Weak radiative hyperon decays in quark model

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1 Introduction

Recently, some new experiments have been performed which measured widths and asymmetry parameters in weak radiative hyperon decays (WRHD). The situation changed significantly for the decays $\Xi^0 \to \Lambda\gamma$ and $\Xi^0 \to \Sigma^0\gamma$, where asymmetry parameters turned out to be negative, while previous experiments indicated positive asymmetry. In this connection, interest was revived in the old problem of describing of WRHD's either with the Hara theorem stating zero asymmetry in $\Sigma^+ \to p\gamma$ decay, or without it.

In the quark model WRHD's are described by three kinds of diagrams (see, e.g., Sharma,Verma 88), namely, 1-quark diagrams with photon emission from the effective $sd\gamma$ – vertex, 2-quark diagrams with bremstrahlung and W-exchange and the 3rd quark as a spectator, and, finally, diagrams where 2 quarks exchange W-boson while the 3rd quark emitts photon. As a rule, the latter class of diagrams can be safely neglected.

However, 1-quark diagram contributions are not able to explain the observed radiative rates. Even contributions of the penguin-like diagrams are not strong enough to enhance sufficiently the $s \to d + \gamma$ decay rate. At the same time, as it was shown in Sharma,Ryazzudin, 2-quark diagram proved to be important. Calculations of these contributions in the quark model simultaneously for parity conserving and parity violating amplitudes without phenomenological parameters do not yield agreement with the data, in particular with taking account of the new experiments.

That is why parity violating amplitudes are often calculated within the quark models of type Sharma,Verma, whereas parity conserving ones are treated with the help of the unitary models of nonleptonic decays, vector-dominance hypothesis and $SU(6)_W$ symmetry (see Zenczykowski). Recently (Zenczykowski06), it was attempted to describe in a unique way weak nonleptonic and radiative hyperon decays. Taking account of the complexity of the problem, the author mainly succeeded in describing of the radiative hyperon decays.

We would like to propose on the basis of the quark model a phenomenological model which however opens the way to theoretical analysis of the problem. We also show that quark models are related to models based on unitary symmetry and pole models. Numerical analysis partly follow the lines of a nice work of Neufeld.

2 Kinematics of the weak radiative hyperon decay

A gauge-invariant form of the amplitude of the weak radiative decay $B_i \to B_f + \gamma$ is usually written in the following way:

$$\mathcal{A}_W = \bar{B}_f(B^{PC} + A^{PV}\gamma_5)\hat{k}\hat{\epsilon}B_i, \tag{1}$$

where B_i B_f are Dirac spinors of the initial and final baryons, q being 4-momentum of the photon ($k_{c.m.} \equiv k_\gamma$ is respectively the 3-momentum of the photon in the rest frame of the initial baryon), ϵ_μ being photon polarization 4-vector.

The partial width of the radiative hyperon decay in terms of the phenomenological parity-violating (PV) A^{PV} and parity-conserving (PC) B^{PC} amplitudes are given by

$$\Gamma_\gamma = \frac{k^3_{c.m.}}{\pi}(|A^{PV}|^2 + |B^{PC}|^2), \tag{2}$$

while the corresponding asymmetry is written as

$$A_\gamma = \frac{2Re(A^{PV*}B^{PC})}{|A^{PV}|^2 + |B^{PC}|^2}. \tag{3}$$

Experimental data on rates and branching ratios $BR = \Gamma_\gamma/\Gamma(\text{total})$ and asymmetry parameters are given in Table 1.

Decay	BR $(\times 10^3)$	$\Gamma_\gamma \times 10^{+15}$	A_γ	$k_\gamma^3 \times 10^{3\ 3}$
$\Sigma^+ \to p\gamma$	$1,23 \pm 0,05$	10.25 ± 0.40	$-0,76 \pm 0,08$	11.4
$\Sigma^0 \to n\gamma$	–	-	–	11.6
$\Lambda^0 \to n\gamma$	$1,75 \pm 0,15$	4.43 ± 0.40	–	4.25
$\Xi^0 \to \Lambda\gamma$	$1,16 \pm 0,08$	2.67 ± 0.20	$-0,78 \pm 0,19$	6.23
$\Xi^0 \to \Sigma^0\gamma$	$3,33 \pm 0,10$	7.65 ± 0.19	$-0,63 \pm 0,09$	1.60
$\Xi^- \to \Sigma^-\gamma$	$0,127 \pm 0,023$	0.502 ± 0.090	–	1.64

Table 1: Weak radiative hyperon decays (WRHD), experiment [1], BR is the branching ratio of the radiative decay, Γ_γ is the radiative partial width, and A_γ is the asymmetry parameter of teh WRHD.

3 On the 1-quark amplitudes of the radiative hyperon decay

Already from the partial decay widths it could be seen that 1-quark amplitudes should give a small contribution to the real partial widths. This can be stated, e.g., from the data on $\Xi^{-,0} \to \Sigma^{-,0}\gamma$ decays. Really, the $\Xi^- \to \Sigma^-\gamma$ decay can be explained (within the quark model) only by the 1-quark diagram, and its reduced partial width in the units $10^{-7}\mu_N$ is about 4.0 (see the 4th column of the Table 5), whereas the reduced partial width of the $\Xi^0 \to \Sigma^0\gamma$ decay, equal to it in the model with the only 1-quark diagram, turns out to be $\sim$60.0 in the same units, which show the necessity to go out of the 1-quark diagram description.

Even an enhancement due to the penguin diagrams does not solve the problem.

The standard 1-quark diagram contribution is given by the effective weak strangeness-violating neutral current

$$\mathcal{J}_\mu^W = (D_{ds} + F_{ds})\bar{B}_2^\alpha \mathcal{O}_\mu B_\alpha^3 + (D_{ds} - F_{ds})\bar{B}_\alpha^3 \mathcal{O}_\mu B_2^\alpha , \qquad (4)$$

where B_β^α is the baryon octet, $\alpha, \beta = 1, 2, 3,\ B_1^3 = p$ etc. Putting $D_{ds} = -b$, $F_{ds}/D_{ds} = 2/3$ one reproduces standard quark model results (see 2nd column of Table 2, and in applications, generally speaking, one takes $b^{PC} \neq b^{PV}$ (the same for D_{ds}, F_{ds}).

Decay	[4]	Eq.(**??**)	Eq.(4)
$\Sigma^+ \to p\gamma$	$-b/3$	0	$-F_{ds} + D_{ds}$
$\Sigma^0 \to n\gamma$	$b/3\sqrt{2}$	$a^d/\sqrt{2}$	$(F_{ds} - D_{ds})/\sqrt{2}$
$\Lambda^0 \to n\gamma$	$3b/\sqrt{6}$	$a^d/\sqrt{6}$	$-(3F_{ds} + D_{ds})/\sqrt{6}$
$\Xi^0 \to \Lambda\gamma$	$b/\sqrt{6}$	$-a^d/\sqrt{6}$	$(3F_{ds} - D_{ds})/\sqrt{6}$
$\Xi^0 \to \Sigma^0\gamma$	$-5b/3\sqrt{2}$	$-a^d/\sqrt{2}$	$-(F_{ds} + D_{ds})/\sqrt{2}$
$\Xi^- \to \Sigma^-\gamma$	$5b/3$	0	$F_{ds} + D_{ds}$

Table 2: Contributions of the 1-quark diagrams and of the effective strangeness-changing neutral $SU(3)_f$ current to WRHD.

4 On the structure of the 2-quark amplitudes

Let us consider now contributions of the 2-quark weak radiative transitions $s + u \to u + d + \gamma$ with W-exchange.

We begin with the analysis of a set of diagrams of the $\Sigma^+ \to p\gamma$–decay. The matrix element of the $\Sigma^+ \to p\gamma$ decay can be put in the form

$$
\begin{aligned}
&6 < p_{\downarrow}, \gamma(+1)|O|\Sigma^{+}_{\uparrow} >= \\
&=< 2u_2u_2d_1 - u_2d_2u_1 - d_2u_2u_1, \gamma(+1)|O| \\
&\qquad\qquad\qquad |2u_1u_1s_2 - u_1s_1u_2 - s_1u_1u_2 >= \\
&= 4 < u_2u_2d_1, \gamma(+1)|O|u_1u_1s_2 > - \\
&4 < u_2u_2d_1, \gamma(+1)|O|u_1s_1u_2 > - \\
&-4 < u_2d_2u_1, \gamma(+1)|O|u_1u_1s_2 > + \\
&4 < u_2d_2u_1, \gamma(+1)|O|u_1s_1u_2 >,
\end{aligned}
\tag{5}
$$

where $q_{1,2}$ means states with definite spin projection $q_{\uparrow,\downarrow}$ of the quark inside the baryon. An explicit form of the operator $O = O^{PV} + O^{PC}$ is for a moment irrelevant to us. The 1st matrix element (m.e.) in the RHS of the last expression, Eq.(6), $< u_2u_2d_1, \gamma(+1)|O|u_1u_1s_2 >$ in the case of the W-exchange between the quarks is described by the 1st diagram of Fig.1, because this m.e. cannot be represented by a diagram with a spectator. It is plausible to assume that its contribution is small.

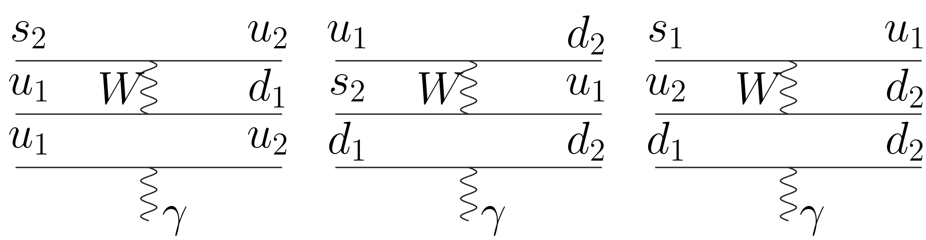


Fig. 1. The 3-quark diagrams without spectator quark (q_1 means $q_{\uparrow}$, q_2 means $q_{\downarrow}$, $q = u, d, s$)

There are three different diagrams of Fig.2 with the u_2 quark as a spectator which give a contribution to the 2nd m.e. of the RHS of Eq.(6) $< u_2u_2d_1, \gamma(+1)|O|u_1s_1u_2 >= A_1$, and

$$A_1 = e^f_u A + e^i_s E + e^i_u B = \frac{2}{3}A - \frac{1}{3}E + \frac{2}{3}B.$$

Here A corresponds to the non-flip transition quark amplitude with all quarks having spin projection $+1/2$:

$s_\uparrow + u_\uparrow \to u_\uparrow + d_\uparrow$;

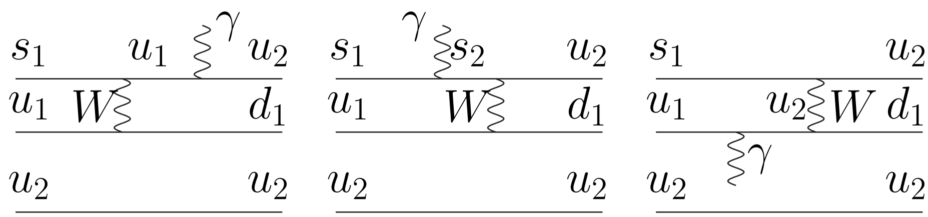


Fig. 2. The 3-quark diagrams of the decay $\Sigma^+ \to p\gamma$ corresponding to the matrix element A_1 with the 3rd quark $u_2 = u_\downarrow$ as a spectator.

E corresponds to the non-flip transition quark amplitude with quarks having different spin projections:

$s_\downarrow + u_\uparrow \to u_\downarrow + d_\uparrow$

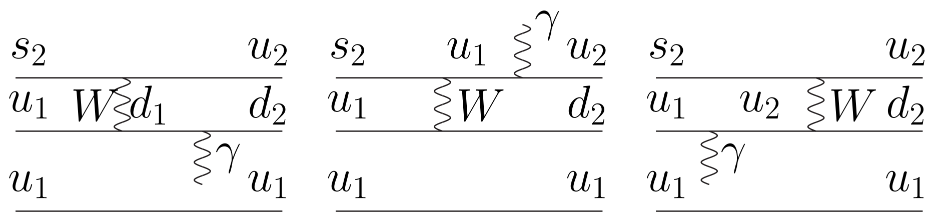
while B corresponds to the spin-flip transition quark amplitude with s quark spin projection equal to $+1/2$

$s_\uparrow + u_\downarrow \to u_\downarrow + d_\uparrow$;

numbers of the coefficients are just electric charges of quarks.

There are three different diagrams of Fig.3 with the u_1 quark as a spectator which give a contribution to the 3rd m.e. of the RHS of Eq.(6) $< u_2 d_2 u_1, \gamma(+1)|O|u_1 u_1 s_2 >= A_3$:

Fig. 3. The 3-quark diagrams of the decay $\Sigma^+ \to p\gamma$ corresponding to the matrix element A_3 with the 3rd quark $u_1 = u_\uparrow$ as a spectator. with quark u_2 as a spectator



$$A_3 = e_u^f C + e_d^f E + e_u^i \tilde{A} = \frac{2}{3}C - \frac{1}{3}E + \frac{2}{3}\tilde{A},$$

where two new coefficients are introduced: C corresponds to the spin-flip transition quark amplitude with s quark spin projection equal to -1/2

$s_\downarrow + u_\uparrow \to u_\uparrow + d_\downarrow$, while $\tilde{A}$ corresponds to the non-flip transition quark amplitude with all quarks having spin projection -1/2: $s_\downarrow + u_\downarrow \to u_\downarrow + d_\downarrow$. To the 4th m.e. of the RHS of Eq.(6) $< u_2 d_2 u_1, \gamma(+1)|O|u_1 s_1 u_2 >$ there are two kinds of contributions, the first given by the diagrams of Fig.4

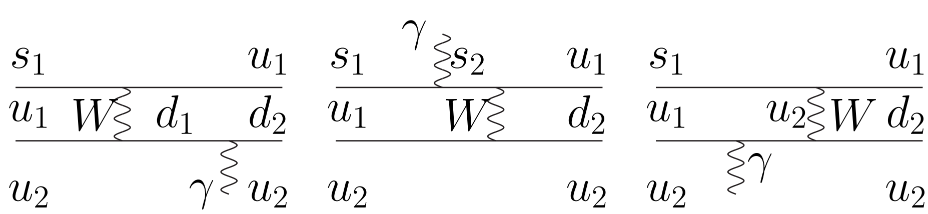


Fig. 4. The 3-quark diagrams of the decay $\Sigma^+ \to p\gamma$ corresponding to the matrix element A_2 with the 3rd quark $u_2 = u_\downarrow$ as a spectator.

$$A_2 = e_d^f A + e_s^i C + e_u^i D = -\frac{1}{3}A - \frac{1}{3}C + \frac{2}{3}D,$$

where D corresponds to the non-flip transition quark amplitude with different spin projections of quarks,

$s_\uparrow + u_\downarrow \rightarrow u_\uparrow + d_\downarrow$, and, the second given by the diagrams of Fig.5 with quark u_1 as a spectator:

$$A_4 = e_s^i \tilde{A} + e_d^f B + e_u^f D = \frac{2}{3} D - \frac{1}{3} \tilde{A} - \frac{1}{3} B.$$

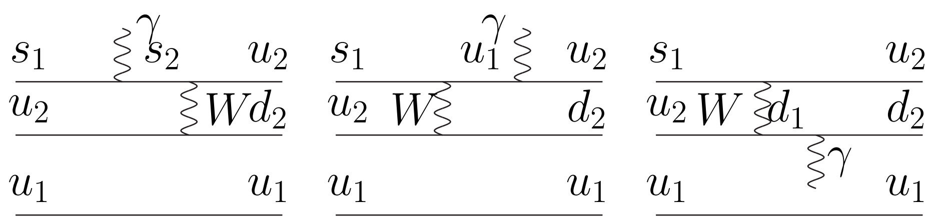


Fig. 5. The 3-quark diagrams of the decay $\Sigma^+ \rightarrow p\gamma$ corresponding to the matrix element A_4 with the 3rd quark $u_1 = u_\uparrow$ as a spectator.

Totally there are 4 combinations of the m.e.'s $A, \tilde{A}, B, C, D, E$ denoted as A_k, $k = 1, 2, 3, 4$. Finally, we have

$$< p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow >= \frac{2}{3}(-2A_1 + A_2 - 2A_3 + A_4). \qquad (6)$$

In a similar way one can obtain expressions for the amplitudes of all the other radiative decays.

Upon assuming that spectator quarks do not change amplitudes A_k, $k = 1, 2, 3, 4$, while going from one decay to another all the amplitudes can be expressed in terms of these quantities

$$\begin{aligned} < p_\downarrow, \gamma(+1)|O|\Sigma^+_\uparrow >&= \frac{2}{3}(-2A_1 + A_2 - 2A_3 + A_4), \qquad (7) \\ < n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >&= \frac{2}{3\sqrt{2}}(A_1 - 2A_2 - 2A_3 + A_4), \\ < n_\downarrow, \gamma(+1)|O|\Lambda_\uparrow >&= \frac{2}{\sqrt{6}}(A_1 - 2A_2 - A_4), \\ < \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >&= \frac{2}{\sqrt{6}}(A_1 - A_2), \\ < \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >&= \frac{2}{3\sqrt{2}}(A_1 + A_2 - 2A_3 + 4A_4). \end{aligned}$$

They depend not on all A_ks, but only on their linear combinations $B_1 = A_1 - A_2,\ B_2 = A_2 + A_3,\ B_3 = A_3 - A_4$.

It is straightforward to show that in the model of [4] A_ks are given by

$$\begin{array}{l} A_1^{PV} = A_1^{PC} = \frac{1}{6}(1-2\zeta)(1+X), \\ A_2^{PV} = A_2^{PC} = -\frac{1}{6}(1-2\zeta)(1-X), \\ A_3^{PV} = A_4^{PC} = \frac{1}{6}(1+X), \\ A_4^{PV} = A_3^{PC} = -\frac{1}{6}(1+X), \end{array} \tag{8}$$

where $X = k/2m_u$ and $6\zeta = (1 - m_u/m_s) = (1-\epsilon)$ Sharma. One can see that up to a factor this formula reproduces their results and the 3rd column in Table 4 of the present work.

Calculations of the quark diagrams along the lines of Sharma allow one to find also the amplitudes $A, \tilde{A}, B, C, D, E$ in the first order in k and linear symmetry breaking by the mass of the strange quark m_s. (The last assertion means that if a photon is emitted by the strange quark of the corresponding amplitudes $A, \tilde{A}, B, C, D, E$, one should put a factor $\epsilon = m_u/m_s, m_u = m_d$.

Namely, in this approximation and in the units of $(eG_F sin\theta_C cos\theta_C/\sqrt{2}m_u)$

$|\psi(k)|^2$ with $X = k/2m_u$ the PC-amplitudes have the form
$A^{PC} = \tilde{A}^{PC} = 0$, $B^{PC} = -(1+X)$, $C^{PC} = -(1+X)$,
$D^{PC} = E^{PC} = 1 + X$,

and these expressions are valid for the photon emitted off the quark with the spin projection $+1/2$.

When a photon is emitted off the quark with the spin projection -1/2 one should change the sign of X in the amplitudes D^{PC} and C^{PC}:

Putting $X = 0$, $\epsilon = 1 - 6\zeta$ in the PV-amplitudes, one get's the results of Zencz for the 2-quark transitions with the 3rd quark as a spectator.

The results are put in the 3rd columns of Tables 3,4, where the overall factor $(\kappa_0/m_u/\sqrt{2})G_F sin\theta_C cos\theta_C|\psi(q)|^2$ is assumed.

Decay	[8]	[4]	Eq.(7)
$\Sigma^+ \to p\gamma$	$-\frac{5+\epsilon}{9\sqrt{2}}b$	$\frac{2}{9}[-3-2X+\zeta(3+X)]$	$\frac{2}{3}(-2A_1^{PV}+A_2^{PV}-2A_3^{PV}+A_4^{PV})$
$\Sigma^0 \to n\gamma$	$-\frac{1-\epsilon}{18}b$	$\frac{2}{9\sqrt{2}}[-2X+\zeta(-3+X)]$	$\frac{2}{3\sqrt{2}}(A_1^{PV}-2A_2^{PV}-2A_3^{PV}+A_4^{PV})$
$\Lambda^0 \to n\gamma$	$\frac{3+\epsilon}{6\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[-2+\zeta(-3+X)]$	$\frac{2}{\sqrt{6}}(A_1^{PV}-2A_2^{PV}-A_4^{PV})$
$\Xi^0 \to \Lambda\gamma$	$-\frac{2+\epsilon}{9\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[1-2\zeta]$	$\frac{2}{\sqrt{6}}(A_1^{PV}-A_2^{PV})$
$\Xi^0 \to \Sigma^0\gamma$	$\frac{1}{3}b$	$\frac{2}{9\sqrt{2}}[-3-2X-2\zeta X]$	$\frac{2}{3\sqrt{2}}(A_1^{PV}+A_2^{PV}-2A_3^{PV}+4A_4^{PV})$
$\Xi^- \to \Sigma^-\gamma$	0	0	0

Table 3: WRHD, 2-quark diagram contributions to PV-amplitudes

At $X = 0$ and $6\zeta = (1-\epsilon)$ these results go into those of [8] with $\sqrt{2}\kappa_0 = b^Z$.

Note that the 2-quark PV-amplitudes of [10] can be written in terms of those of [4] through another relation, namely, as a superposition of the PV- and PV-amplitudes of [4], $A^{PV,Zen} = x \cdot A^{PV,Sh} + A^{P,Sh}$; this fact is due to the inclusion of states of

Decay	[8]	[4]	Eq.(7)
$\Sigma^+ \to p\gamma$	$-\frac{1-\epsilon}{9\sqrt{2}}b$	$\frac{2}{9}[X+\zeta(3+X)]$	$\frac{2}{3}(-2A_1^{PC}+A_2^{PC}-2A_3^{PC}+A_4^{PC})$
$\Sigma^0 \to n\gamma$	$-\frac{5+\epsilon}{18}b$	$\frac{2}{9\sqrt{2}}[3+X+\zeta(-3+X)]$	$\frac{2}{3\sqrt{2}}(A_1^{PC}-2A_2^{PC}-2A_3^{PC}+A_4^{PC})$
$\Lambda^0 \to n\gamma$	$\frac{1+\epsilon}{6\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[1-X+\zeta(-3+X)]$	$\frac{2}{\sqrt{6}}(A_1^{PC}-2A_2^{PC}-A_4^{PC})$
$\Xi^0 \to \Lambda\gamma$	$\frac{2+\epsilon}{9\sqrt{3}}b$	$\frac{2}{3\sqrt{6}}[1-2\zeta]$	$\frac{2}{\sqrt{6}}(A_1^{PC}-A_2^{PC})$
$\Xi^0 \to \Sigma^0\gamma$	$\frac{1}{3}b$	$\frac{2}{9\sqrt{2}}[3+4X-2\zeta X]$	$\frac{2}{3\sqrt{2}}(A_1^{PC}+A_2^{PC}-2A_3^{PC}+4A_4^{PC})$
$\Xi^- \to \Sigma^-\gamma$	0	0	0

Table 4: WRHD, 2-quark diagram contributions to PC-amplitudes

different parities (x being a parameter equal to zero in the exact unitary symmetry scheme and taken equal to 1/3 in calculations with $SU(3)$ breaking [10]).

The total PV-amplitudes A^{PV} are described practically in the same way in various models as a sum of 2- and 1-quark transition contributions.

Our best fit is: $b^{PV}/3 = 0.96$, $d^{PV} = -0.5$ (in units $10^{-7}\mu_N$), $\epsilon^{PV} = -5/16$. (In the interval $|\epsilon^{PV}| \leq 5/16$ the results are practically the same.)

$$A^{PV}(\Sigma^+ \to p\gamma) = -\frac{(5+\epsilon^{PV})}{9\sqrt{2}}b^{PV} + \frac{1}{\sqrt{2}}d^{PV} = -\frac{1}{\sqrt{2}}(1.50 + 0.50) = -1.41; \quad (9)$$

$$A^{PV}(\Sigma^0 \to n\gamma) = -\frac{(1-\epsilon^{PV})}{18}b^{PV} - \frac{1}{2}d^{PV} = \frac{1}{2}(-0.42 + 0.50) = 0.04; \quad (10)$$

$$A^{PV}(\Lambda \to n\gamma) = \frac{(3+\epsilon^{PV})}{6\sqrt{3}}b^{PV} - \frac{3\sqrt{3}}{2}d^{PV} = \frac{1}{\sqrt{3}}(1.35 + 2.25) = 2.08; \quad (11)$$

$$A^{PV}(\Xi^0 \to \Lambda\gamma) = -\frac{(2+\epsilon^{PV})}{9\sqrt{3}}b^{PV} + \frac{\sqrt{3}}{2}d^{PV} = \frac{1}{\sqrt{3}}(-0.54 - 0.75) = -0.75; \quad (12)$$

$$A^{PV}(\Xi^0 \to \Sigma^0\gamma) = \frac{1}{3}b^{PV} - \frac{5}{2}d^{PV} = 0.95 + 1.25 = 2.20; \quad (13)$$

$$A^{PV}(\Xi^- \to \Sigma^-\gamma) = 0 \cdot b^{PV} + \frac{5}{\sqrt{2}}d^{PV} = -1.75. \quad (14)$$

One can see that all observed decays have contributions of the same order of magnitude to the PV-amplitudes (but the last one, as it is obvious) from the 2- and 1-quark diagrams.

The case of the PC-amplitudes proves to be more difficult. Indeed, if in the quark model of [4] the PV-amplitudes A^{PV}'s have the same structure as A^{PC}'s (see the 3rd columns in Tables 3,4), in a series of works the PC-amplitudes were analysed in terms of pole models and unitary symmetry models, elaborated for description of nonleptonic hyperon decays without a direct appeal to quark models (Zencz-2006):

$$
\begin{aligned}
B(\Sigma^+ \to p\gamma) &= \frac{\sqrt{2}}{3}(\frac{f}{d} - 1)(1-\epsilon)((\frac{F}{D} - 1))C \quad (15)\\
B(\Sigma^0 \to n\gamma) &= \frac{4}{3}C - \frac{1}{3}(\frac{f}{d} - 1)(1-\epsilon)(\frac{F}{D} - 1)C\\
B(\Lambda \to n\gamma) &= \frac{4}{3\sqrt{3}}C + \frac{1}{9\sqrt{3}}(\frac{3f}{d} + 1)(1-\epsilon)(\frac{3F}{D} + 1)C\\
B(\Xi^0 \to \Lambda\gamma) &= -\frac{4}{3\sqrt{3}}C - \frac{1}{9\sqrt{3}}(\frac{3f}{d} - 1)(1-\epsilon)(\frac{3F}{D} - 1)C\\
B(\Xi^0 \to \Sigma^0\gamma) &= -\frac{4}{3}C + \frac{1}{3}(\frac{f}{d} + 1)(1-\epsilon)(\frac{F}{D} + 1)C\\
B(\Xi^- \to \Sigma^-\gamma) &= -\frac{\sqrt{2}}{3}(\frac{f}{d} + 1)(1-\epsilon)(\frac{F}{D} + 1)C
\end{aligned}
$$

At first sight, it seems impossible to compare the conclusions of this and similar models with the quark models except as in the limit of the exact $SU(6)$ model.

Nevertheless, we shall show now in what way it is possible to expand this expression into a sum of 1- and 2- quark contributions for all the PC-amplitudes.

For this purpose, it is sufficient to put $f/d = -1 + z$, $F/D = 2/3 + Z$ in the PC-amplitudes of Eq.(16) (see Eq.(5.2b) in [8]). With this step the PC-amplitudes are splitted into the part corresponding to 2- quark contributions (the 1st term without z and/or Z) and three parts corresponding to 1- quark contributions proportional to the factors z, Z and $z \cdot Z$.

One can easily be conviced that with $z = Z = 0$ relations, Eq.(16) reduces to redefinitions of the 2nd column of Table 3.

Instead, at $Z = 0$ the 1-quark contributions proportional to z coincide up to redefinitions with those obtained in [4] and Eq.(4) with $F_{ds}/D_{ds} = 2/3$ (see the 2nd and the last columns of Table 2). The terms with $Z \neq 0$ also correspond to the 1-quark contributions but at a different effective value of the ratio F_{ds}/D_{ds} in Eq.(4), namely, the terms proportional to Z at $z = 0$ correspond to the choice of the effective ds - current with $F_{ds}/D_{ds} = -1$, while the terms proportional to $z \cdot Z$ correspond to pure F_{ds} - current (see the last column of Table 2).

Consider now one-by-one all the hyperon decays beginning from the transformation of the PC-amplitude of the decay $\Sigma^+ \to p\gamma$ from [8] and putting also our best fit results ($C = 6.3$ in units $10^{-7}\mu_N$, $z = 0.05$, $Z = 0.16$, $\epsilon = -2.0$) for every contribution and their sum.

$$B(\Sigma^+ \to p\gamma) = [\frac{4}{9\sqrt{2}} - \frac{\sqrt{2}}{9}z - \frac{2\sqrt{2}}{3}Z + \frac{\sqrt{2}}{3}z \cdot Z](1-\epsilon)Z \cdot C \Rightarrow (16)$$
$$\frac{\sqrt{2}}{3}6.3(2 - 0.05 - 0.96 + 0.024) = 3.00.$$

It is seen that the main contribution comes from the 2-quark amplitudes and from the 1-quark amplitude with the effective ds-current with $F_{ds}/D_{ds} = -1$, whereas the standard 1- quark contribution, corresponding to the choice of the effective ds - current with $F_{ds} = (2/3)D_{ds}$ is small and the last 1- quark contribution with the pure F_{ds} effective current could be safely neglected.

Formally, we write expansion also for non-observable decay $\Sigma^0 \to n\gamma$:

$$B(\Sigma^0 \to n\gamma) = \frac{4}{3}C - \frac{1}{3}(\frac{f}{d} - 1)(1-\epsilon)(\frac{F}{D} - 1)C$$
$$\Rightarrow [\frac{4(5+\epsilon)}{18} + \frac{1}{9}z(1-\epsilon) + \frac{2}{3}(1-\epsilon)Z - \frac{1}{3}z(1-\epsilon)Z] \cdot C \Rightarrow \quad (17)$$
$$\frac{1}{3}6.3(2.00 + 0.05 + 0.96 - 0.024) = 6.33;$$

and all we have said previously about the decay $\Sigma^+ \to p\gamma$ is valid for this decay too.

Next we consider the decay $\Lambda \to n\gamma$.

$$B(\Lambda \to n\gamma) = \frac{4}{3\sqrt{3}}C + \frac{1}{9\sqrt{3}}(\frac{3f}{d} + 1)(1-\epsilon)(\frac{3F}{D} + 1)C \quad (18)$$
$$\Rightarrow [\frac{4(1+\epsilon)}{6\sqrt{3}} + \frac{1}{\sqrt{3}}z(1-\epsilon) - \frac{2}{3\sqrt{3}}(1-\epsilon)Z + \frac{1}{\sqrt{3}}z(1-\epsilon)Z] \cdot C \Rightarrow$$
$$\frac{1}{\sqrt{3}}6.3(-0.667 + 0.15 - 0.32 + 0.024) = -2.96.$$

It is seen that here the main contribution comes from the 2-quark amplitudes and from the 1-quark amplitude with the effective ds - current with $F_{ds}/D_{ds} = -1$, whereas the standard 1-quark contribution is small. The last 1-quark contribution with the pure F_{ds} effective current proportional to $z \cdot Z$ from now on is neglected.

Now let us consider radiative decays of the cascade hyperons.

$$B(\Xi^0 \to \Lambda\gamma) = -\frac{4}{3\sqrt{3}}C - \frac{1}{9\sqrt{3}}(\frac{3f}{d} - 1)(1-\epsilon)(\frac{3F}{D} - 1)C \quad (19)$$
$$\Rightarrow [-\frac{4(2+\epsilon)}{9\sqrt{3}} - \frac{1}{3\sqrt{3}}z(1-\epsilon) + \frac{4}{3\sqrt{3}}(1-\epsilon)Z - \frac{1}{\sqrt{3}}z(1-\epsilon)Z] \cdot C \Rightarrow$$
$$\frac{1}{\sqrt{3}}6.3(-0.05 + 0.64 - 0.024) = 2.06.$$

Here the main contribution comes from the 1-quark amplitude with the effective ds - current with $F_{ds}/D_{ds} = -1$ while the 2-quark amplitudes vanish by the choice of ϵ. This solution is dictated by observed negative asymmetry in this decay.

The next PC-amplitude has the form:

$$B(\Xi^0 \to \Sigma^0\gamma) = -\frac{4}{3}C + \frac{1}{3}(\frac{f}{d} + 1)(1-\epsilon)(\frac{F}{D} + 1)C \qquad (20)$$

$$\Rightarrow [-\frac{4}{3} + \frac{5}{9}z(1-\epsilon) + 0 \cdot Z + \frac{1}{3}z(1-\epsilon)Z] \cdot C \Rightarrow$$

$$\frac{1}{3}6.3(-4.0 + 0.25 + 0.024) = -7.82.$$

In this decay the main contribution to the PC-amplitude comes from the 2-quark amplitudes while the 1-quark contribution with the effective ds - current with $F_{ds}/D_{ds} = -1$ is small and the standard 1- quark contribution corresponding to the choice of the effective ds - current with $F_{ds} = (2/3)D_{ds}$ is zero.

In conclusion we give the PC-amplitude of the decay $\Xi^- \to \Sigma^-\gamma$.

$$B(\Xi^- \to \Sigma^-\gamma) = -\frac{\sqrt{2}}{3}(\frac{f}{d} + 1)(1-\epsilon)(\frac{F}{D} + 1)C \qquad (21)$$

$$\Rightarrow 0 \cdot C - \frac{5\sqrt{2}}{9}z(1-\epsilon)C + 0 \cdot Z \cdot C - \frac{\sqrt{2}}{3}z(1-\epsilon)Z \cdot C \Rightarrow$$

$$-\frac{\sqrt{2}}{3}6.3 \cdot (0.25 + 0.024) = -0.81,$$

and here there are no 2-quark contributions, the 1-quark contribution with the effective ds-current with $F_{ds}/D_{ds} = -1$ is zero while the standard 1-quark term gives the main contribution.

So the 1st term in every formula for the PC-amplitudes (i.e., for PC-amplitudes at $z = Z = 0$ or, which is the same, at $f/d = -1$, $F/D = 2/3$) corresponds exactly to standard expressions fir 2-quark contributions at $X = 0$ (see Table 4).

The 1-quark contributions, proportional to z, also coincide with those in Sharma and Zenczykowski. However, and it is important, if usualy in the quark model the PV- and PC- amplitudes had identical factors of the 2-quark terms, in recent works and in the present one this is not the case. (Even more, the parameter ϵ is negative here and different for the PV- and PC-amplitudes.) It can be thought that in this way phenomenological models effectively take into account the difference between the dynamics of the processes going with and without parity conservations. Our results are given in Table 5.

Unfortunately, at the present time, it seems to be impossible to perform dynamical calculations in an unambiguous way.

Deacy	A^{PV}	B^{PC}	$\pi\Gamma_\gamma/k_\gamma^3$	A_γ	$k_\gamma^3 \times 10^3$ 3
$\Sigma^+ \to p\gamma$	-1.41	3.00	11.4 $(11.0 \pm 0.4)^{exp.}$	-0.74 $(-0,76 \pm 0,08)$	11.4
$\Sigma^0 \to n\gamma$	0.04	6.27	40.0	0.01	11.6
$\Lambda^0 \to n\gamma$	2.08	-2.95	13.0 $(13.0 \pm 1.1)^{exp.}$	-0.94	4.25
$\Xi^0 \to \Lambda\gamma$	-0.75	2.06	4.08 $(5.4 \pm 0.4)^{exp.}$	-0.64 $(-0,78 \pm 0,19)$	6.23
$\Xi^0 \to \Sigma^0\gamma$	2.20	-7.82	66.0 $(59.75 \pm 2.0)^{exp.}$	-0.52 $(-0,63 \pm 0,09)$	1.60
$\Xi^- \to \Sigma^-\gamma$	-1.75	-0.81	3.7 $(3.82 \pm 0.8)^{exp.}$	0.76	1.64

Table 5: WRHD, phenomenological model and experiment [1]. Amplitudes A^{PV} and B^{PC} are in units of $10^{-7}\mu_N$, $\pi\Gamma_\gamma/k_\gamma^3 = |A^{PV}|^2 + |B^{PC}|^2$ in units of $(10^{-7}\mu_N)^2$.

5 Conclusion

It is shown that many models describing weak radiative hyperon decays can be reduced to rather a simple quark model including 1-quark transitions with the effective $sd\gamma$- vertex and 2-quark process with the W – exchange $s + u \to u + d + \gamma$. As an example, quark and unitary models are considered.

Using them as a basis, a phenomenological model is constructed which describes the data and gives clear predictions for the asymmetry parameters of the decays $\Lambda \to n + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$.

For the 2-quark processes with the W – exchange rather general expressions are obtained which could be used not only for the hyperon decays but also for the decay of the new heavy baryons.

We do not discuss here a traditional problem connected with the Hara theorem prediction of zero asymmetry in the decay $\Sigma^+ \to p + \gamma$, as already in the GIM model this problem can be overcome.

Nevertheless, there are many theoretical problems unresolved in the models of weak radiative hyperon decays (together with those of hyperon nonleptonic decays) which expect a more thorough analysis.

References

[1] Particle Data Group, Journal of Physics G **33**, 1 (2006).

[2] Particle Data Group, The Eur. Phys. J. C **3**,613 (1998); Phys. Rev. D **54**, 1-I (1996).

[3] Y. Hara, Phys. Rev. Lett. **12**, 378 (1964).

[4] R.C. Verma and A. Sharma, Phys. Rev. D **38**, 1443 (1988).

[5] A.N. Kamal and R.C. Verma, Phys. Rev. D **26**, 190 (1982).

[6] A.N. Kamal and Riazuddin, Phys. Rev. D **28**, 2317 (1983).

[7] J. Lach and P. Zenczykowski, Int. J. Mod. Phys. A **10**, 3817 (1995).

[8] P. Zenczykowski, Phys. Rev. D **40**, 2290 (1989).

[9] P. Zenczykowski, Phys. Rev. D **44**, 1485 (1991).

[10] P. Zenczykowski, hep-ph/0512122.

[11] H.Neufeld, Nucl. Phys. B **402**, 166 (1993).

[12] R.E. Behrends, Phys. Rev. **111**, 1691 (1958).

[13] R.H. Graham and S. Pakvasa, Phys. Rev. 140, B1144 (1965).

[14] S.G. Kamath, Nucl. Phys. B **198**, 61 (1982); J.O. Eeg, Z. Phys. C **21**, 253 (1984).

[15] E.N. Bukina, V.M. Dubovik, V.S. Zamiralov, Nucl. Phys. B (Proc. Suppl.) **93**, 34 (2001).

Appendix A

The 2-quark contributions to neutral hyperon decays

1. Let us analyse the decay $\Sigma^0 \to n\gamma$, although it cannot be seen soon experimentally. The 2-quark amplitude of this decay can be expressed via the following matrix elements:

$$6\sqrt{2} < n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >= \tag{.1}$$

$$=< 2d_2d_2u_1 - d_2u_2d_1 - u_2d_2d_1, \gamma(+1)|O|2u_1d_1s_2 + 2d_1u_1s_2 -$$

$$-u_1s_1d_2 - s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2 >=$$

$$= 8 < d_2d_2u_1, \gamma(+1)|O|u_1d_1s_2 > -8 < d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 > -$$

$$-4 < d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 > -8 < d_2u_2d_1, \gamma(+1)|O|u_1d_1s_2 > +$$

$$+4 < d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 > +4 < d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >$$

The 1st and 2nd matrix elements (m.e.'s) in the RHS of this expression correspond to the 2nd and 3rd diagrams of Fig.1, so we do not consider them (see [5]). The 2nd m.e. in the RHS of Eq.(A.1) $< d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 >= A_2$ is described by three diagrams of Fig.3 but with the quark d_2 as a spectator. To the 4th m.e. in the RHS of Eq.(A.1) $< d_2u_2d_1, \gamma(+1)|O|u_1d_1s_2 >= A_3$ three diagrams of Fig.3 contribute but with the quark d_1 as a spectator.

To the 5th m.e. in the RHS of Eq.(A.1) $< d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 >= A_1$ three diagrams of Fig.2 contribute but with the quark d_2 as a spectator.

To the 6th m.e. in the RHS of Eq.(A.1) $< d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >= A_4$ three diagrams of Fig.5 contribute but with the quark d_1 as a spectator. Their sum gives

$$< n_\downarrow, \gamma(+1)|O|\Sigma^0_\uparrow >= \frac{2}{3\sqrt{2}}(A_1 - 2A_2 - 2A_3 + A_4).$$

2. Now let us describe the decay $\Lambda \to n\gamma$:

$$2\sqrt{6} < n_{\downarrow}, \gamma(+1)|O|\Lambda_{\uparrow} >= \qquad (.2)$$

$$=< 2d_2d_2u_1 - d_2u_2d_1 - u_2d_2d_1, \gamma(+1)|O|u_1s_1d_2 + s_1u_1d_2 - d_1s_1u_2 - s_1d_1u_2$$

$$= 4 < d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 > -4 < d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 > -$$

$$-4 < d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 > +4 < u_2d_2d_1, \gamma(+1)|O|d_1s_1u_2 > .$$

The 1st m.e. in the RHS of Eq.(A.2) $< d_2d_2u_1, \gamma(+1)|O|u_1s_1d_2 >= A_2$ is given by the contributions of three diagrams of Fig.3 with the spectator d_2. The 2nd m.e. in the RHS of Eq.(A.2) $< d_2d_2u_1, \gamma(+1)|O|d_1s_1u_2 >$ in the case of W-exchange between quarks can be described by the diagram of Fig.1 and should be very small. To the 3rd m.e. in the RHS of Eq.(A.2) $< d_2u_2d_1, \gamma(+1)|O|u_1s_1d_2 >= A_1$ three diagrams of Fig.2 contribute but with the quark d_2 as a spectator.

To the 4th m.e. in the RHS of Eq.(A.2) $< d_2u_2d_1, \gamma(+1)|O|d_1s_1u_2 >= A_4$ three diagrams of Fig.5 contribute but with the quark d_1 as a spectator.

Finally one obtains:

$$< n_{\downarrow}, \gamma(+1)|O|\Lambda_{\uparrow} >= \frac{2}{\sqrt{6}}(A_1 - 2A_2 - A_4).$$

3. For the decay $\Xi^0 \to \Lambda\gamma$ we have

$$2\sqrt{6} < \Lambda_{\downarrow}, \gamma(+1)|O|\Xi^0_{\uparrow} >= \qquad (.3)$$

$$=< u_2s_2d_1 + s_2u_2d_1 - d_2s_2u_1 - s_2d_2u_1, \gamma(+1)|O|2s_1s_1u_2 - s_1u_1s_2 - u_1s_1s_2$$

$$= 4 < u_2s_2d_1, \gamma(+1)|O|s_1s_1u_2 > -4 < u_2s_2d_1, \gamma(+1)|O|s_1u_1s_2 > -$$

$$-4 < d_2s_2u_1, \gamma(+1)|O|s_1s_1u_2 > +4 < d_2s_2u_1, \gamma(+1)|O|s_1u_1s_2 > .$$

The 1st and the 3rd matrix elements (m.e.'s) in the RHS of this expression correspond to the 2nd and 3rd diagrams of

Fig.1, and we neglect them both. The 2nd m.e. in the RHS of Eq.(A.3) $< u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_1$ is given by the contributions of three diagrams of Fig.2 with the spectator s_2. To the 4th m.e. in the RHS of Eq.(A.3) $< d2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_2$ three diagrams of Fig.3 contribute but with the quark s_2 as a spectator. Finally,

$$< \Lambda_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >= \frac{2}{\sqrt{6}}(A_1 - A_2).$$

4. For the decay $\Xi^0 \to \Sigma^0\gamma$ one has

$$6\sqrt{2} < \Sigma^0_\downarrow, \gamma(+1)|O|\Xi^0_\uparrow >=< 2u_2 d_2 s_1 + 2d_2 u_2 s_1 - \qquad (.4)$$

$$-u_2 s_2 d_1 - s_2 u_2 d_1 - d_2 s_2 u_1 - s_2 d_2 u_1, \gamma(+1)|O|2s_1 s_1 u_2 - s_1 u_1 s_2 - u_1 s_1 s_2 >=$$

$$8 < u_2 d_2 s_1, \gamma(+1)|O|s_1 s_1 u_2 > -8 < u_2 d_2 s_1, \gamma(+1)|O|s_1 u_1 s_2 > -$$

$$-4 < u_2 s_2 d_1, \gamma(+1)|O|s_1 s_1 u_2 > +4 < u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 > -$$

$$-4 < d_2 s_2 u_1, \gamma(+1)|O|s_1 s_1 u_2 > +4 < d_2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 > .$$

The 1st m.e. in the RHS of Eq.(A.4) $< u_2 d_2 s_1, \gamma(+1)|O|s_1 s_1 u_2 >= A_4$ is described by three diagrams of Fig.5 with the spectator s_1.

The 2nd m.e. in the RHS of Eq.(A.4) $< u_2 d_2 s_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_3$ is given by the contributions of three diagrams of Fig.3 with the spectator s_1.

The 3rd and the 5th matrix elements (m.e.'s) in the RHS of this expression correspond to the diagrams of Fig.1 type, and we neglect them both.

To the 4th m.e. in the RHS of Eq.(A.4) $< u_2 s_2 d_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_1$ three diagrams of Fig.2 contribute but with the quark s_2 as a spectator. And, finally, to the 6th m.e. in the RHS of Eq.(A.4) $< d_2 s_2 u_1, \gamma(+1)|O|s_1 u_1 s_2 >= A_2$ three diagrams of

Fig.3 contribute but with the quark s_2 as a spectator. Their sum gives:

$$< \Sigma^0_{\downarrow}, \gamma(+1)|O|\Xi^0_{\uparrow} >= \frac{2}{3\sqrt{2}}(A_1 + A_2 - 2A_3 + 4A_4).$$

The analysis performed shows that decays of all the neutral hyperons as well as that of $\Sigma^+ \rightarrow p\gamma$ can be expressed in terms of the same amplitudes A_k, $k = 1, 2, 3, 4$, assumig that spectator quarks do not change them.

Appendix B

Relation between two different representations of the PC-amplitudes

The form of the PC-amplitudes considered here is not obviously unique in the framework of unitary symmetry models. In one of the recent works [10] another form was used which was close to the old form presented by [13]:

$$B(\Sigma^+ \to p\gamma) = \sqrt{2}(\frac{f}{d} - 1)(\mu_{\Sigma^+} - \mu_p)\frac{N}{\mu_p}, \tag{.1}$$

$$B(\Sigma^0 \to n\gamma) = [-(\frac{f}{d} - 1)(\mu_{\Sigma^0} - \mu_n) + \frac{1}{\sqrt{3}}(\frac{3f}{d} + 1)\mu_{\Sigma\Lambda}]\frac{N}{\mu_p},$$

$$B(\Lambda \to n\gamma) = [\frac{1}{\sqrt{3}}(\frac{3f}{d} + 1)(\mu_\Lambda - \mu_n) - (\frac{f}{d} - 1)\mu_{\Sigma\Lambda}]\frac{N}{\mu_p},$$

$$B(\Xi^0 \to \Lambda\gamma) = [-\frac{1}{\sqrt{3}}(\frac{3f}{d} - 1)(\mu_{\Xi^0} - \mu_\Lambda) - (\frac{f}{d} + 1)\mu_{\Sigma\Lambda}]\frac{N}{\mu_p},$$

$$B(\Xi^0 \to \Sigma^0\gamma) = [(\frac{f}{d} + 1)(\mu_{\Sigma^0} - \mu_{\Sigma^0}) + \frac{1}{\sqrt{3}}(\frac{3f}{d} - 1)\mu_{\Sigma\Lambda}]\frac{N}{\mu_p},$$

$$B(\Xi^- \to \Sigma^-\gamma) = -\sqrt{2}(\frac{f}{d} + 1)(\mu_{\Xi^-} - \mu_{\Sigma^-})\frac{N}{\mu_p}.$$

They can be related between them, as was noted in [8], with the help of the simple representation of the broken unitary model of the baryon magnetic moments (We give also values of the magnetic moments which could be obtained in this simple model at $F = 1.8$, $D = 2.7$ $(1 - \epsilon) = 1/3$):

$$\mu_p = F + \frac{1}{3}D = 2.7, \tag{.2}$$

$$\mu_n = -\frac{2}{3}D = -1.8,$$

$$\mu_{\Sigma^+} = F + \frac{1}{3}D + \frac{1}{3}(1 - \epsilon)(F - D) = 2.6,$$

$$\mu_{\Sigma^-} = -F + \frac{1}{3}D + \frac{1}{3}(1-\epsilon)(F-D) = -1.0,$$
$$\mu_{\Xi^-} = -F + \frac{1}{3}D + \frac{2}{3}(1-\epsilon)F - 0.5,$$
$$\mu_{\Xi^0} = -\frac{2}{3}D + \frac{2}{3}(1-\epsilon)F = -1.4,$$
$$\mu_{\Lambda} = -\frac{1}{3}D + \frac{1}{9}(1-\epsilon)(3F+D) = -0.6.$$

However, this representation does not lead to a better description of the data and does not change our conclusion as to strong differences in the coefficients in the description of the PV- and PC- amplitudes.