From atmospheric neutrinos to the neutrino mass hierarchy
Neutrino oscillation

If neutrinos have small and different masses → flavor eigenstates of weak interactions $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$

≠ propagation mass eigenstates $\nu_i = \nu_1, \nu_2, \nu_3$

Connected by unitary matrix (Pontecorvo-Maki-Nakagawa-Saka PMNS)

$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} & U_{\alpha 3} \\ U_{\beta 1} & U_{\beta 2} & U_{\beta 3} \\ U_{\gamma 1} & U_{\gamma 2} & U_{\gamma 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\delta} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}s_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}s_{13} \end{pmatrix}$$

$c_{ij} = \cos(\theta_{ij}), s_{ij} = \sin(\theta_{ij}), \delta = \text{CP-violating phase}$

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

Transition probability

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |\langle \nu_\beta(t) | \nu_\alpha(t=0) \rangle|^2$$

$$= \delta_{\alpha \beta} - 4 \sum_{j > i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E_\nu} \right)$$

(U only real components; no CP violation; neutrinos Dirac type)
3-flavor oscillation in vacuum

Oscillation probabilities for an initial electron neutrino

$\nu_e$, $\nu_\mu$, $\nu_\tau$

$\Delta m_{21}^2$ (solar)

$\Delta m_{32}^2$ (atm.)

$\theta_{13}$ reactor

$\theta_{12}$ (solar)

$\theta_{23}$ (atm.)

L/E (km/GeV)
3-flavor oscillation in vacuum

Oscillation probabilities for an initial electron neutrino

\[ P_{\nu_e \rightarrow \nu_\mu} = \delta_{e\mu} - 4 \sum_{j>i} \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E_{\nu}} \right) \]

Neutrino oscillation in vacuum not sensitive to hierarchy

Mass hierarchy

\[ \Delta m_{32}^2 \]

\[ \Delta m_{21}^2 \]

\[ \Delta m_{31}^2 \]

\[ m_1^2 \]

\[ m_2^2 \]

\[ m_3^2 \]

\[ m(\nu_e) \]

\[ m(\nu_\mu) \]

\[ m(\nu_\tau) \]
Matter effects

- In matter particles can gain “effective mass” in presence of interactions (known from solid state physics)

**muon, tau neutrinos**

\[
\begin{align*}
\nu_{\mu,\tau} & \quad \nu_{\mu,\tau} \\
\Downarrow & \quad \Downarrow \\
e^- & \quad e^- \\
(p,n) & \quad (p,n)
\end{align*}
\]

**electron neutrinos**

\[
\begin{align*}
\nu_e & \quad \nu_e \\
\Downarrow & \quad \Downarrow \\
e^- & \quad e^- \\
(p,n) & \quad (p,n)
\end{align*}
\]

\[
\begin{align*}
\nu_e & \quad \nu_e \\
\Downarrow & \quad \Downarrow \\
e^- & \quad e^- \\
W & \quad \nu_e
\end{align*}
\]

\[
\begin{align*}
\nu_e & \quad \nu_e \\
\Downarrow & \quad \Downarrow \\
e^- & \quad e^- \\
W & \quad \nu_e
\end{align*}
\]

\[
\begin{align*}
\nu_e & \quad \nu_e \\
\Downarrow & \quad \Downarrow \\
e^- & \quad e^- \\
W & \quad \nu_e
\end{align*}
\]
Matter effects

- In matter particles can gain “effective mass” in presence of interactions (known from solid state physics)

### Effective parameters for 2-flavor mixing:

\[
\begin{align*}
\Delta m^2_M &= \xi \times \Delta m^2 \\
\sin 2\theta_M &= \frac{\sin 2\theta}{\xi} \\
\xi &= \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{A_{CC}}{\Delta m^2}\right)^2}
\end{align*}
\]

\[
\rightarrow A_{CC} = \pm 2\sqrt{2} E_\nu G_F N_e
\]

\[\uparrow \quad (+) \, \nu_e \quad ; \quad (-) \, \bar{\nu}_e\]
Matter effects – the MSW effect

Effective parameters for 2-flavor mixing:

\[ \Delta m^2_M = \xi \times \Delta m^2 \]
\[ \sin 2\theta_M = \frac{\sin 2\theta}{\xi} \]

\[ \xi = \sqrt{\sin^2 2\theta + \left( \cos 2\theta - \frac{A_{CC}}{\Delta m^2} \right)^2} \]

Mixing becomes maximal (sin 2\theta_M = 1) if \( A_{CC}^R = \Delta m^2 \cos 2\theta \) i.e.

\[ N_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} E G_F} \]

\[ \rightarrow \] total transition from one to other flavor possible!
Matter effects – the MSW effect

Effective parameters for 2-flavor mixing:

\[
\Delta m^2_M = \xi \times \Delta m^2 \\
\sin 2\theta_M = \frac{\sin 2\theta}{\xi}
\]

\[
\xi = \sqrt{\sin^2 2\theta + \left( \cos 2\theta - \frac{A_{CC}}{\Delta m^2} \right)^2}
\]

Mixing becomes maximal (\(\sin 2\theta_M = 1\)) if \(A_{CC}^R = \Delta m^2 \cos 2\theta\) i.e. \(N_e^R = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}E G_F}\)

\(\rightarrow\) total transition from one to other flavor possible!

Example: Neutrinos from the Sun

▷ Propagation along smoothly decreasing density profile \(\rightarrow\) at some point \(N_e = N_e^R\)

▷ Effect is energy dependent ...

![Graph showing survival probability vs. energy](image.png)
Matter effects – the MSW effect

Effective parameters for 2-flavor mixing:

\[ \Delta m^2_M = \xi \times \Delta m^2 \]
\[ \sin 2\theta_M = \frac{\sin 2\theta}{\xi} \]

\[ \xi = \sqrt{\sin^2 2\theta + \left( \cos 2\theta - \frac{A_{CC}}{\Delta m^2} \right)^2} \]

Mixing becomes maximal ($\sin 2\theta_M = 1$) if $A_{CC}^R = \Delta m^2 \cos 2\theta$ i.e. $N_{eR} = \frac{\Delta m^2 \cos 2\theta}{2 \sqrt{2} E G_F}$

→ total transition from one to other flavor possible!

Example: Neutrinos from the Sun

- Propagation along smoothly decreasing density profile → at some point $N_e = N_e^R$
- Effect is energy dependent ...
- ... and differs for $\Delta m_{21}^2 > (<) 0$ !
  → sign of $\Delta m_{21}^2$ (octant of $\theta_{12}$) measured
MSW-effect and the octant of the mixing angle

- Two-flavor oscillation:
  \[ |\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \]
  \[ |\nu_{\mu}\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \]
  \[ \theta < 45^\circ \text{ (1st octant)} \rightarrow \nu_e \text{ mostly } \nu_1 \]
  \[ \theta > 45^\circ \text{ (2nd octant)} \rightarrow \nu_e \text{ mostly } \nu_2 \]

- Octant not measurable in vacuum:

- In matter:
  \[ P_{\nu_e \rightarrow \nu_{\mu}} = \frac{1}{2} \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \]

- In matter:
  \[ \xi = \sqrt{\sin^2 2\theta + \left( \cos 2\theta - \frac{A_{CC}}{\Delta m^2} \right)^2} \]
  Three flavor:
  \[ \rightarrow \theta_{12} = 34^\circ \text{ (1st octant)} \]
  \[ \rightarrow \theta_{13} = 9^\circ \text{ (1st octant)} \]
  \[ \rightarrow \theta_{23} = 39^\circ \text{ (? octant)} \]

(Capozzi et al. (2013), arXiv:1312.2878)
Matter effects – parametric resonances

- Occur in systems with periodically varying density profile
- Based on modulation of oscillation phase ($\Delta m^2$) (in contrast to MSW, mixing NEVER has to be large)

\[
P_{\nu_e \rightarrow \nu_\mu} = \frac{1}{2} \sin^2 2\theta_M \cdot \sin^2 \left( \frac{\Delta m^2_M L}{4E_\nu} \right)
\]

\[
\Delta m^2_M = \xi \times \Delta m^2 \quad \sin 2\theta_M = \frac{\sin 2\theta}{\xi}
\]

$\Delta m^2_{M,1} < \Delta m^2_{M,2}$  $\theta_{M,1} > \theta_{M,2}$

Matter effects – parametric resonances

- Occur in systems with periodically varying density profile

- Based on modulation of oscillation phase ($\Delta m^2$) (in contrast to MSW, mixing NEVER has to be large)

\[ P_{\nu_e \rightarrow \nu_\mu} = \frac{1}{2} \sin^2 2\theta_M \cdot \sin^2 \left( \frac{\Delta m^2_M L}{4E_\nu} \right) \]

\[ \Delta m^2_M = \xi \times \Delta m^2 \quad \sin 2\theta_M = \frac{\sin 2\theta}{\xi} \]

\[ \Delta m^2_{M,1} < \Delta m^2_{M,2} \quad \theta_{M,1} > \theta_{M,2} \]

- Works also for “truncated” periodic profiles like Earth mantel → core → mantel transition (large effect requires density close to MSW resonance)

Neutrino mass hierarchy with neutrino telescopes

- Oscillation pattern modified by matter effects in Earth (MSW effect + parametric resonances)
- Effect differs for normal and inverted hierarchy (sign of $\Delta m^2$ changes)
Neutrino mass hierarchy with neutrino telescopes

$P(\nu_e \rightarrow \nu_\mu)$ with Travel Through the Earth - 10 GeV, 179
Neutrino mass hierarchy with neutrino telescopes

$P(\nu_e \rightarrow \nu_\mu)$ with Travel Through the Earth - 10 GeV, 179

$P(\nu_e \rightarrow \nu_\mu)$ with Travel Through the Earth - 6 GeV, 126

Alexander Kappes, NOW 2014, Conca Specchiulla, 12.09.2014
Neutrino mass hierarchy with neutrino telescopes

- Maximum effect NH ↔ IH for θ=130° at 7 GeV
- For ν NH and IH approximately swapped

Recap: \( \xi = \sqrt{\sin^2 2\theta + \left(\frac{\cos 2\theta - \frac{A_{CC}}{\Delta m^2}}{\sin 2\theta + \frac{A_{CC}}{\Delta m^2}}\right)^2} \)

→ effect cancels if detected \( N(\nu) = N(\bar{\nu}) \)
- Fortunately, flux(\(\nu_{atm}\)) \( \approx 1.3 \times \) flux(\(\bar{\nu}_{atm}\))
  and \( xsec(\nu) \approx 2 \times xsec(\bar{\nu}) \)

\( \implies \) Count \( N_\mu(E,\theta) \) from \( \nu_\mu + N \rightarrow \mu + X \)
and compare with NH / IH predictions

Remark:
neutrino telescopes inherently insensitive to \( \nu \leftrightarrow \bar{\nu} \)
Neutrino mass hierarchy with neutrino telescopes

$P(\nu_\mu \rightarrow \nu_\mu)$ with Travel Through the Earth - 10 GeV, 179

$P(\nu_\mu \rightarrow \nu_\mu)$ with Travel Through the Earth - 6 GeV, 126

Akhmedov et al., arXiv:1205.7071
Atmospheric neutrino fluxes

- Fluxes $\lesssim 10$ GeV depend on location
- In general, also azimuth dependence
  - factor 3 for $\nu_e$ at INO
  - nearly flat for all $\nu$ at South Pole
- For mass hierarchy ratio $\nu / \bar{\nu}$ particularly important

Horizontal component geomagnetic field (IGRF2010)

(Athar et al. (2012), arXiv:1210.5154)
In relevant energy range (~4–20 GeV):
- Mixture of (quasi-)elastic + Resonant (single $\pi$ production) + Deep-inelastic
- Nuclear corrections small

New calculation of total $\nu$-N scattering cross-sections by Gazizov et al.
→ differences to GENIE up to 30% (at 2 GeV)

Gazizov et al. (2014)
Systematic uncertainties: mixing parameters

- Uncertainties on oscillation parameters distort oscillation pattern

$\nu_\mu$ survival probability

IH $\cos$ (zenith) = -0.6

NH

Resconi (2013)
Systematic uncertainties: mixing parameters

- Uncertainties on oscillation parameters distort oscillation pattern

$\nu_\mu$ survival probability

Resconi (2013)
Systematic uncertainties: mixing parameters

- Uncertainties on oscillation parameters distort oscillation pattern

2/3 \( \nu_\mu \) + 1/3 \( \bar{\nu}_\mu \)

Resconi (2013)
Systematic uncertainties: mixing parameters

- Uncertainties on oscillation parameters distort oscillation pattern

\[ \frac{2}{3} \nu_\mu + \frac{1}{3} \bar{\nu}_\mu \]

- Uncertainties for NH/IH correlated
- PINGU/ORCA will use “control regions” to measure oscillation parameters

Resconi (2013)
Effect of systematic uncertainties (in PINGU)

- External uncertainties
- PINGU uncertainties

$$\Delta m_{31}^2$$
$$\theta_{13}$$
$$\nu$$ cross-section
$$A_{\text{eff}}$$ energy dependence
$$\theta_{23}$$
$$\bar{\nu}$$ cross-section
Energy scale

Impact [σ] (one year)

PINGU LoI: arXiv: 1401.2046
Sensitivity to mass-hierarchy

IH true, Multichannel

- 1st Octant
- 2nd Octant

PINGU Loi: arXiv:1401.2046
Summary

- Matter effects in atmospheric neutrino oscillations allow to determine the ordering of the neutrino mass eigenstates

- Important to have a good handle on “external” systematic uncertainties
  - uncertainties on oscillation parameters will be significantly constrained by ORCA/PINGU
  - uncertainties on (anti-) neutrino cross sections give significant contribution (a.o. limited by precision of available data)

- $\theta_{23}$ lying in the 2nd octant (> 45°) will lead to a dramatic increase of ORCA/PINGU’s sensitivity to the mass hierarchy
The mass hierarchy is not just a sign!

World best-fit values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>1σ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m^2/10^{-5}$ eV$^2$ (NH or IH)</td>
<td>7.54</td>
<td>7.32 – 7.80</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}/10^{-1}$ (NH or IH)</td>
<td>3.08</td>
<td>2.91 – 3.25</td>
</tr>
<tr>
<td>$\Delta m^2/10^{-3}$ eV$^2$ (NH)</td>
<td>2.44</td>
<td>2.38 – 2.52</td>
</tr>
<tr>
<td>$\Delta m^2/10^{-3}$ eV$^2$ (IH)</td>
<td>2.40</td>
<td>2.33 – 2.47</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (NH)</td>
<td>2.34</td>
<td>2.16 – 2.56</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-2}$ (IH)</td>
<td>2.39</td>
<td>2.18 – 2.60</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (NH)</td>
<td>4.25</td>
<td>3.98 – 4.54</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$ (IH)</td>
<td>4.37</td>
<td>4.08 – 5.31</td>
</tr>
<tr>
<td>$\delta/\pi$ (NH)</td>
<td>1.39</td>
<td>1.12 – 1.72</td>
</tr>
<tr>
<td>$\delta/\pi$ (IH)</td>
<td>1.35</td>
<td>0.96 – 1.59</td>
</tr>
</tbody>
</table>

(Capozzi et al. (2013), arXiv:1312.2878)
The mass hierarchy is not just a sign!

![Graphs showing inverted and normal neutrino mass hierarchy with fit and best-fit curves.](image-url)
1st Octant, IH true

- Tracks
- Cascades
- Multichannel

\[ \sqrt{t} \]

Preliminary