LFV in Models with A4 Flavour Symmetry

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based on work with Ferruccio Feruglio, Claudia Hagedorn and Yin Lin:

Neutrino Oscillation Workshop
Otranto, September 6-13, 2008
Low-Energy Effective Lagrangian

- Solar and atmospheric neutrino anomalies $\Rightarrow$ $\nu$ oscillations
- Appealing description $\Rightarrow$ low-energy effective Lagrangian

$$L_{eff} = L_{SM} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + \ldots$$

$$\frac{L_5}{\Lambda} = \frac{(\bar{H}^+ \ell)(\bar{H}^+ \ell)}{\Lambda} = \frac{1}{2} \frac{v^2}{\Lambda} \nu\nu + \ldots \Rightarrow \Lambda \approx 10^{15} \text{ GeV}$$

**neutrinos provide a window in GUT physics**

- Low Energy Observables:
  - $\nu$ masses
  - $\nu$ oscillations

- (g-2)$_\mu$ discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

- GUTs
- flavour symmetries
- $\nu^c$
- superheavy gauge bosons

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Luca Merlo

Neutrino Oscillation Workshop
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Lepton Mixing Angles and the Tri-bimaximal Mixing

- Neutrino Oscillation Parameters

\[
U_{PMNS} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

- Indications for \( \theta_{13} > 0 \)

\[
\sin^2 \theta_{23} = 0.45^{+0.16}_{-0.09}, \quad \theta_{23} = (42.1^{+9.2}_{-5.3})^\circ
\]

\[
\sin^2 \theta_{12} = 0.326^{+0.05}_{-0.04}, \quad \theta_{12} = (34.8^{+3.0}_{-2.5})^\circ
\]

\[
\sin^2 \theta_{13} < 3.2 \times 10^{-2}, \quad \theta_{13} < 10.3^\circ
\]

- Tri-bimaximal Neutrino Mixing (TBM) [Rodejohan's and Chen's Talks]

\[
U_{TBM} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\]

\[
\sin^2 \theta_{23}^{TB} = \frac{1}{2}, \quad \theta_{23}^{TB} = 45^\circ
\]

\[
\sin^2 \theta_{12}^{TB} = \frac{1}{3}, \quad \theta_{12}^{TB} \approx 35.3^\circ
\]

\[
\sin^2 \theta_{13}^{TB} = 0, \quad \theta_{13}^{TB} = 0^\circ
\]

[Palazzo's Talk]
The Symmetry Group $A_4$

$A_4$ is a subgroup of $SO(3)$ leaving a regular tetrahedron invariant.

It is generated by two elements which obey to

$$S^2 = T^3 = (ST)^3 = 1$$

$S$ generates a subgroup $G_S \cong Z_2$ of $A_4$

$T$ generates a subgroup $G_T \cong Z_3$ of $A_4$

$G_f = A_4 \times Z_3 \times U(1)_{FN}$

$G_e \equiv G_T$

$G_\nu \equiv G_S$

$m_e$ diagonal

$U_{TBM}^T m_\nu U_{TBM} = m_\nu^{\text{diag}}$

[Ma & Rajasekaran 2001; Ma 0409075; Altarelli & Feruglio 0504165 + 0512103; Altarelli, Feruglio & Lin 0610165]
$G_f = A_4 \times Z_3 \times U(1)_{FN}$

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Can also be extended to the quark sector

[Feruglio, Hagedorn, Lin & L.M. 0702194; Altarelli, Feruglio & Hagedorn 0802090]

TBM requires a well determined vacuum alignment

\[ \langle \varphi_T \rangle / \Lambda = (u, 0, 0) + O(u^2) \]
\[ \langle \varphi_S \rangle / \Lambda = c_b(u, u, u) + O(u^2) \]
\[ \langle \xi \rangle / \Lambda = c_\alpha u + O(u^2) \]
\[ \langle \theta \rangle / \Lambda \equiv \tau \]

\[ t \approx 0.05 \]
\[ 0.001 < u < 0.05 \]
\[ \tau \text{ Yukawa coupling} < 4\pi \]

\[ m_\ell(\langle \varphi \rangle) = \frac{v}{\sqrt{2}} \left[ \begin{pmatrix} c_\ell l^2 u & 0 & 0 \\ 0 & c_\mu l u & 0 \\ 0 & 0 & c_\tau u \end{pmatrix} + O(u^2) \right] \]
\[ m_\nu(\langle \varphi \rangle) = \frac{v^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix} u + O(u^2) \]
Dipole Moments and LFV Decays

After integrating out all the d.o.f. related to $\Lambda$ and to $M$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \delta \mathcal{L}(m_\nu) + i \frac{e}{M^2} e^c I^I \left( \sigma^{\mu \nu} F_{\mu \nu} \right) \mathcal{M} \left( \langle \varphi \rangle \right) e^c I^I | h.c. | \ldots$$

Note that:
- $M < \Lambda$ implies it is possible to measure observables
- the flavour structure of $\mathcal{M}(\langle \varphi \rangle)$ is controlled by the same parameters which give rise to the Yukawa couplings, indeed $[\mathcal{M}(\langle \varphi \rangle)]_{ij} = \alpha_{ij} [\gamma_e]_{ij}$ where $\alpha_{ij} = O(1)$

In the basis in which charged leptons are diagonal

$$\text{Im} \left[ \mathcal{M} \left( \langle \varphi \rangle \right) \right]_{ii}$$

$$\text{Re} \left[ \mathcal{M} \left( \langle \varphi \rangle \right) \right]_{ii}$$

$$\left| [\mathcal{M} \left( \langle \varphi \rangle \right)]_{ij} \right|^2 \quad (i \neq j)$$

- $d_i$ electric dipole moment (EDM)
- $a_i$ magnetic dipole moment (MDM)

$$R_{ij} = \frac{BR(\ell_i \rightarrow \ell_j \gamma)}{BR(\ell_i \rightarrow \ell_j \nu_i \bar{\nu}_j)}$$

LFV processes

4-fermion operators:
- $\mu \rightarrow eee$
- $\tau \rightarrow eee$
- $\tau \rightarrow \mu \mu \mu$
- $\ldots$
Constraints on $M$

Note: $M$ corresponds to the scale of new physics, but the mass scale of the new particles is $M'$, which is in a weakly interacting theory $M' \approx g M/4\pi$

Dipole Matrix

$$M(\langle \varphi \rangle) = \begin{pmatrix} O(t^2 u^2) & O(t^2 u^2) & O(t^2 u^2) \\ O(t u^2) & O(t u) & O(t u^2) \\ O(u^2) & O(u^2) & O(u) \end{pmatrix}$$

up to $O(1)$ coefficient $R_{\mu e} \approx R_{\tau \mu} \approx R_{\tau e}$ independently from $\theta_{13}$

$\tau$ decays are below the expected future sensitivity

$$R_{\mu e} < 1.2 \times 10^{-11}(10^{-13}) \implies \frac{|u|}{M^2} < 1.2 \times 10^{-11}(1.1 \times 10^{-12}) \text{ GeV}^{-2}$$

- Strongest constraint from $d_e$: (cancellations in $\text{Im}[M]$: accidental or due to CP-conservation)
- Interesting indication from $\delta a_{\mu}$

$M > 80 \text{ TeV}$

{| $d_\mu < 2.8 \times 10^{-19} \text{ e cm}$ | $M > 80 \text{ GeV}$ |
| $\delta a_e < 3.8 \times 10^{-12}$ | $M > 350 \text{ GeV}$ |
| $\delta a_\mu \approx 30 \times 10^{-10}$ | $M \approx 2.7 \text{ TeV}$ |

$M > 10 \text{ (30) TeV}$

$|u| \approx 0.001 \implies M > 10 \text{ (30) TeV}$

$|u| \approx 0.05 \implies M > 70 \text{ (200) TeV}$

Probably above the region of interest for $(g-2)_\mu$ and for LHC
SUSY Case

The $M_{ij}$ ($i \neq j$) come from two sources:
- NLO corrections to $\langle \varphi_T \rangle$
- double flavon insertion of the type $\xi^+ \varphi_S$, $\xi \varphi_S^+$

In a (soft broken) SUSY version, a chirality flip requires an insertion of $\varphi_T$, at the LO in the SUSY breaking parameters

$$\mathcal{M} (\langle \varphi \rangle) = \begin{pmatrix}
O(t^2 u) & O(t^2 u^2) & O(t^2 u^2) \\
O(tu^2) & O(tu) & O(tu^2) \\
O(u^2) & O(u^2) & O(u)
\end{pmatrix} \rightarrow \mathcal{M} (\langle \varphi \rangle) = \begin{pmatrix}
O(t^2 u) & O(t^2 u^2) & O(t^2 u^2) \\
O(tu^3) & O(tu) & O(tu^2) \\
O(u^3) & O(u^3) & O(u)
\end{pmatrix}$$

- The constraints from EDM and MDM are the same
- In most of the allowed range for $u$, $R_{\mu e} \approx R_{\tau \mu} > R_{\tau e}$

$$|u| \approx 0.001 \quad \Rightarrow \quad M > 0.7 \ (2) \ \text{TeV}$$
$$|u| \approx 0.05 \quad \Rightarrow \quad M > 14 \ (48) \ \text{TeV}$$

There is a range for $u$ in which it is possible to explain both $\text{BR}(\mu \rightarrow e \gamma)$ and $\delta a_\mu$
Relations between Observables

It is easy to find a relation between $\text{BR}(\mu \rightarrow e\gamma)$, the $\delta a_\mu$ and the $\theta_{13}$

$$\frac{\text{BR}(\mu \rightarrow e\gamma)}{\text{BR}(\mu \rightarrow e\nu_\mu\nu_\mu)} = \frac{12\pi^3 \alpha_{cm}}{G_F^2 m_\mu^4} (\delta a_\mu)^2 \left[ |\gamma^{(1)}|^2 |u|^4 + \frac{m_e^2}{m_\mu^2} |\gamma^{(2)}|^2 |u|^2 \right]$$

$$|u| \approx \theta_{13}$$

Only a sharp limit on $\theta_{13}$, which cannot be larger than few degrees
Conclusions (I)

- $A_4$ models can accommodate the experimental data with a very good approximation. Among the others, $A_4 \times Z_3 \times U(1)_{FN}$ represents a sort of minimal choice.
- We have analysed a set of low-energy observables, which can give information about the BSM theory. These are lepton EDM, MDM and LFV $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma$.
- General Case:
  - The strongest bound on the energy scale $M$ comes from the EDM of the electron: $M > 80$ TeV. This suggests a cancellation in the imaginary part of the dipole matrix.
  - For LFV processes $R_{\mu e} \approx R_{\tau \mu} \approx R_{\tau e}$, which is a distinctive aspect of the model. It implies that $\tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma$ have rates smaller than present and future sensitivity. The bound on the energy scale is $M > 10^{-70}$ TeV.
  - The anomalous MDM of the muon provides indication for $M$ of few TeV, interesting for LHC.
Conclusions (II)

- **SUSY Case:**
  - There is a cancellation in the elements of the dipole matrix below the diagonal. As a result the bound on $M$ becomes less severe: $M > 0.7\text{-}14 \text{ TeV}$
  - There is a range of values of $|u|$ for which the scale $M$ can be such small to explain the discrepancy in $\delta a_\mu$ and the BR($\mu \rightarrow e\gamma$)
  - The model indicates a value for $\theta_{13}$ of few degrees, which is close but probably just below the future expected sensitivity.

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**Graph:**
- **SUSY x $A_4$**
  - $M$ can be few TeV
  - only $\mu \rightarrow e\gamma$ can be above the future sensitivity
  - disfavoured by $A_4$
Thanks

Tri-bimaximal Mixing

\[ U_{PMNS} = U_{TBM} \]

Consider a flavour symmetry \( G_f \), that is broken into two different subgroups: \( G_e \) in the charged lepton sector, and \( G_\nu \) in the neutrino sector. \( m_e \) and \( m_\nu \) are invariant under \( G_e \) and \( G_\nu \) respectively. It is possible to get

\[ U_{TBM}^T m_\nu U_{TBM} = m_\nu^{\text{diag}} \]

\[ U_{PMNS} \equiv U_e^+ U_\nu = U_{TBM} \]
Extension to Quarks

\[ A_4 \]

[Altarelli & Feruglio 0512103]

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It is the double covering of \( A_4 \).
It has 24 elements and 7 representations: 3, 1, 1', 1" & 2, 2', 2"

\[ T' \]

[Feruglio, Hagedorn, Lin & L.M. 0702194]

- same assignment as in the lepton sector
- quark mass matrices diagonal and @ LO \( V_{\text{CKM}} = 1 \)
- Corrections induce by higher dimensional operators are negligible
- top mass from dim 5 operator

- Lepton sector as in \( A_4 \) model
- \( t \) and \( b \) masses at the renormalizable level
- First step: the second 2 generations with \( m_t, m_b > m_c, m_s \neq 0 \)
- and \( V_{cb} \)
- Second step: remaining mass and mixing angles

\[ m_u = \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \]

\[ m_d = \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \]
The superpotential of the model is given by

\[ w_d = N (\varphi_0^T \varphi_T) + g (\varphi_0^T \varphi_T \varphi_T) + g_1 (\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi} (\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2 \]

and the minimum is at

\[
\begin{align*}
\langle \varphi_T \rangle / \Lambda &= (v_T, 0, 0) \\
\langle \varphi_S \rangle / \Lambda &= (v_S, v_S, v_S) \\
\langle \xi \rangle / \Lambda &= u \\
\langle \tilde{\xi} \rangle / \Lambda &= 0 \\
\end{align*}
\]

\[ v_T = -\frac{3N}{2g}, \quad v_S^2 = -\frac{g_4}{3g_3} u^2 \]

\[ u \text{ undetermined} \]
Vacuum Alignment (ED's)

The Model has 1 compactified ED and 2 branes
(The assignments of the quantum numbers are slightly different)

\[
\frac{\langle \varphi \rangle}{\Lambda} = \nu(1, 1, 1) \\
\frac{\langle \varphi' \rangle}{\Lambda} = \nu'(1, 0, 0) \\
\frac{\langle \xi \rangle}{\Lambda} = u
\]

\[
F(x, y) = (F_1, \overline{F}_2) \sim 3
\]

[Altarelli & Feruglio 0504165]
4-Fermion Operators

We consider here only the leading order 4-fermion operators, which are invariant under the Lorentz x SU(2) x U(1)_{em} x G_{f}.

- **Conserve** the individual lepton number. Their bound on \( M \) is of several TeV

\[
\frac{1}{M^2} \bar{f}^c f^c \bar{f'}^c f'^c \quad \text{and} \quad \frac{1}{M^2} \bar{f}^c f^c (\bar{l}l) \quad (f, f' = e, \mu, \tau)
\]

- **Violate** the individual lepton number with the selection rule \( \Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2 \)

\[
\frac{1}{M^2} \bar{e}^c \mu^c \bar{\tau}^c \mu^c \quad \text{and} \quad \frac{1}{M^2} (\bar{l}l \bar{l}l) \]

The second one stands for several independent invariants, obtained through different contractions of \( A_4 \): for instance

\[
(\bar{l}l)'(\bar{l}l)'' = \bar{l}_e l_e \bar{l}_\mu l_\mu + \bar{l}_\mu l_\mu \bar{l}_\tau l_\tau + \bar{l}_\tau l_\tau \bar{l}_e l_e + \left[ \bar{l}_e l_\tau \bar{l}_\mu l_\tau + \bar{l}_\mu l_e \bar{l}_\tau l_e + \bar{l}_\tau l_\mu l_e l_\mu + \text{h.c.} \right]
\]

These operators contribute to \( \tau^- \rightarrow \mu^+ e^- e^- \), \( \tau^- \rightarrow e^+ \mu^- \mu^- \).

From the current experimental input the bound on \( M \) is 15 TeV.
SUSY Extension

The $M_{ij}$ ($i \neq j$) come from two sources:

- NLO corrections to $\langle \phi_T \rangle$
- double flavon insertion of the type $\xi^+ \phi_S$, $\xi \phi_S^+$

In a (soft broken) SUSY version, a chirality flip requires an insertion of $\phi_T$, at the LO in the SUSY breaking parameters:

\[
\begin{align*}
   & m_\ell & \iff & \int d^2 \theta_{SUSY} c h_d \left( \frac{\phi_T}{\Lambda} \ell \right) \\
   & m_{\tilde{\ell}_{RL}} & \iff & \int d^2 \theta_{SUSY} c h_d \left( \frac{\phi_T}{\Lambda} \ell \right) \theta_{SUSY}^2 m_{SUSY}
\end{align*}
\]

Other type of chirality-flip insertions are suppressed

\[
\frac{1}{\Lambda} \int d^2 \theta_{SUSY} d^2 \bar{\theta}_{SUSY} c h_d \left( \frac{\xi^+ \phi_S}{\Lambda^2} \ell \right) \theta_{SUSY}^2 \bar{\theta}_{SUSY}^2 m_{SUSY}^2 \iff (m_{\tilde{\ell}_{RL}})\text{corrections}
\]

The only sources of chirality flip are the lepton and slepton (RL) masses, at the LO: the main effects come from $\phi_T$ alone. We take this as a definition of SUSY case.
Comparison with MFV

- The flavour group $G_f$ is the largest possible: $G_f = SU(3)_\ell \times SU(3)_{e^c} \times ...$

- The fields transform as

$$\ell = (\bar{3}, 1) \quad e^c = (1, 3)$$

- $G_f$ is broken only by the Yukawa coupling

$$\mathcal{L}_{mass} = \bar{e}^c H^+ y_e \ell + \frac{(\tilde{H}^+ \ell) Y (\tilde{H}^+ \ell)}{\Lambda} \implies y_e = (3, 3)$$

$$Y = (6, 1)$$

- The Yukawa couplings can be written as

$$y_e = \sqrt{2} \frac{m_e^{diag}}{v} \quad Y = \frac{\Lambda}{v^2} U^* m_\nu^{diag} U^+$$

- The diagonal elements of the dipole matrix have the same order of $[\mathcal{M}(\langle \varphi \rangle)]_{ij}$ in $A_4$ models, thus the constraints on $M$ from MDM are similar (EDM are zero because $[\mathcal{M}(\langle \varphi \rangle)]_{ij}$ are assumed real)

[Dambrosio, Giudice, Isidori & Strumia 2002; Cirigliano, Grinstein, Isidori & Wise 2005]
The off-diagonal elements of the dipole matrix

\[ [\mathcal{M}((\mathcal{O}))]_{ij} = \beta (\mathcal{O}e Y^+ Y)_{ij} + \ldots \]

\[ = \sqrt{2}\beta \left( \frac{m_\ell}{v} \right) \frac{\Lambda^2}{v^4} \left[ \Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3} \right] + \ldots \]

✓ a possible signal @ MEG can always be accommodated (exception for \( \theta_{13} \approx 0.02 \) when \( R_{\mu e} = 0 \))
✓ a non-observation of \( R_{ij} \) can be explained by lowering \( \Lambda \)

\[
\left( \frac{R_{\mu e}}{R_{\tau \mu}} \right) \approx \frac{2}{3} r \pm \sqrt{2} \sin \theta_{13} e^{i\delta} < 1 \quad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}
\]

MFV (\( M \) can be few TeV)

both \( \mu \to e\gamma \) and \( \tau \to \mu\gamma \) could be above the future sensitivity

SUSY x \( A_4 \) (\( M \) can be few TeV)

only \( \mu \to e\gamma \) can be above the future sensitivity
disfavoured by \( A_4 \)

\[ R_{\mu e} < 1.2 \times 10^{-11} \]

\[ \Rightarrow \quad R_{\tau \mu} < 10^{-9} \]