$2\nu\beta\beta$ decay of deformed nuclei with realistic NN forces

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Introduction

Nuclear $0\nu\beta\beta$-decay ($\bar{\nu} = \nu$)

Light neutrino exchange mechanism

Virtual excitation of states of all multipolarities in $(A,Z+1)$ nucleus
Nuclear $2\nu\beta\beta$-decay

second order weak process within SM
The measured $T^{2\nu \nu}_{1/2}$ (compilation of A. Barabash, 2005) is shown in the table below:

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$T^{2\nu \nu}_{1/2}$, in $10^{19}$ y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{48}$Ca</td>
<td>$4.2^{+2.1}_{-1.0}$</td>
</tr>
<tr>
<td>$^{76}$Ge</td>
<td>$150 \pm 10$</td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>$9.2 \pm 0.7$</td>
</tr>
<tr>
<td>$^{96}$Zr</td>
<td>$2.0 \pm 0.3$</td>
</tr>
<tr>
<td>$^{100}$Mo</td>
<td>$0.71 \pm 0.04$</td>
</tr>
<tr>
<td>$^{116}$Cd</td>
<td>$3.0 \pm 0.2$</td>
</tr>
<tr>
<td>$^{128}$Te</td>
<td>$(2.5 \pm 0.3) \times 10^5$</td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>$90 \pm 10$</td>
</tr>
<tr>
<td>$^{136}$Xe</td>
<td>$&gt; 81$ (90% CL)</td>
</tr>
<tr>
<td>$^{150}$Nd</td>
<td>$0.78 \pm 0.07$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$200 \pm 60$</td>
</tr>
</tbody>
</table>
Inverse Half-Lives \([T_{1/2}(0^+ \rightarrow 0^+)]^{-1}\)

\[\begin{align*}
G^{2\nu}(Q, Z) \left| M^{2\nu}_{GT} \right|^2 & \quad m_{\beta\beta}^2 \ G^{0\nu}(Q, Z) \left| M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2} M^{0\nu}_F \right|^2 \\
\text{Eff. neutrino mass } m_{\beta\beta} = \sum_j m_j U_{e j}^2 & \\
U_{e j} — \text{first raw of the neutrino mixing matrix} &
\end{align*}\]
Nuclear Matrix Elements

\[ M_{2\nu}^{GT} = \sum_s \frac{\langle 0_f | \hat{\beta}^- | s \rangle \langle s | \hat{\beta}^- | 0_i \rangle}{E_s - (M_i + M_f)/2} \]

\[ \hat{\beta}^- = \sum_k \sigma_k \tau_k^- \]

\[ M_{0\nu}^{GT} = \langle 0_f | \sum_{ik} P_{\nu}(r_{ik}, \bar{\omega}) \tau_i^- \tau_k^- \sigma_i \cdot \sigma_k | 0_i \rangle \]

Neutrino potential: \[ P_{\nu}(r, \bar{\omega}) = \]

\[ \frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \bar{\omega})} \approx \frac{R}{r} \phi(\bar{\omega}r) \]
Introduction

V.R., A. Faessler, F. Simkovic, P. Vogel, PRC 68 (2003); NPA 766 (2006); NPA 793 (2007)

\( g_{pp} \) fitted to \( 2\nu\beta\beta \)-decay half-life \( \Rightarrow \) stable \( M^{0\nu} \)

\( 0\nu\beta\beta \) half-lives \( T^{0\nu}_{1/2} \) (in years) assuming \( \langle m_{\beta\beta} \rangle = 50 \) meV.

<table>
<thead>
<tr>
<th>Transition</th>
<th>( M^{0\nu} )</th>
<th>( T^{0\nu}_{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{76}\text{Ge} \rightarrow ^{76}\text{Se} )</td>
<td>3.92</td>
<td>8.6 ( 10^{26} )</td>
</tr>
<tr>
<td>(^{82}\text{Se} \rightarrow ^{82}\text{Kr} )</td>
<td>3.49</td>
<td>2.4 ( 10^{26} )</td>
</tr>
<tr>
<td>(^{130}\text{Te} \rightarrow ^{130}\text{Xe} )</td>
<td>2.95</td>
<td>2.2 ( 10^{26} )</td>
</tr>
<tr>
<td>(^{150}\text{Nd} \rightarrow ^{150}\text{Sm} )</td>
<td>4.16</td>
<td>2.2 ( 10^{25} )</td>
</tr>
</tbody>
</table>

\[ \uparrow \]

strongly deformed \( \Leftarrow \) SM (Nowacki’04): “you can forget about it!”

SNO+ (\& SuperNEMO): 0.1% of \(^{nat}\text{Nd} \) \( \Rightarrow \) 56 kg of \(^{150}\text{Nd} \) \( \Rightarrow m_{\beta\beta} \approx 0.1 \) eV

But: \( M^{0\nu} = 1.57 \) pseudo-SU(3) model of Hirsch \textit{et al.} NPA\textbf{582} (1995)

\( M^{0\nu} = 1.61 \) projected HFB appr. of K. Chaturvedi \textit{et al.} arXiv:0805.4073 [nucl-th]
QRPA in deformed nuclei

• $2\nu\beta\beta$ in deformed nuclei; QRPA with schematic separable forces

• QRPA with realistic forces in deformed nuclei ($0\nu\beta\beta$ of $^{150}$Nd)
  applied first to $2\nu\beta\beta$ (PhD thesis of M. Saleh Yousef)

$0\nu\beta\beta$ (PhD thesis of D. Fang, work in progress)
QRPA in deformed nuclei

Basic relationships

\[ |1 M(K), m \rangle = \sqrt{\frac{3}{16\pi^2}} [D_{MK}^1(\phi, \theta, \psi) Q_{m,K}^\dagger + (-1)^{1+K} D_{M-K}^1(\phi, \theta, \psi) Q_{m,-K}^\dagger] |0^+_{g.s.}\rangle \quad (K = \pm 1), \]

\[ |1 M(K), m \rangle = \sqrt{\frac{3}{8\pi^2}} D_{MK}^1(\phi, \theta, \psi) Q_{m,K}^\dagger |0^+_{g.s.}\rangle \quad (K = 0) \]

\[ M_{GT}^{2\nu} = \sum_{K=0,\pm 1} \sum_{m_i m_f} \langle 0^+_f | \tilde{\beta}_K^- | K^+, m_f \rangle \langle K^+, m_f | K^+, m_i \rangle \langle K^+, m_i | \beta_K^- | 0^+_i \rangle \]

\[ \bar{\omega}_{K,m_i m_f} = (\omega_{K,m_f} - \omega_{K,1_f} + \omega_{K,m_i} - \omega_{K,1_i})/2 + \bar{\omega}_{1_i, exp} \]

case II (unshifted QRPA spectrum)

\[ \bar{\omega}_{K,m_i m_f} = (\omega_{m_f} + \omega_{m_i})/2 \]
Deformed Woods-Saxon s.p. wave functions $|\tau\Omega_\tau\rangle$
decomposed over the spherical harmonic oscillator ones $|\eta\Omega\rangle$

$$|\tau\Omega_\tau\rangle = \sum_{\eta} B^\tau_{\eta}|\eta\Omega_\tau\rangle$$

$|\eta\Omega\rangle = \sum_{\Sigma} C_{i,\Omega-\Sigma}^{j\Omega,\Sigma} \frac{1}{\sqrt{2}} |N/\Lambda = \Omega_\tau - \Sigma\rangle|\Sigma\rangle$ is the spherical harmonic oscillator wave function in the $j$-coupled scheme
QRPA in deformed nuclei

Two-body deformed wave function

\[ |p\bar{n}\rangle = \sum_{\eta_p\eta_n,J} F^{JK}_{p\eta_p, n\eta_n, J} |\eta_p\eta_n, J\rangle \]

\[ |\eta_p\eta_n, J\rangle = C^{JK}_{j_p\Omega_{p,j_n\Omega_n}} |\eta_p\Omega_p\rangle |\eta_n\Omega_n\rangle \]

\[ F^{JK}_{p\eta_p, n\eta_n} = B^{n}_{\eta_p} B^{n}_{\eta_n} (-1)^{j_n - \Omega_n} C^{JK}_{j_p\Omega_{p,j_n\Omega_n}} \]

Two-body residual interaction m.e.

\[ V_{p\bar{n}, p'\bar{n}'} = -2 \sum_{J} \sum_{\eta_p\eta_n} \sum_{\eta_{p'}\eta_{n'}} F^{JK}_{p\eta_p, n\eta_n} F^{JK}_{p'\eta_{p'}, n'\eta_{n'}} G(\eta_p\eta_n\eta_{p'}\eta_{n'}, J) \]

\[ V_{pn', p'n} = 2 \sum_{J} \sum_{\eta_p\eta_n} \sum_{\eta_{p'}\eta_{n'}} F^{JK'}_{p\eta_p, \bar{n}'\eta_{n'}} F^{JK'}_{p'\eta_{p'}, \bar{n}'\eta_{n}} G(\eta_p\eta_{n'}\eta_{p'}\eta_n, J) \]

\[ K'_{pn'} = \Omega_p + \Omega_{n'} = \Omega_{p'} + \Omega_n \]
Results

GT strength functions \((g_{ph} = 1.15; \chi = 3.73/A^{0.7} \text{ MeV})\)

![Graph showing GT strength functions for realistic and separable cases with different shapes and energies.](image-url)
Results

\[ M^{2V}_{\text{GT}} \text{[MeV}^{-1}] \]

\[ \text{Realistic} \quad ^{76}\text{Ge} \rightarrow ^{76}\text{Se} \quad \text{Separable} \]

\[ g_{pp} \quad \kappa[\text{MeV}] \]

\[ \text{def.} = \text{exp. defor.: } \beta_2(^{76}\text{Ge}) = 0.1, \beta_2(^{76}\text{Se}) = 0.16 \text{ (P. Raghavan, At. Data Nucl. Data Tabl. 42 (1989))} \]
Results

GT strength functions

Realistic

Separable

\[ S(\text{GT}^-) \]

- \( \beta_2 = 0.0 \)
- \( \beta_2 = 0.37 \)
- \( \beta_2 = 0.24 \)

\[ S(\text{GT}^+) \]

- \( \beta_2 = 0.0 \)
- \( \beta_2 = 0.23 \)
- \( \beta_2 = 0.21 \)
def. (1) — exp. defor.: $\beta_2^{(150}\text{Nd}) = 0.37 \pm 0.09$, $\beta_2^{(150}\text{Sm}) = 0.23 \pm 0.03$

(P. Raghavan, At. Data Nucl. Data Tabl. 42 (1989))

def. (2) — calc. defor.: $\beta_2^{(150}\text{Nd}) = 0.24$, $\beta_2^{(150}\text{Sm}) = 0.21$

(P. Moeller et al., At. Data Nucl. Data Tabl. 59 (1995))
Conclusions

• Realistic (G-matrix based) NN interaction is implemented in the QRPA equations for deformed nuclei

• GT strength functions and $2\nu\beta\beta$ decay matrix element are calculated for $^{76}$Ge and $^{150}$Nd.

• Prospect for QRPA calculation of $M^{0\nu}$ for $^{150}$Nd is opened

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