \mathcal{R}_D \sim \text{CKM's} \right)$$

# Tree and Penguin Contributions

$$R_1^2 = |\textcolor{red}{P} - C_1^\Delta|^2, \quad R_2^2 = |\textcolor{red}{P} - C_2^\Delta|^2$$



-Hadronic Parameters:

$$\text{Re}[P] = -c_1^{(D)} \Delta \pm \sqrt{-\text{Im}[P]^2 - \left(\frac{c_0^{(D)} \Delta}{c_2^{(D)}}\right)^2 + \frac{\widetilde{BR}}{c_2^{(D)}}}$$

$$\text{Im}[P] = \frac{\widetilde{BR} \mathcal{A}_{\text{dir}}}{2c_0^{(D)} \Delta}$$

$$T = P + \Delta$$

$$\left( \widetilde{BR} \equiv BR/g_{PS}; \quad c_i^{(D)} \sim \text{CKM's} \right)$$

# Tree and Penguin Contributions

## SUMMARY

- Amplitudes:  $A_{SM}(B_q \rightarrow M_1 M_2) = \lambda_u^{(D)*} \textcolor{red}{T} + \lambda_c^{(D)*} \textcolor{red}{P}$
- IR-safe quantity:  $\Delta \equiv \textcolor{blue}{T} - \textcolor{red}{P}$
- Consistency condition:  $|\mathcal{A}_{\text{dir}}| \leq \sqrt{\frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \left( 2 - \frac{\mathcal{R}_D^2 \Delta^2}{2\widetilde{BR}} \right)}$
- Hadronic Parameters:

$$\begin{aligned} \text{Re}[\textcolor{red}{P}] &= -\textcolor{green}{c}_1^{(D)} \Delta \pm \sqrt{-\text{Im}[P]^2 - \left(\frac{c_0^{(D)} \Delta}{c_2^{(D)}}\right)^2 + \frac{\widetilde{BR}}{c_2^{(D)}}} \\ \text{Im}[\textcolor{red}{P}] &= \frac{\widetilde{BR} \mathcal{A}_{\text{dir}}}{2c_0^{(D)} \Delta} \end{aligned} ; \quad \textcolor{red}{T} = \textcolor{red}{P} + \Delta$$

## $\sin 2\beta$ from $B \rightarrow \phi K_s$

$$\Delta S_{\phi K_s} = 2 \left| \frac{\lambda_u^{cs}}{\lambda_c^{cs}} \right| \operatorname{Re} \left( \frac{T_{\phi K_s}}{P_{\phi K_s}} \right) \sin \gamma \sin 2\beta \lesssim 0.044 \operatorname{Re} \left( \frac{T_{\phi K_s}}{P_{\phi K_s}} \right)$$

- FROM  $\operatorname{BR}(B \rightarrow \phi K_s) + \Delta_{\phi K_s} \rightsquigarrow \underline{T_{\phi K_s}, P_{\phi K_s}}$

We get :

$$\operatorname{Re} \left( \frac{T}{P} \right) \leq 1 + \left( -0.011 + \sqrt{-3.62 \cdot 10^{-4} + 612 \tilde{\operatorname{BR}} / \Delta^2} \right)^{-1}$$

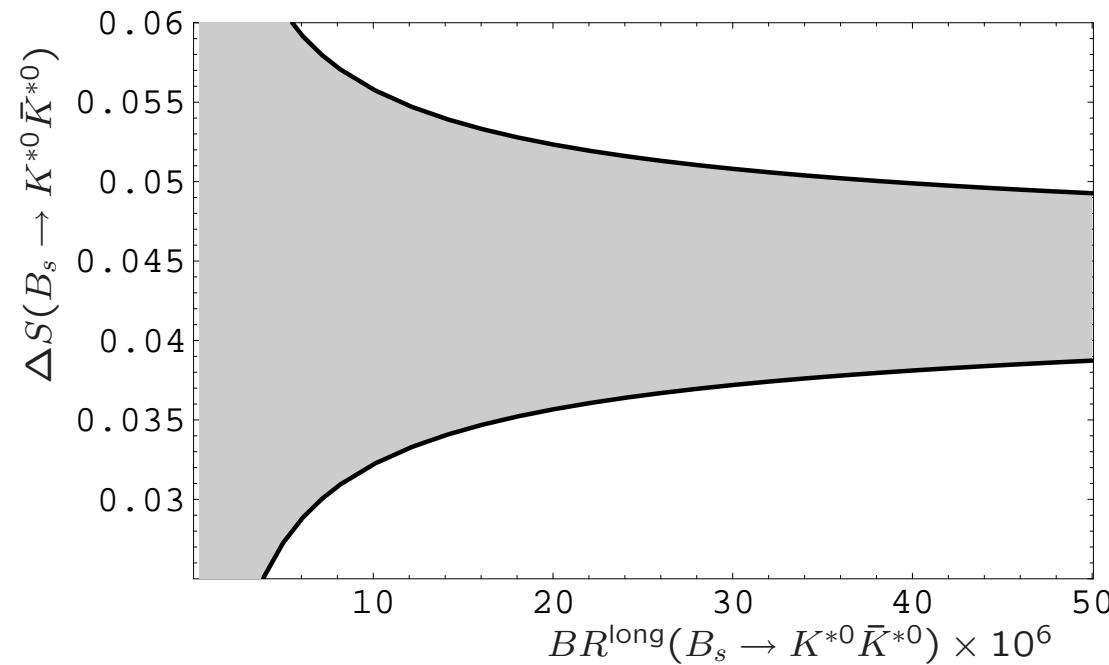
$$\operatorname{Re} \left( \frac{T}{P} \right) \geq 1 + \left( -0.011 - \sqrt{-3.62 \cdot 10^{-4} + 612 \tilde{\operatorname{BR}} / \Delta^2} \right)^{-1}$$

- $\operatorname{BR}(B \rightarrow \phi K_s) = 8.3_{-1.0}^{+1.2} \cdot 10^{-6}$
  - $\Delta_{\phi K_s} = (2.29 \pm 0.67) \cdot 10^{-7} \text{ GeV}$
- $\left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow$

$0.03 < \Delta S_{\phi K_s} < 0.06$

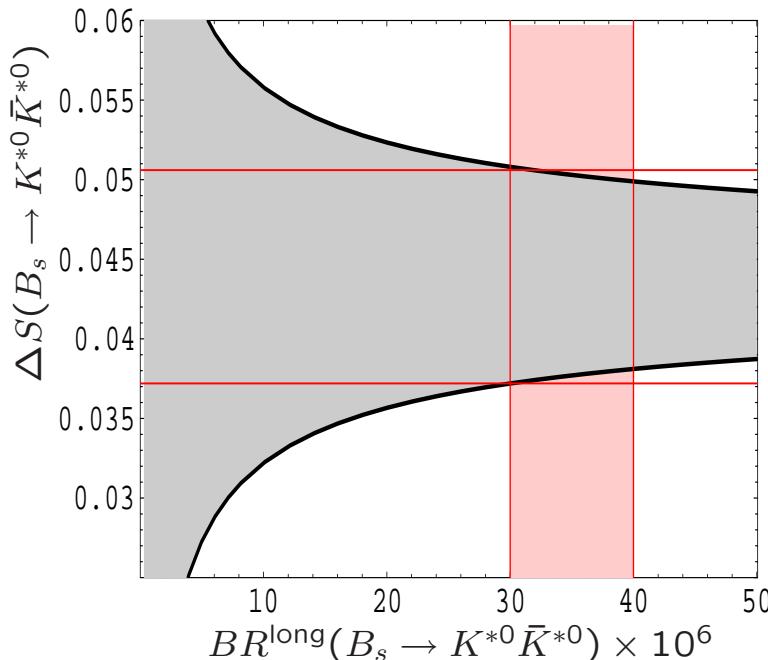
# $\phi_s$ from $B_s \rightarrow VV$

$$\mathcal{A}_{\text{mix}}^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0}) \simeq \underbrace{\sin \phi_s + 2 \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \text{Re} \left( \frac{T_{K^* K^*}^s}{P_{K^* K^*}^s} \right) \sin \gamma \cos \phi_s}_{\Delta S(B_s \rightarrow K^{*0} \bar{K}^{*0})} + \dots$$



# $\phi_s$ from $B_s \rightarrow VV$

$$\mathcal{A}_{\text{mix}}^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0}) \simeq \underbrace{\sin \phi_s + 2 \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \text{Re} \left( \frac{T_{K^* K^*}^s}{P_{K^* K^*}^s} \right) \sin \gamma \cos \phi_s}_{\Delta S(B_s \rightarrow K^{*0} \bar{K}^{*0})} + \dots$$



Example:

For  $BR^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0}) \sim (30 - 40) \times 10^{-6}$

Then

$$(\mathcal{A}_{\text{mix}}^{\text{long}} - 0.051) < \sin \phi_s < (\mathcal{A}_{\text{mix}}^{\text{long}} - 0.037)$$

# Other angles from data & $\Delta$

In the case of a  $B_d$  meson decaying through a  $b \rightarrow D$  process ( $D = d, s$ ):

$$\begin{aligned}\sin^2 \alpha &= \frac{\widetilde{BR}}{2|\lambda_u^{(D)}|^2 |\Delta|^2} \left( 1 - \sqrt{1 - (\mathcal{A}_{\text{dir}})^2 - (\mathcal{A}_{\text{mix}})^2} \right) \\ \sin^2 \beta &= \frac{\widetilde{BR}}{2|\lambda_c^{(D)}|^2 |\Delta|^2} \left( 1 - \sqrt{1 - (\mathcal{A}_{\text{dir}})^2 - (\mathcal{A}_{\text{mix}})^2} \right)\end{aligned}$$

In the case of a  $B_s$  meson decaying through a  $b \rightarrow D$  process ( $D = d, s$ ):

$$\begin{aligned}\sin^2 \frac{\phi_s}{2} &= \frac{\widetilde{BR}}{2|\lambda_c^{(D)}|^2 |\Delta|^2} \left( 1 - \sqrt{1 - (\mathcal{A}_{\text{dir}})^2 - (\mathcal{A}_{\text{mix}})^2} \right) \\ \sin^2 \left( \frac{\phi_s}{2} + \gamma \right) &= \frac{\widetilde{BR}}{2|\lambda_u^{(D)}|^2 |\Delta|^2} \left( 1 - \sqrt{1 - (\mathcal{A}_{\text{dir}})^2 - (\mathcal{A}_{\text{mix}})^2} \right)\end{aligned}$$

# Other angles from data & $\Delta$

**Even better:** Measure the time-dependent “untagged” rate

$$\Gamma^{\text{long}}(B_s(t) \rightarrow VV) + \Gamma^{\text{long}}(\overline{B}_s(t) \rightarrow VV) \propto R_H e^{-\Gamma_H^{(s)} t} + R_L e^{-\Gamma_L^{(s)} t}$$

Which allows to extract  $\mathcal{A}_{\Delta\Gamma}^{\text{long}}$ :

$$\mathcal{A}_{\Delta\Gamma}^{\text{long}}(B_s \rightarrow VV) = \frac{R_H - R_L}{R_H + R_L}$$

$$\sin^2 \frac{\phi_s}{2} = \frac{\widetilde{BR} (1 - \mathcal{A}_{\Delta\Gamma}^{\text{long}})}{2|\lambda_c^{(D)}|^2 |\Delta|^2} ; \quad \sin^2 \left( \frac{\phi_s}{2} + \gamma \right) = \frac{\widetilde{BR} (1 - \mathcal{A}_{\Delta\Gamma}^{\text{long}})}{2|\lambda_u^{(D)}|^2 |\Delta|^2}$$

# $B_s \rightarrow K^* \bar{K}^*$ observables

- Assume no NP in  $B_d \rightarrow K^{*0} \bar{K}^{*0}$ .
- Extract  $P_{K^* K^*}^d$ ,  $T_{K^* K^*}^d$  from  $BR_{K^* K^*}^{d, \text{long}}$ ,  $A_{\text{dir}, K^* K^*}^{d, \text{long}}$  and  $\Delta_{K^* K^*}^d$ .
- Relate  $B_s \rightarrow K^{*0} \bar{K}^{*0}$  to  $B_d \rightarrow K^{*0} \bar{K}^{*0}$  by U-spin:

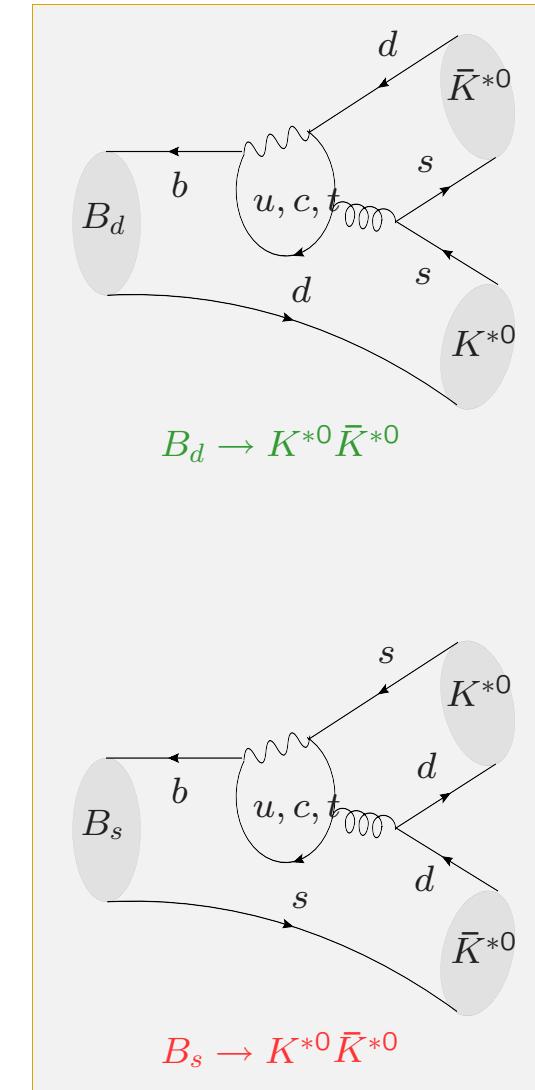
$$P_{K^* K^*}^s = f P_{K^* K^*}^d (1 + \delta_{K^* K^*}^P)$$

$$T_{K^* K^*}^s = f T_{K^* K^*}^d (1 + \delta_{K^* K^*}^T)$$

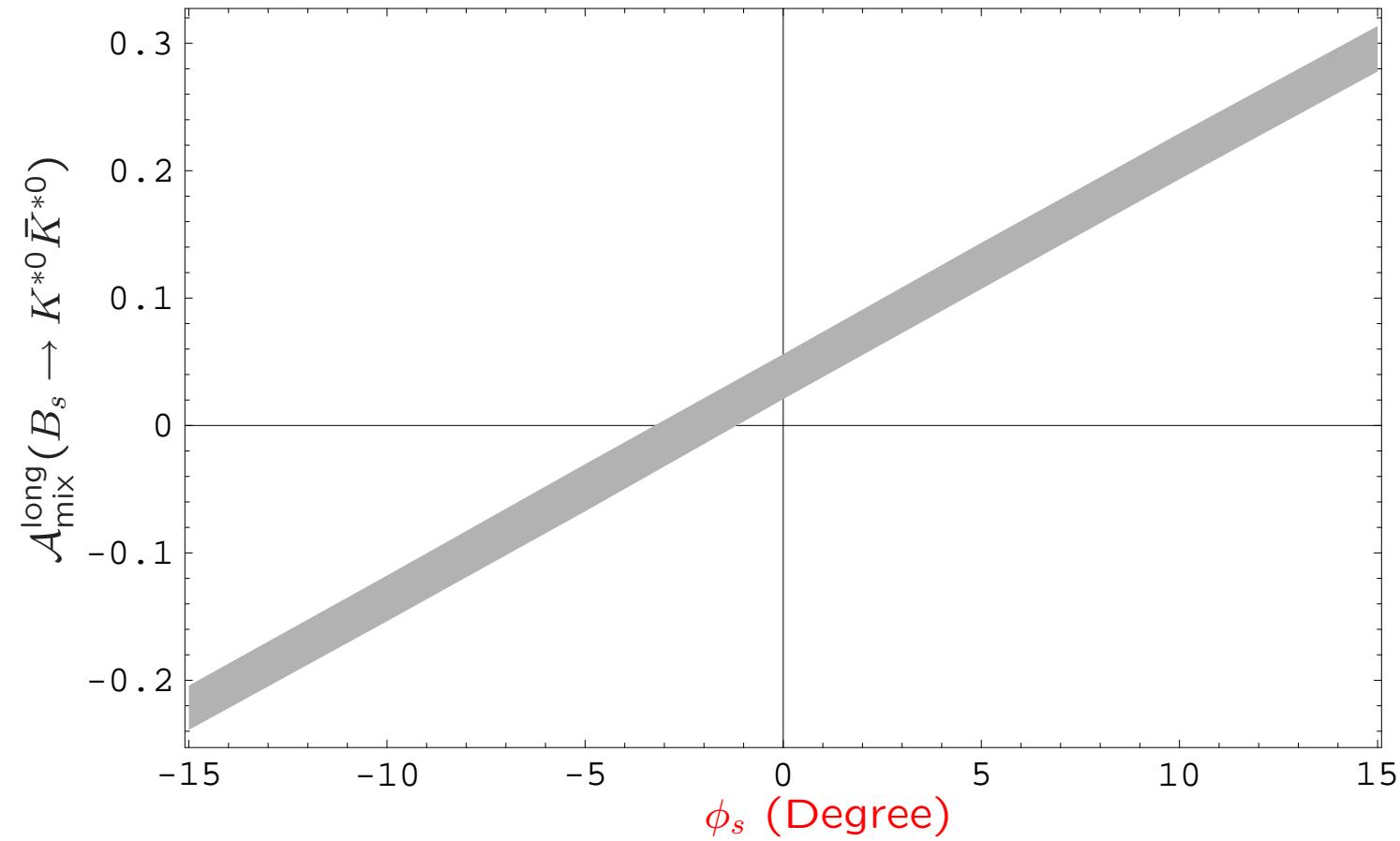
Factorizable SU(3) : (lattice)  $f = \frac{m_{B_s}^2 A_0^{B_s \rightarrow K^*}}{m_B^2 A_0^{B \rightarrow K^*}} = 0.88 \pm 0.19$

Non-Factorizable SU(3) : (QCDF)  $|\delta_{K^* K^*}^P| \leq 0.12$ ,  $|\delta_{K^* K^*}^T| \leq 0.15$

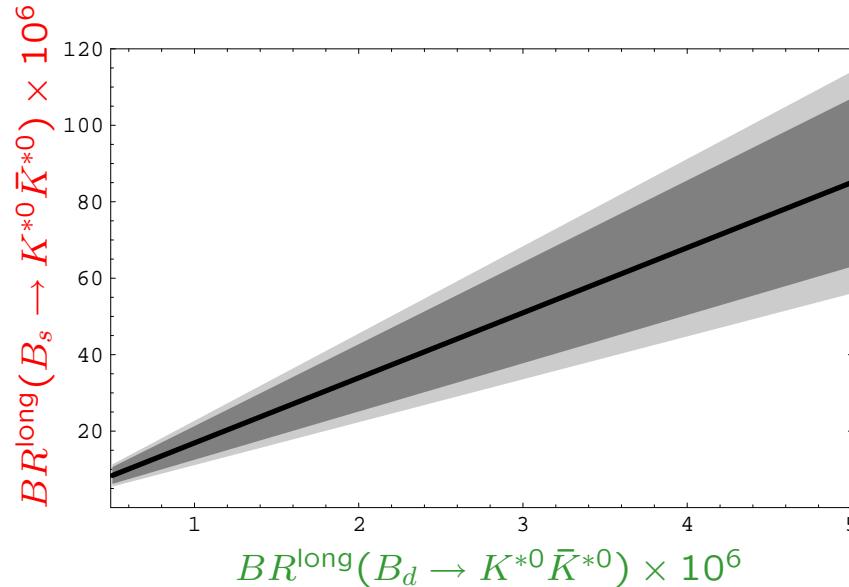
- Compute observables for  $B_s \rightarrow K^{*0} \bar{K}^{*0}$  as a function of  $\phi_s$ .



# $B_s \rightarrow K^* \bar{K}^*$ observables



# $B_s \rightarrow K^* \bar{K}^*$ observables



Results within SM, obtained taking

$$\gamma = 62 \pm 6$$

$$\phi_s = -2^\circ$$

- Central values
- Input uncertainties
- Error from  $f$

$$\left( \frac{BR^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0})}{BR^{\text{long}}(B_d \rightarrow K^{*0} \bar{K}^{*0})} \right)_{SM} = 17 \pm 6$$

$$\mathcal{A}_{\text{dir}}^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0})_{SM} = 0.000 \pm 0.014$$

$$\mathcal{A}_{\text{mix}}^{\text{long}}(B_s \rightarrow K^{*0} \bar{K}^{*0})_{SM} = 0.004 \pm 0.018$$

# To Summarize...

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- ★ An approach based on the QCDF-inspired quantity  $\Delta$  improves precision of **Flavor Symmetries** and reliability of **QCDF**:
  - $B_s \rightarrow KK$  modes
  - $B_d$  mixing:  $\sin 2\beta$
- ★  $B_s - \bar{B}_s$  mixing: The hope for a clear NP signal in flavor physics.
- ★ Longitudinal observables in penguin-mediated  $B_s \rightarrow VV$  decays allow for nice ways of extracting  $\phi_s$ .
- ★ In this talk I have presented the phenomenological implications of  $\Delta$  for  $B_{d,s}$  mixing.

Ass slide