A new way of comparing DBD experiments

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Outline

• DBD sensitivity and experimental parameters

• Why do we need a “new” way of comparing experiments?

• Redefinition of key parameters: *performance and scale*

• Comparison in the *P-S* space

• The future of DBD experiments: movements in the *P-S* space
$0\nu\beta\beta$ decay sensitivity

Neutrino-less double beta decay half life can be expressed (for light neutrino scenario) as:

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2 / m_e^2$$
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- NME
- Beyond SM physics
0νββ decay sensitivity

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- phase space (well known, calculated with some approximations)
- NME (large theoretical uncertainties, strong dependance on nuclear model, no straightforward extrapolation from SM processes)
- axial vector constant suppression (large uncertainties)
- which beyond SM physics has to be considered (light neutrinos, heavy neutrinos, Majorons etc.)
- possible correlations between phase space and NME
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All experimental techniques can be compared in terms of the \( F_{0\nu} \) figure of merit.
Experiments can measure the decay total half life, with a sensitivity that can be expressed as:

$$F_{0\nu} = \tau_{1/2}^{\text{Back.Fluct.}} = \ln 2 \ N_{\beta\beta} \epsilon \frac{T}{n_B} =$$

$$= \ln 2 \times \frac{x \ \eta \ \epsilon \ N_A}{A} \sqrt{\frac{M}{B} \frac{T}{\Delta}} \ (68\% CL)$$

and is the process half-life corresponding to the maximum signal $n_B = \sqrt{M \cdot T \cdot B \cdot \Delta}$ that could be hidden by the background fluctuations at 68% confidence level.
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- \( A \): molecular mass
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- $M$ detector mass (after fiducial volume cuts)
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NB: only signal efficiency has to be included; efficiencies (like fiducial volume cuts)
that affect both signal and background in the same way only reduce the effective
detector mass as far as the \( B \) parameter is normalised to the mass after the cut
Why a “new” way?

- Given the large theoretical uncertainties, sensitivity to the decay half life $F_{0\nu}$ is used as a figure of merit to describe the “quality” and the potential of an experimental technique.

- $F_{0\nu}$ depends on many parameters, but only two of them at a time are usually used to compare experiments, by representing them as a point in $(\Delta,B)$ or $(M,B)$ or $(\Delta,M)$ planes.

- Most of the time, this method gives an incomplete view of the problem and can lead to a wrong interpretation of the experiments potential:
  
  - the sensitivity of a small detector with an excellent energy resolution and a technology that allows for a complete rejection of the background can be small, due to the small number of available DBD nuclei and correspondingly small signal.
  
  - the sensitivity of a very large detector with a bad energy resolution and large background can also be small if the (maybe large) signal is diluted over a large energy region and hidden by the background fluctuations.
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Need for a tool to compare experiments by simultaneously using the available information about all the parameters affecting the sensitivity.
0νββ decay sensitivity

The minimum detectable signal at a given confidence level depends on the background level:

- “finite background” FB: the average number of background events in the ROI collected during the experiment live time is larger than one; the minimum detectable signal at a given C.L. depends on the fluctuation of the number of background events
- “zero background” ZB: the probability of collecting more than one event in the ROI during the live time is negligible; the minimum detectable signal only depends on signal fluctuations and is fixed for a given C.L. ($n_{CL} = 1.14$ for 68% C.L.)

The sensitivity formula for the two cases becomes:

$$M \cdot T \cdot B \cdot \Delta$$
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\[
M \cdot T \cdot B \cdot \Delta \leq 1
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\[
F_{0\nu}^{ZB} = \ln 2 \cdot N_{\beta\beta} \epsilon \frac{T}{n_{CL}} = \ln 2 \times \frac{x \eta \epsilon N_{A} M T}{A n_{CL}}
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Parameters redefinition

In order to represent experiments in a 3D space two parameters has to be chosen that gather all the experiment features.

First we define:

\[ \zeta = \frac{x\eta e}{A} \]

intrinsic properties of the source: usually don’t change from one generation to the next

The dimensions of this parameter are:

\[ [\zeta] = \frac{\text{# of moles of “efficient” } \beta\beta \text{ isotope}}{\text{mass}} = \frac{n_{\beta\beta}}{\text{kg}} \]

By multiplying mass and background by it:

\[ \zeta M = \tilde{M} \quad \text{and} \quad \frac{B}{\zeta} = \tilde{B} \]

This parameter allows us to express the mass and the background with the dimensions of “number of moles of DBD isotopes that can produce a signal”, no matter which is the material or the experimental technique.
Parameters redefinition

The sensitivity becomes:

\[ F_{0\nu} = \ln 2 \, N_A \times \frac{\zeta MT}{n_L} = \ln 2 \, N_A \times \frac{\tilde{M}T}{n_L} \]

The separation between the two regimes is conserved because

\[ B \Delta \cdot MT = \tilde{B} \Delta \cdot \tilde{M}T \]

A further simplification is obtained by replacing

\[ S = \tilde{M} \cdot T \]

SCALE

and

\[ P = \tilde{B} \cdot \Delta \]

PERFORMANCE
Parameters redefinition

The two new parameters:

• SCALE: represents the “dimension” of the experiment, both in terms of size and live time. It has the same dimensions of an exposure, expressed as number of moles of detectable emitting isotope per year of live time. It’s a measure of how much signal can be expected in the experiment

• PERFORMANCE: measures how good is the experiment in measuring the signal compared to the background level. It’s expressed in counts per mole of detectable emitting isotope per year

With this redefinition, the sensitivity is simply:

\[ F_{0\nu} = \begin{cases} 
\ln 2 \times \frac{N_A}{n_L} \times S, & \text{if } P \cdot S \lesssim 1 \\
\ln 2 \times \frac{N_A}{n_L} \times \sqrt{\frac{S}{P}}, & \text{if } P \cdot S > 1 
\end{cases} \]
The \((P,S,F_{0\nu})\) space

Each experiment can be represented in the \((P,S,F_{0\nu})\) space as a point on the \(F_{0\nu}(P,S)\) surface just defined.
Critical comparison

Each experiment can be represented in the \((P,S,F_{0\nu})\) space as a point on the \(F_{0\nu}(P,S)\) surface just defined.
In a 2D projection of the sensitivity surface (in log-log scale):

- the “golden region” is a straight line with slope -1
- iso-sensitivity curves are straight lines parallel to the P axis in the ZB region
- iso-sensitivity curves are straight lines with unitary slope in the FB region

\[ P \cdot S \approx 1 \]
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Critical comparison

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Isotope</th>
<th>$\tilde{M}$</th>
<th>$\tilde{B}[\times 10^{-3}]$</th>
<th>$\Delta$</th>
<th>$P[\times 10^{-3}]$</th>
<th>$S(5y)$</th>
<th>$F_{0\nu}[\times 10^{26}y]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUORE[10,11]</td>
<td>$^{130}$Te</td>
<td>1389.5</td>
<td>5.3</td>
<td>5</td>
<td>26.7</td>
<td>6947.5</td>
<td>2.13$^{b}$</td>
</tr>
<tr>
<td>CUORE-0[10]</td>
<td>$^{130}$Te</td>
<td>66.3</td>
<td>44</td>
<td>5.6</td>
<td>244</td>
<td>331.5</td>
<td>0.15$^{a}$</td>
</tr>
<tr>
<td>GERDA[12]</td>
<td>$^{76}$Ge</td>
<td>119.2</td>
<td>1.9</td>
<td>4.8</td>
<td>9.2</td>
<td>596</td>
<td>1.06$^{a}$</td>
</tr>
<tr>
<td>GERDA-II[12]</td>
<td>$^{76}$Ge</td>
<td>328.2</td>
<td>1.8/0.11</td>
<td>3.2</td>
<td>5.7/0.34</td>
<td>1641</td>
<td>2.24$^{b}$ / 6.01$^{c}$</td>
</tr>
<tr>
<td>KamLAND-Zen[14]</td>
<td>$^{136}$Xe</td>
<td>1318.2</td>
<td>1.0</td>
<td>243.2</td>
<td>243.2</td>
<td>6591</td>
<td>0.69$^{a}$</td>
</tr>
<tr>
<td>EXO-200[13]</td>
<td>$^{136}$Xe</td>
<td>481.6</td>
<td>0.31</td>
<td>96.5</td>
<td>30.3</td>
<td>2408</td>
<td>1.18$^{a}$</td>
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<tr>
<td>MJD[18]</td>
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<td>237.6</td>
<td>0.095</td>
<td>4</td>
<td>0.4</td>
<td>1188</td>
<td>4.35$^{c}$</td>
</tr>
<tr>
<td>SuperNEMO-D[15]</td>
<td>$^{82}$Se</td>
<td>23</td>
<td>0.15</td>
<td>120</td>
<td>18.2</td>
<td>115</td>
<td>0.33$^{c}$</td>
</tr>
<tr>
<td>SNO+[19]</td>
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<td>1252.8</td>
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<td>26.9</td>
<td>6264</td>
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<tr>
<td>NEXT-100[16]</td>
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<td>5.4</td>
<td>827</td>
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<tr>
<td>Lucifer[17]</td>
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<td>125.1</td>
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<td>20</td>
<td>4</td>
<td>636.5</td>
<td>1.65$^{c}$</td>
</tr>
</tbody>
</table>

$^{a}$ The experiment is running and the parameters have been measured

$^{b}$ The experiment feasibility has been demonstrated and the parameters values have been measured with demonstrators (realistic estimation)

$^{c}$ The experiment is in a conceptual design phase; the parameters values are theoretical estimations

10. R. Ardito et al., arXiv:hep-ex/0501010
11. CUORE Collaboration, arXiv:nucl-ex/1109.0494
17. A. Giuliani et al, Proceedings of BEYOND Conference, Cape Town, South Africa, 2010
Movements in the P-S plane

The performance-scale representation shows the optimal strategy to increase the sensitivity, depending on the region of the plane where an experiment lies.

ZB
- **P**: no improvement, background already negligible
- **S**: sensitivity maximally improved by increasing the signal

FB
- same (non-maximal) improvement both improving **P** or **S**
Movements in the P-S plane

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*Optimal paths* for increasing the sensitivity are perpendicular to the iso-sensitivity lines in each point of the plane (in linear scale).

Once an experiment reaches the *golden region* the maximal improvement is obtained by simultaneously reducing P and increasing S.
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**Diagram**

The diagram illustrates the movements in the P-S plane, where the performance-scale representation highlights the optimal paths for increasing sensitivity. The golden region indicates the area where maximal improvement is achieved by adjusting $P$ and $S$.
Conclusions

• Most of the plots usually produced to compare sensitivity of neutrino-less double beta decay experiments only consider a subset of the critical experimental parameters, giving incomplete information.

• A redefinition of the “experimental” variables entering the sensitivity calculation is proposed that leads to a unified representation in the performance-scale space.

• Thanks to this representation the sensitivity to neutrino-less double beta decay half life of completely different experimental techniques can be compared directly.

• Thanks to this representation the paths leading to the maximum improvement of sensitivity can be fully described for experimental techniques laying in different regions of the parameters space.

• Short term plans: finalise a web based tool where everybody can produce its own comparison plot, with constantly up-to-date experiments database built upon publications and proceedings.