

AdS/QCD Correspondence:

Novel Approach to

Old Problems

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Logic of the talk

2.

Traditionally: - Lattice
 - Sum rules

Common: confinement \Leftrightarrow soft fields
 much success

Both have problems with short distances

- in lattice measurements with high resolution ($a \rightarrow 0$) non-pert. fields are squeezed into "vacuum cracks" of high intensity

- a short-distance power correction is needed in sum rules

AdS/QCD might provide with

a language, if not solutions

Sum rules

$$\int \exp(iq \cdot x) \langle j(x), j(0) \rangle d^4x = f(Q^2)$$

$$\int ds [\text{Im} f(s)] \exp(-s/M^2) =$$

$$= (\text{parton model}) (1 + a_1 d_S(M^2) + \frac{\langle G^2 \rangle}{M^4} C_G^+)$$

Sum rules \equiv short pert series
+ power corrections

Power corrections \Rightarrow soft "fields"
(confinement)

Analysis: a few sources of 1983
-1998
power corrections

the actual confinement-related
power correction **is missing**

Quadratic correction

$$\int [\text{Im } f_i(s)] \exp(-s/M^2) ds$$

$$= (\text{parton model}) \left(1 + a_j d_j(M^2) + b_j \frac{\Lambda_{\text{QCD}}^2}{M^2} + \dots \right)$$

Change in philosophy:

- $\frac{\Lambda_{\text{QCD}}^2}{M^2}$ comes from short distances
(or "mixed" $(M \cdot \Lambda_{\text{QCD}})^{1/4}$)

- $\frac{\Lambda_{\text{QCD}}^2}{M^2}$, in principle, is calculable perturbatively

- $\frac{\Lambda_{\text{QCD}}^2}{M^2}$ can be traded for or dual to
a long pert. series

$$\frac{\Lambda_{\text{QCD}}^2}{M^2} \iff \sum_1^{N_2} a_n^j d_5^n(M^2)$$

$$N_2 \sim \ln M^2$$

Gluon condensate

Field theory:

measured directly

$$\langle (G_{\mu\nu}^a)^2 \rangle = \frac{\text{const}}{a^4} \left[1 + \sum_{n=1}^N d_S^n a_n + (\text{rest})_N \right]$$

lattice spacing ($1/a = \Lambda_{UV}$)

If $N \leq 10$

$$(\text{rest})_N \sim (\Lambda_{QCD} \cdot a)^2$$

Di Giacomo
Parma group
Rakow '06

Once $N \sim 16$ (!)

$$(\text{rest})_N \sim (\Lambda_{QCD} \cdot a)^4$$

and one recovers the standard
"gluon condensate"

No factorial growth of the
expansion coefficients (so far)

No infrared renormalon

Short-distance $V_{Q\bar{Q}}(R)$

Traditional "sum-rules like":

$$\lim_{R \rightarrow 0} V_{Q\bar{Q}}(R) \approx \frac{\text{const}}{R} + (\text{const}) \Lambda_{\text{QCD}}^3 R^2$$

('78-'98

Voloshin
-Lentwiler,
Balitsky

'Novel' power correction:

$$\lim_{R \rightarrow 0} V_{Q\bar{Q}}(R) \approx \frac{\text{const}}{R} + \text{const}' \Lambda_{\text{QCD}}^2 R^{\frac{1}{2}}$$

Akhoury
v.3. '97

Dual representation: long pert. series

+ no power correction

$$\lim_{R \rightarrow 0} V_{Q\bar{Q}}(R) \simeq \frac{1}{R} \sum_1^N d_s^n Q_n + \text{no power corr.}$$

References
in Vairo's talk

YM vacuum, lattice

~ '95 - '03

7.

Confining fields:

closed 2d surfaces, "thin"

$$\underline{(a \cdot \Lambda_{QCD})^2} (\text{Area}) = (\text{const}) \Lambda_{QCD}^2 V_{\text{tot}}$$

$$(\text{Action}) = (\text{const})' \frac{(\text{Area})}{a^2}$$

ITEP
group

a^2 — lattice spacing

Hierarchy of 'gluon condensates':

$$\langle G^2 \rangle_{\text{pert}} \sim \frac{1}{a^4} \quad \text{pert. th.}$$

$$\text{observed} \rightarrow \langle G^2 \rangle_{\text{surfaces}} \sim \frac{1}{a^4} (a \cdot \Lambda_{QCD})^2$$

$$\text{conjectured} \rightarrow \langle G^2 \rangle_{\text{roft}} \sim \frac{1}{a^4} (a \cdot \Lambda_{QCD})^4$$

Vortices also provide linear potentials

$$\left[V_{Q\bar{Q}}(R) \right]_{\text{vortices}} \sim \sigma \cdot R$$

at all distances

Summary on field th. side

- Sum rules need Λ_{QCD}^2 correction,
associated with short distances
- no simple parameterization for Λ_{QCD}^2
but hint for duality:
 $(\Lambda_{QCD} a)^2 \Leftrightarrow$ long pert. series
- Lattice identifies thin vortices
as confining fields
- the vortices do reproduce Λ_{QCD}^2 terms
in $\langle G^2 \rangle$, $V_{Q\bar{Q}}(r)$
in agreement with the conclusions above
- the price for explanation of Λ_{QCD}^2
is very high: the very object
(2d surfaces, thin) is foreign to
field-theory language

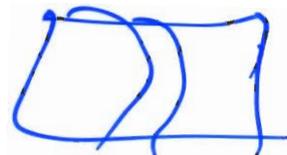
AdS/QCD correspondence

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for references
see review

defined, for our purposes, in terms of electric and magnetic strings (thin) which can be open or Wilson, 't Hooft lines, respectively.

$$\langle W \rangle \sim \sum_{A_c} \exp(-A_c f(A_c))$$

$$\langle H \rangle \sim \sum_{A_H} \exp(-A_H \tilde{f}(A_H))$$



where A_c , A_H are area of surfaces, spanned on the loops,

f, \tilde{f} - weight functions

The idea is borrowed from Z_2 gauge theory

where $f, \tilde{f} = \text{constants}$.

In non-Abelian case, f, \tilde{f} are determined heuristically (model dependent)

Running string tension

Tension depends on the length of string:

(no short strings
in 4d)

$$\sigma(\ell) \sim \frac{1}{\ell^2}, \ell \ll \Lambda_{QCD}^{-1}$$

$$\sigma(\ell) \sim \Lambda_{QCD}^2, \ell \sim \Lambda_{QCD}^{-1}$$

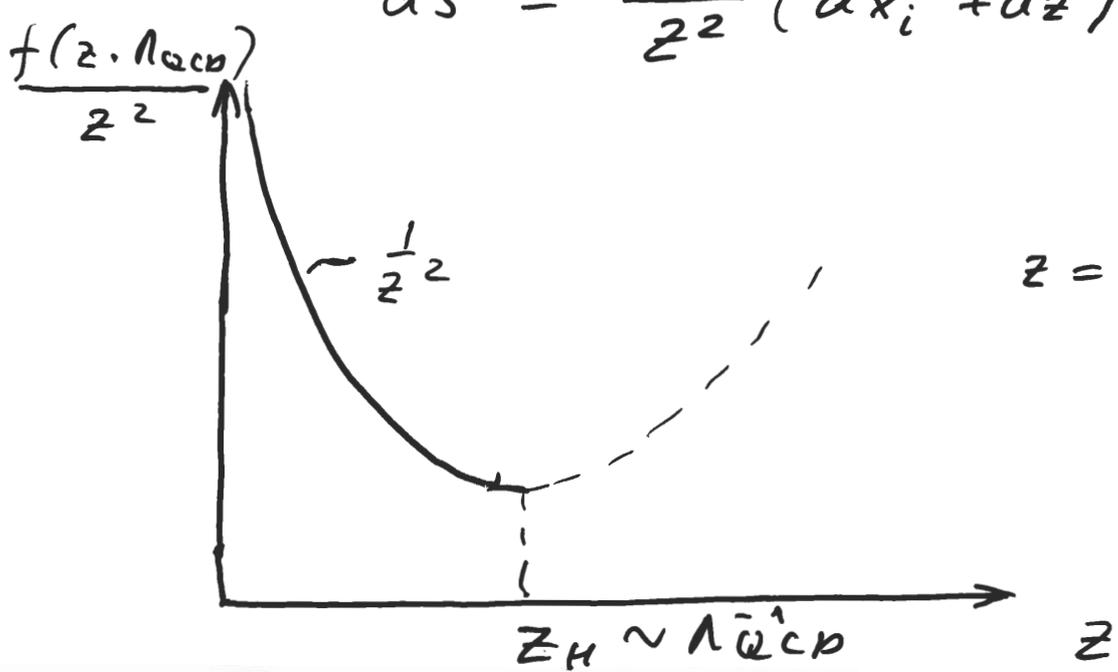
("horizon")

$$\sigma(\ell) \sim \Lambda_{QCD}^2, \ell \gg \Lambda_{QCD}^{-1}$$

Geometrical language:

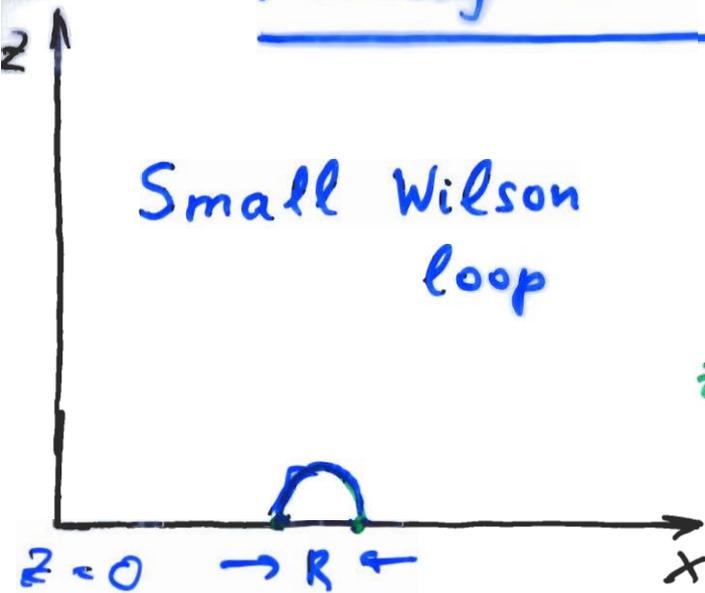
- trade ℓ for fifth coordinate, z
- calculate Nambu-Goto with metric

$$ds^2 = \frac{const}{z^2} (dx_i^2 + dz^2) f(z \cdot \Lambda_{QCD})$$

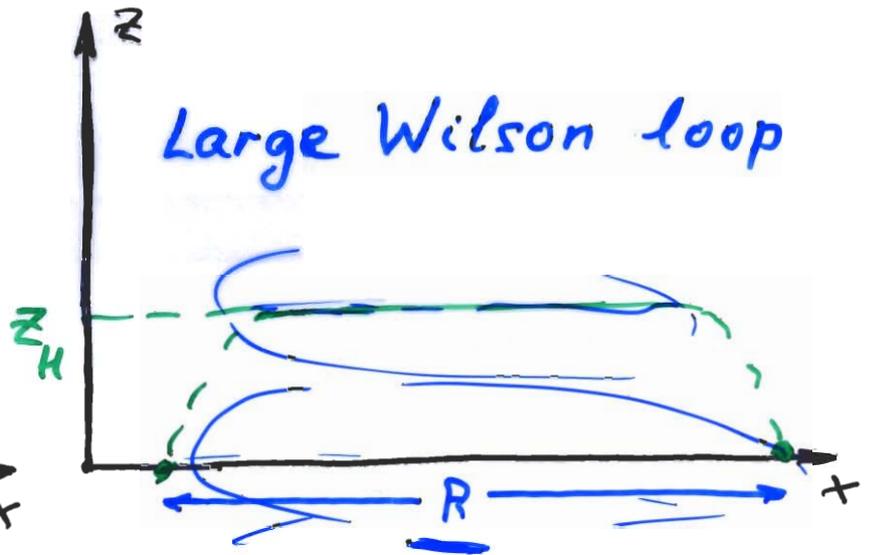


$z = 0$ is "our" 4d

Fixing the model



Small Wilson loop



Large Wilson loop

$$V_{Q\bar{Q}}(R) \sim \frac{\text{const}}{R} \\ + (\text{self energy})$$

$$V_{Q\bar{Q}}(R) \sim \sigma_H R \\ + (\text{self energy})$$

There are infinitely many metrics which have the same limits $R \rightarrow 0$, $R \rightarrow \infty$

To fix the model one can trade the Cornell potential for the metric

$$ds^2 = \frac{\exp(cz^2)}{z^2} (dx_i^2 + dz^2) \cdot \text{const}$$

Produces linear piece $\sigma \cdot R$ from $R \rightarrow 0$

The metric was actually introduced by O. Andreev and P. Son et al. to reproduce Regge trajectories, ($c \simeq 0.9 \text{ GeV}^2$)

Back to our problems

most important, language is emerging to discuss "thin vortices"

They seem to be "magnetic strings", closed in vacuum, can be open on 't Hooft line

[The original definition is in (very) specific lattice language; one should look backwards and check that the lattice phenomenology fits continuum-theory magnetic strings]

Basic geometrical structures might be (?)

the same in three different approaches.

- string living in extra dimensions

- string-like solutions in SUSY gauge th

- lattice phenomenology.

Much more to be done

$\langle G^2 \rangle$ and AdS/QCD

Metric encodes now Λ_{QCD}^2 correction at short distances:

$$\lim_{z^2 \rightarrow 0} \frac{\exp(c z^2)}{z^2} \approx \frac{1}{z^2} + c \quad \left. \begin{array}{l} \downarrow \\ \sim \Lambda_{\text{QCD}}^2 \end{array} \right\}$$

great relief: gauge-inv. language for Λ_{QCD}^2 term (replaces, say, "gluon mass")

Small-size Wilson loop is related to $\langle G^2 \rangle$

$$\langle G^2 \rangle_{\text{AdS/QCD}}$$

$$\sim \underbrace{\frac{\text{const}}{a^4}}_{\text{pure pert.}} + \underbrace{(\text{const})' \frac{1}{a^2} \Lambda_{\text{QCD}}^2}_{\text{imitates 'strings'}} + \underbrace{(0.010) \text{GeV}^4}_{\text{"standard"}}$$

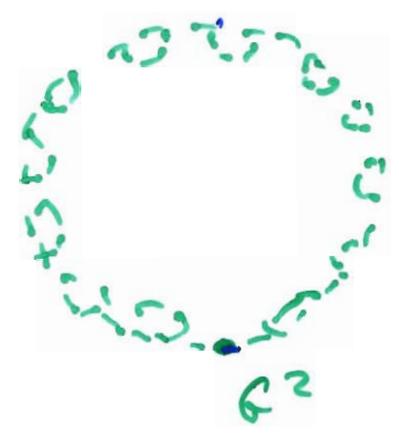
$\langle G^2 \rangle \sim \Lambda_{\text{QCD}}^4$ piece

The model produces

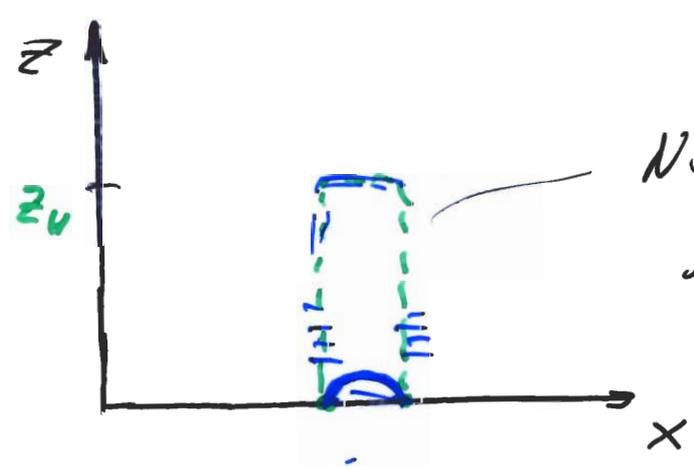
"short pert. series + power corrections"
relates Cornell potential to $\langle G^2 \rangle \sim \Lambda_{\text{QCD}}^4$

AdS/QCD and IR renormalon

In field theory, IR renormalons are associated with pert. propagator travelling far off, $r \sim \Lambda_{QCD}^{-2}$



In stringy picture, $\langle G^2 \rangle \sim \Lambda_{QCD}^4$ piece is not due to surfaces reaching z_k :

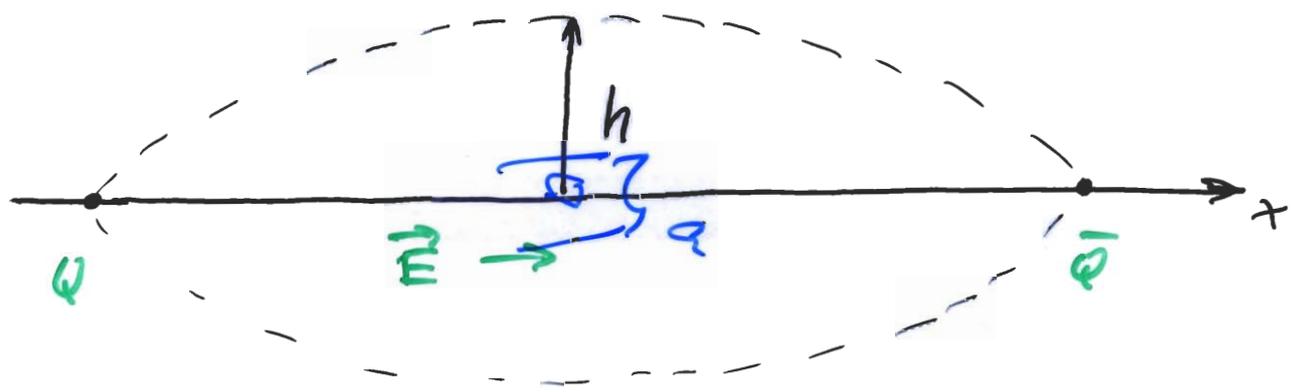


No such contribution, strongly suppressed

But due to second-order short-distance effect: fits nicely absence of IR renormalon in pert. series

see below

Confining string



Standard picture:

$$\langle G^2 \rangle_w - \langle G^2 \rangle_0 \sim - \Lambda_{QCD}^4$$

$$h \sim \Lambda_{QCD}^{-1} \ln R_{q\bar{q}}$$

Luscher et al. '81

$\langle G^2 \rangle_w$ measured directly through

$$\frac{\langle W(C), (Plaquette) \rangle_{\text{connected}}}{\langle W(C) \rangle \langle (Plaquette) \rangle}$$

Recent measurement indicates

Gubarev et al. '07

if $a \rightarrow 0$ $h \sim a \cdot \ln R$

lattice spacing

Correlator of two cocentric Wilson loops

$$R_2 \gg \Lambda_{QCD}^{-1} \gg R_1 \quad (\text{limitate plaquette})$$

Expectation:



$$\{ \langle G^2 \rangle_0 \gg \Lambda_{QCD}^4$$

$$\frac{\langle W(R_2), W(R_1) \rangle_{\text{connected}}}{\langle W(R_2) \rangle \langle W(R_1) \rangle} \sim (\Lambda_{QCD} R_1)^d \sim$$

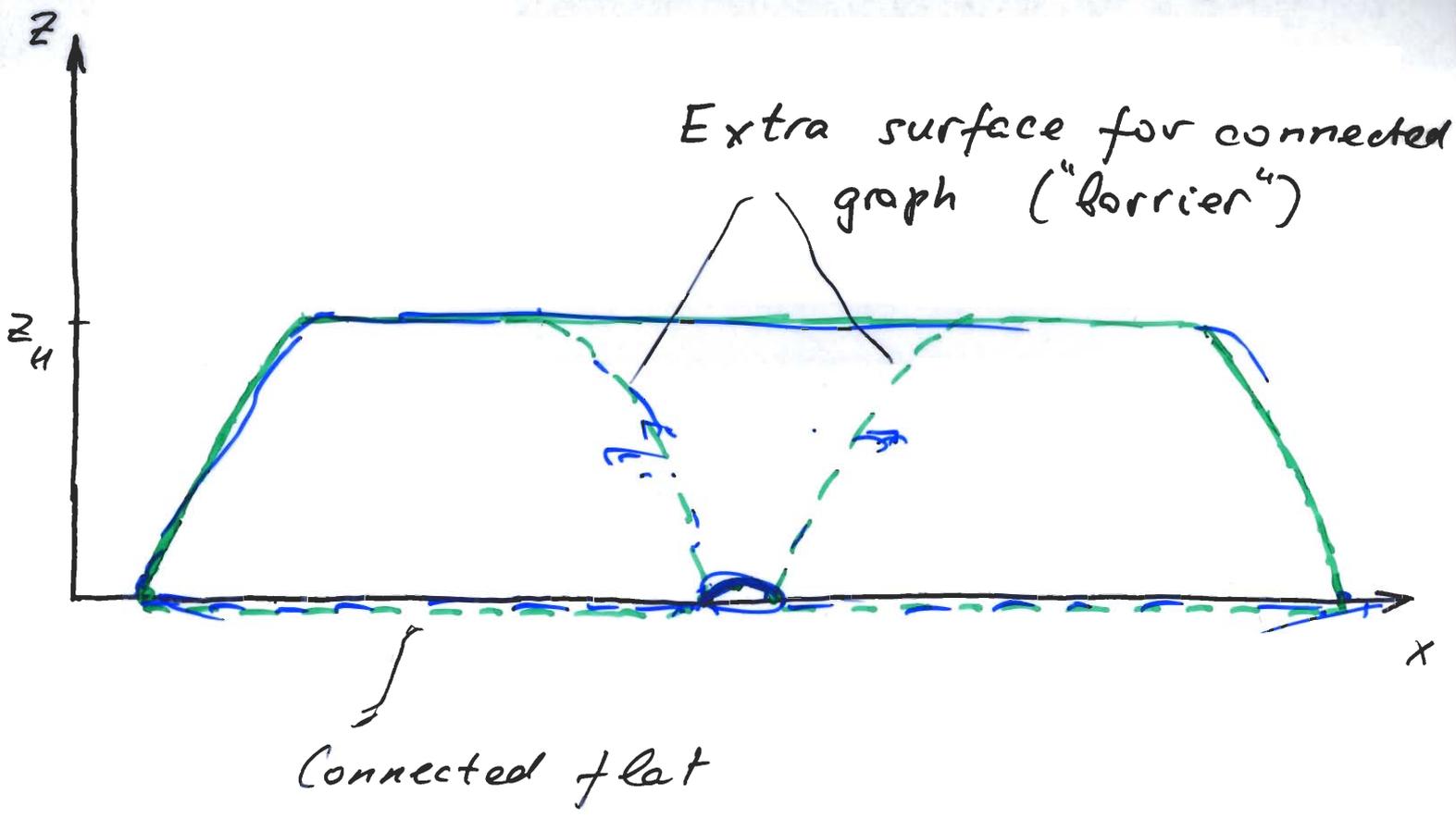
$$\sim \exp\left(-\frac{d b_0}{a_s(R_1)}\right)$$

$d = 4$ standard

$d = 2$ Gubarev

Looks like a barrier transition

- Field theory does not know about the barrier
- flat string geometry produces opposite effect ($g_h = 0$)
- extra dim. fits well (qualitatively)



Reproduces $\exp\left(-\frac{\text{const}}{2\beta(R_i)}\right)$

But evaluation of 'const' seems unreliable in the model used

O. Andreev
U. Z
in progress

Summary on AdS/QCD side

- equips with gauge-inv. language for Λ_{QCD}^2 corrections
- explains naturally absence of IR renormalon, structure of the confining string
- but: the model is not derived, pure heuristic
- the model is rather crude
(corresponds, say, to the Cornell potential)