Control of decoherence

S. Pascazio[1,*] and P. Facchi[1,†]

[1]*Dipartimento di Fisica, Università di Bari*
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari
I-70126 Bari, Italy

We discuss three control strategies aimed at countering the effects of decoherence: the first hinges on frequent projective measurements, the second on frequent unitary "kicks" ("bang-bang" pulses) and the third on a strong continuous coupling. Decoherence is suppressed if the frequency N of the measurements/kicks is large enough or if the coupling K is sufficiently strong: in all these cases, the Hilbert space of the system splits into invariant subspaces, among which any transition is hindered. However, if N or K are large, but not extremely large, all these control procedures accelerate decoherence.

PACS numbers: 03.67.Pp; 03.65.Xp, 03.65.Yz; 03.67.Lx

Keywords: Decoherence, noise, fluctuations, control, quantum Zeno effect

1. INTRODUCTION

Decoherence [1] is the dynamical deterioration of the coherence features of quantum mechanical systems and represents one of the most serious obstacles against the preservation of quantum superpositions and entanglement over long periods of time. The possibility of controlling (and eventually halting) decoherence is a key problem with important applications, e.g. in quantum computation [2]. We focus here on three schemes that have been recently proposed in order to counter the effects of decoherence. The first is based on the quantum Zeno effect (QZE) [3–5]), the second on "bang-bang" (BB) pulses and their generalization, quantum dynamical decoupling [6] and the third on a strong, continuous coupling [7]. These apparently different methods are in fact deeply related to each other [8] and their comparison is interesting [9].

We will focus on the key role played by the form factors of the interaction and clarify under which circumstances and physical conditions these three controls may *accelerate*, rather

*`saverio.pascazio@ba.infn.it`
†`paolo.facchi@ba.infn.it`

than hinder decoherence. The method we propose is general and can be applied to diverse situations of practical interest, such as atoms and ions in cavities, organic molecules, quantum dots and Josephson junctions [10].

2. THE GENERAL SETTING AND AN EXAMPLE

We will consider a quantum system living in a Hilbert space $\mathcal{H}$ and an orthogonal resolution of the identity

$$\{P_n\}_{n\in\mathbb{N}}, \qquad P_n P_m = \delta_{mn} P_n, \qquad \sum_n P_n = 1. \tag{1}$$

The associated partition on the total Hilbert space is

$$\mathcal{H} = \bigoplus_n \mathcal{H}_n, \tag{2}$$

where $\mathcal{H}_n = P_n \mathcal{H}$. We will take for simplicity $\mathcal{H}_1$ to be two-dimensional (a qubit) and will discuss some possible strategies in order to hinder the transitions out of this subspace.

Let us start by looking at an elementary example in order to understand how the qubit subspace can be protected from decoherence. Consider a 3-level system in $\mathcal{H} = \mathbb{C}^3$

$$\langle a| = (1,0,0), \quad \langle b| = (0,1,0), \quad \langle c| = (0,0,1) \tag{3}$$

and the Hamiltonian

$$H = \Omega_1 \sigma_{ab} + \Omega_2 \sigma_{bc} = \begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1 & 0 & \Omega_2 \\ 0 & \Omega_2 & 0 \end{pmatrix}, \tag{4}$$

where $\sigma_{ab} = |a\rangle\langle b| + |b\rangle\langle a|$, and σ_{bc} is analogously defined.

We first consider the case of *bona fide* measurements, described by projection operators *à la* von Neumann [3]. The measurements are taken to be "incomplete" and "nonselective" [11, 12]: incomplete, in the sense that some outcomes may be lumped together (for instance because the measuring apparatus has insufficient resolution); this means that the projection operator that selects a particular lump is multidimensional (and therefore the information gained on the measured observable is incomplete); "nonselective," in the sense that the measuring apparatus does not select the different outcomes, but simply destroys the phase correlations between some states, provoking the transition from a pure state to a mixture.

Let the measurement be described by the superoperator

$$\hat{P}\rho = \sum_n P_n \rho P_n \tag{5}$$

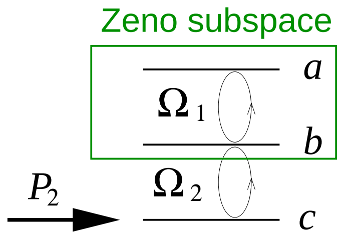

FIG. 1: Three level system undergoing measurements (P_1 not indicated). We explicitly showed the Zeno subspace $\mathcal{H}_1$.

in terms of the (incomplete, nonselective) projections ($P_1 + P_2 = \mathbf{1}$)

$$P_1 = \mathbb{1}_{ab} = |a\rangle\langle a| + |b\rangle\langle b| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_2 = \mathbb{1}_c = |c\rangle\langle c| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

The Zeno dynamics is given by

$$\rho(t) = \lim_{N\to\infty} \left(\hat{P}\hat{U}_{t/N}\right)^N \rho_0 = \hat{P}\exp(-i\hat{H}_{\mathrm{Z}}t)\rho_0 = \sum_n \mathcal{V}_n(t)\rho_0\mathcal{V}_n^\dagger(t), \tag{7}$$

where $\hat{U}_t\rho = \exp(-i\hat{H}t)\rho = \exp(-iHt)\rho\exp(iHt)$. In the Zeno limit (frequency of measurements $N \to \infty$, period $\tau = t/N \to 0$) the Hilbert space is partitioned in two sectors (*Zeno subspaces*) [13], according to (2), with $\dim\mathcal{H}_1 = 2$, $\dim\mathcal{H}_2 = 1$: the subspaces $\mathcal{H}_1$ and $\mathcal{H}_2$ decouple. The dynamics is governed by the Zeno Hamiltonian

$$H_{\mathrm{Z}} = P_1HP_1 + P_2HP_2 = \Omega_1\sigma_{ab} = \begin{pmatrix} 0 & \Omega_1 & 0 \\ \Omega_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

and the evolution operators within each Zeno subspace read [13]

$$\begin{aligned} \mathcal{V}_1 &= P_1\exp(-iP_1HP_1t) = \mathbb{1}_{ab}\exp(-i\Omega_1\sigma_{ab}t) = \begin{pmatrix} \cos\Omega_1 t & -i\sin\Omega_1 t & 0 \\ -i\sin\Omega_1 t & \cos\Omega_1 t & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \mathcal{V}_2 &= P_2\exp(-iP_2HP_2t) = P_2 = \mathbb{1}_c. \end{aligned} \tag{9}$$

If the coupling Ω_2 is viewed as a caricature of the loss of quantum mechanical coherence, the qubit subspace $\mathcal{H}_1$ is protected from decoherence (and one can suggestively say that it becomes "decoherence free" [14]). See Fig. 1.

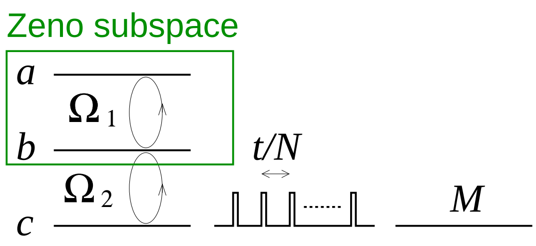

FIG. 2: Three level system undergoing frequent unitary kicks that couple one of its levels to an "external" system M. We explicitly indicated the Zeno subspace $\mathcal{H}_1$.

In general the Zeno evolution (7) due to a generic measurement (5) reads [13]

$$\mathcal{V}_n(t) = P_n \exp(-iH_{\mathrm{Z}}t), \qquad H_{\mathrm{Z}} = \hat{P}H = \sum_n P_n H P_n, \tag{10}$$

H_{Z} being the Zeno Hamiltonian.

In order to understand how unitary kicks yield the Zeno subspaces, consider the 4-level system in the enlarged Hilbert space $\mathcal{H}_{\mathrm{sys}} \oplus \mathrm{span}\{|M\rangle\}$

$$\langle a| = (1,0,0,0), \quad \langle b| = (0,1,0,0), \quad \langle c| = (0,0,1,0), \quad \langle M| = (0,0,0,1) \tag{11}$$

and the Hamiltonian

$$H = \Omega_1 \sigma_{ab} + \Omega_2 \sigma_{bc} = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{12}$$

This is the same example as (3)-(4), but we added a fourth level $|M\rangle$. We now couple $|M\rangle$ to $|c\rangle$ by performing the unitary kicks ($\lambda \neq 0 \bmod 2\pi$)

$$U_{\mathrm{kick}} = \exp(-i\lambda\sigma_{cM}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\lambda & -i\sin\lambda \\ 0 & 0 & -i\sin\lambda & \cos\lambda \end{pmatrix} = \sum_{n=1,\pm} e^{-i\lambda_n} P_n, \tag{13}$$

where $\sigma_{cM} = |c\rangle\langle M| + |M\rangle\langle c|$, $\lambda_1 = 0, \lambda_\pm = \pm\lambda$, and

$$P_1 \; = \; \mathbb{1}_{ab} = |a\rangle\langle a| + |b\rangle\langle b| = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{14}$$

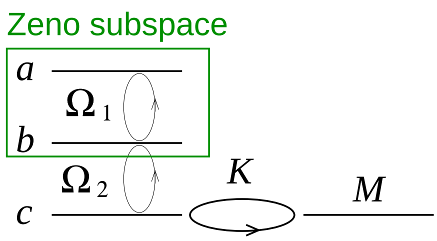

FIG. 3: Three level system with one of its levels strongly coupled to an "external" system M. We explicitly indicated the Zeno subspace $\mathcal{H}_1$.

$$P_\pm = \frac{(|c\rangle \pm |M\rangle)(\langle c| \pm \langle M|)}{2} = \frac{\mathbb{1}_{cM} \pm \sigma_{cM}}{2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \pm 1 \\ 0 & 0 & \pm 1 & 1 \end{pmatrix} \tag{15}$$

form an orthogonal resolution of the identity as in (1) ($P_1 + P_- + P_+ = \mathbf{1}$), defining the partition (2) of the total Hilbert space.

In the Zeno limit (frequency of kicks $N \to \infty$, period $\tau = t/N \to 0$),

$$\rho(t) = \lim_{N\to\infty} \hat{U}_{\rm kick}^{-N} \left(\hat{U}_{\rm kick}\hat{U}_{t/N}\right)^N \rho_0 = \exp(-i\hat{H}_{\rm Z}t)\rho_0, \tag{16}$$

where $\hat{U}_{\rm kick}\rho = U_{\rm kick}\rho U_{\rm kick}^\dagger$, the subspaces $\mathcal{H}_1$, $\mathcal{H}_+$ and $\mathcal{H}_-$ decouple due to the wildly oscillating phases $\mathrm{O}(N)$. See Fig. 2. The Zeno Hamiltonian reads

$$H_{\rm Z} = \hat{P}H = \sum_n P_n H P_n = \Omega_1 \sigma_{ab} = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{17}$$

and the evolution within each Zeno subspace reads

$$\begin{aligned} \mathcal{V}_1 &= P_1 \exp(-iP_1HP_1t) = \mathbb{1}_{ab}\exp(-i\Omega_1\sigma_{ab}t) = \begin{pmatrix} \cos\Omega_1 t & -i\sin\Omega_1 t & 0 & 0 \\ -i\sin\Omega_1 t & \cos\Omega_1 t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mathcal{V}_\pm &= P_\pm \exp(-iP_\pm HP_\pm t) = P_\pm. \end{aligned} \tag{18}$$

Finally, in order to understand how the scheme involving continuous coupling works, add to (12) the Hamiltonian (acting on $\mathcal{H}_{\rm sys} \oplus \mathrm{span}\{|M\rangle\}$)

$$KH_{\rm c} = K\sigma_{cM} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K \\ 0 & 0 & K & 0 \end{pmatrix} = K\,(P_+ - P_-)\ , \tag{19}$$

where $P_\pm$ are the same as in (15). The fourth level $|M\rangle$ is now "continuously" coupled to level $|c\rangle$, $K \in \mathbb{R}$ being the strength of the coupling [15]. As K is increased, level $|M\rangle$ performs a better "continuous observation" of $|c\rangle$, yielding the Zeno subspaces. The eigenprojections of $H_{\rm c}$

$$H_{\rm c} = \eta_1 P_1 + \eta_- P_- + \eta_+ P_+ \tag{20}$$

are again (14)-(15), with $\eta_1 = 0, \eta_\pm = \pm 1$. Once again, in the Zeno limit ($K \to \infty$),

$$\rho(t) = \lim_{K\to\infty} \exp(iK\hat{H}_c t)\exp(-i\hat{H}_K t)\rho_0 = \exp(-i\hat{H}_{\rm Z} t)\rho_0, \tag{21}$$

where $H_K = H + KH_c$, the subspaces $\mathcal{H}_1$, $\mathcal{H}_+$ and $\mathcal{H}_-$ decouple due to the wildly oscillating phases $\mathrm{O}(K)$. See Fig. 3. The Zeno Hamiltonian $H_{\rm Z}$ turns out to be identical to (17), while the evolution within each Zeno subspace is given by (18).

3. SPONTANEOUS DECAY IN VACUUM

As we explained in the previous section, one of the most interesting potential applications of the quantum Zeno subspaces concerns the possibility of freezing decoherence, viewed as loss of phase correlation and/or probability leakage to the environment. The model outlined in the previous section is too simple to schematize a genuine decoherence process. For instance, take (12)+(19), exemplified in Fig. 3: the continuous coupling K does not freeze the *decay* of level $|b\rangle$ onto level $|c\rangle$, it simply hinders the *Rabi transition* $|b\rangle \leftrightarrow |c\rangle$. A better model would be

$$H_K = H_{\rm decay} + KH_{\rm c} = \Omega_1\sigma_{ab} + \Omega_2\sigma_{bc} - i\Delta\mathbb{1}_{cM} + K\sigma_{cM} = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & -i\Delta & K \\ 0 & 0 & K & -i\Delta \end{pmatrix}. \tag{22}$$

This describes the spontaneous emission of level $|b\rangle$ into a (structured) continuum, which in turn is resonantly coupled to a fourth level $|M\rangle$ [5]. The decay rate into the continuum is $\gamma = 2/(\tau_{\rm Z}^2\Delta)$, where $\tau_{\rm Z} = \Omega_2^{-1}$ is the Zeno time [16] (convexity of the initial quadratic region).

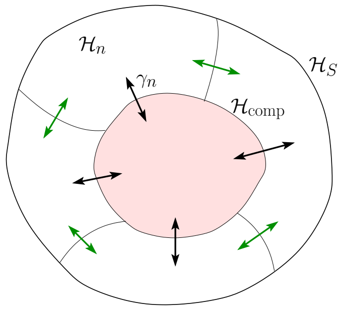

FIG. 4: The Zeno subspaces are formed when the frequency τ^{-1} of measurements or BB pulses or the strength K of the continuous coupling tend to ∞. The shaded region represents the "computational" subspace $\mathcal{H}_{\rm comp} \subset \mathcal{H}_S$. The transition rates γ_n depend on τ or K.

This case is relevant for quantum computation, if one is interested in protecting a given subspace ($\mathcal{H}_1$) from decoherence, by inhibiting spontaneous emission. A somewhat related example is considered in [17]. A proper analysis of this model yields to the following main conclusions: as expected, when the Rabi frequency K is increased, the spontaneous emission from level $|b\rangle$ (to be "protected" from decay/decoherence) is hindered. However, the real problem are the relevant timescales: in order to get an effective "protection" of level $|b\rangle$, one needs $K > 1/\tau_{\rm Z} = \Omega_2$. More to this, if the decaying state $|b\rangle$ has energy $\omega_b \neq 0$, an inverse Zeno effect [18, 19] may take place and the requirement for obtaining the QZE becomes even more stringent, yielding $K > 1/\tau_{\rm Z}^2\gamma$. Both these conditions can be very demanding for a real system subject to dissipation. For instance, typical values for spontaneous decay in vacuum are $\gamma \simeq 10^9{\rm s}^{-1}$, $\tau_{\rm Z}^2 \simeq 10^{-29}{\rm s}^2$ and $1/\tau_{\rm Z}^2\gamma \simeq 10^{20}{\rm s}^{-1}$ [16]. The situation can be made more favorable by using cavities. In this context, model (22) yields some insights in the examples analyzed in [14] and [20], but we will not further elaborate on this point here. Related interesting proposals, making use of kicks or continuous coupling in cavity QED, can be found in [21].

4. DYNAMICAL SUPERSELECTION RULES: THE ZENO SUBSPACES

The three different procedures described in Section **2** yield, by different physical mechanisms, the formation of invariant Zeno subspaces. This is shown in Fig. 4. If one of these invariant subspaces is the "computational" subspace $\mathcal{H}_{\rm comp}$, the possibility arises of inhibiting decoherence in this subspace.

In principle, in the $\tau, K^{-1} \to 0$ limit, decoherence can be completely *halted*; however, as we shall see, it is important to understand *how* the limit is attained, as for τ, K^{-1} small, but not extremely small, decoherence can be enhanced. We now turn our attention to the study of the transition rates γ_n between different subspaces and in particular their τ and K dependence (see Fig. 4).

5. GENERALITIES AND NOTATION

We only briefly summarize the main results [9]. Let the total system consist of a target system S and a reservoir B and its Hilbert space be expressed as the tensor product $\mathcal{H}_{\rm tot} = \mathcal{H}_S \otimes \mathcal{H}_B$. The total Hamiltonian

$$H_{\rm tot} = H_0 + H_{SB} = H_S + H_B + H_{SB} \tag{23}$$

is the sum of the system Hamiltonian H_S, the reservoir Hamiltonian H_B and their interaction H_{SB}, which is responsible for decoherence; the operators H_S and H_B act on $\mathcal{H}_S$ and $\mathcal{H}_B$, respectively. H_0 is the free total Hamiltonian.

We assume that the interaction Hamiltonian H_{SB} in (23) can be written as [22]

$$H_{SB} = \sum_m \left(X_m \otimes A_m^\dagger + X_m^\dagger \otimes A_m \right), \tag{24}$$

where the X_m are the eigenoperators of the system Liouvillian $\hat{H}_S$, satisfying

$$\hat{H}_S X_m = [H_S, X_m] = -\omega_m X_m \qquad (\omega_m \neq \omega_n, \quad \text{for} \quad m \neq n) \tag{25}$$

and A_m are the destruction operators of the bath

$$A_m = \int d\omega \, \sqrt{\kappa_m(\omega)} \, a(\omega) \;, \tag{26}$$

expressed in terms of the bosonic operators $a(\omega)$ and the bare spectral density functions (form factors) $\kappa_m(\omega)$, which are taken to be nonvanishing only for $\omega > 0$. We focus on two particular *Ohmic* cases: an exponential and a polynomial form factors:

$$\kappa_m^{(E)}(\omega) = g^2 \omega \exp(-\omega/\Lambda_E), \qquad \kappa_m^{(P)}(\omega) = g^2 \frac{\omega}{[1+(\omega/\Lambda_P)^2]^2}, \tag{27}$$

respectively, where g is a coupling constant and Λ a cutoff. In order to properly compare these two cases, we require that the bandwidth be the same

$$W = \langle\omega\rangle_E = \langle\omega\rangle_P \,, \tag{28}$$

where $\langle f(\omega)\rangle = \int f(\omega)\kappa_m(\omega)d\omega / \int \kappa_m(\omega)d\omega$.

The initial state of the total system is taken to be the tensor product of the system and reservoir initial states $\rho(0) = \sigma(0) \otimes \rho_B$, where the reservoir equilibrium state has an inverse temperature β

$$\rho_B = \frac{1}{Z}\exp(-\beta H_B), \qquad Z = \mathrm{tr}_B e^{-\beta H_B}. \tag{29}$$

The system state $\sigma(t)$ at time t is given by the partial trace

$$\sigma(t) \equiv \mathrm{tr}_B \rho(t). \tag{30}$$

There is decoherence when $\sigma(t)$ is not unitarily equivalent to $\sigma(0)$ for a given class of initial states. In the Markov approximation the state of the system (30) satisfies the master equation

$$\dot{\sigma}(t) = \left(-i\hat{H}_S + \mathcal{L}\right)\sigma(t)\ , \tag{31}$$

where, up to a renormalization of the free Liouvillian $\mathcal{L}_S$ by Lamb and Stark shift terms, $\mathcal{L}$ engenders the dissipation due to the interaction with the bath,

$$\mathcal{L}\sigma = \sum_m \gamma_m \left(X_m \sigma X_m^\dagger - \frac{1}{2}\left\{X_m^\dagger X_m, \sigma\right\}\right), \tag{32}$$

where $X_{-m} = X_m^\dagger$ and

$$\gamma_m = 2\pi\kappa_m^\beta(\omega_m) \tag{33}$$

are the dissipation rates. The key quantities are the thermal spectral density functions

$$\kappa_m^\beta(\omega) = \frac{1}{1 - e^{-\beta\omega}}\left[\kappa_m(\omega) - \kappa_m(-\omega)\right] \tag{34}$$

that extend along the whole real axis, due to the counter-rotating terms, and satisfy the KMS symmetry [23].

6. CONTROL PROCEDURES

When one of the three methods of control outlined in Sec. **2** is applied, the dynamics of the system is modified. In particular, in the master equation (31) the dissipative term $\mathcal{L}$, responsible for dissipation and decoherence, is modified: it preserves the Lindblad form (32), but with modified dissipation rates that depend on the period $\tau = t/N$ of the measurements/kicks or on the strength K of the continuous interaction. The modified rates will be denoted by $\gamma^{\mathrm{Z}}(\tau)$, $\gamma^{\mathrm{k}}(\tau)$ and $\gamma^{\mathrm{c}}(K)$ for the case of Zeno control, bang-bang control and continuous-coupling control, respectively.

6.1. Zeno Control

The controlled dissipation rates read

$$\gamma_m^{\mathrm{Z}}(\tau) = \tau \int_{-\infty}^{\infty} d\omega\ \kappa_m^{\beta}(\omega)\ \mathrm{sinc}^2\left(\frac{\omega - \omega_m}{2}\tau\right)\ , \tag{35}$$

with $\mathrm{sinc}(x) = \sin(x)/x$. Notice that, in the $\tau \to 0$ limit, the dissipative part disappears, $\gamma^{\mathrm{Z}}(\tau) \to 0$, and decoherence is suppressed, as expected. On the other hand, $\gamma^{\mathrm{Z}}(\tau) \to \gamma$, when $\tau \to \infty$ (uncontrolled evolution).

In general, (35) yields both Zeno and inverse Zeno effects as τ is changed. The transition between the two regimes takes place at $\tau = \tau^*$, where τ^* is defined by the equation [19]

$$\gamma^{\mathrm{Z}}(\tau^*) = \gamma. \tag{36}$$

The key issue is to understand *how small* τ should be in order to get suppression (control) of decoherence (QZE), rather than its enhancement (IZE). The ratio $\gamma^{\mathrm{Z}}(\tau)/\gamma$ is shown in Fig. 5 as a function of τ [in units W–the bandwidth defined in Eq. (28)].

6.2. Bang-Bang Control

The controlled dissipation rates read

$$\gamma_{mn}^{\mathrm{k}}(\tau) \sim 2\pi\kappa_m^{\beta}\left(\frac{2\pi n}{\tau}\right). \tag{37}$$

Again, in the $\tau \to 0$ limit, the dissipative part disappears (with a law that depends on the form factor), $\gamma^{\mathrm{k}}(\tau) \to 0$, and decoherence is suppressed, as expected. The ratio $\gamma^{\mathrm{k}}(\tau)/\gamma$ is shown in Fig. 5 as a function of τ. Once again, the transition between the two regimes takes place at $\tau = \tau^*$, where τ^* is defined by the equation

$$\gamma^{\mathrm{k}}(\tau^*) = \gamma. \tag{38}$$

6.3. Continuous-coupling Control

The dissipative rates read

$$\gamma_{mn}^{\mathrm{c}}(K) = 2\pi\kappa_m^{\beta}\left(K\Omega_n + \omega_m\right), \tag{39}$$

where Ω_n are the eigenfrequencies of $\hat{H}_{\mathrm{c}}$. Hence, in the $K \to +\infty$ limit, the dissipative part disappears (with a law that depends on the form factor), $\gamma^{\mathrm{c}}(K) \to 0$, and decoherence

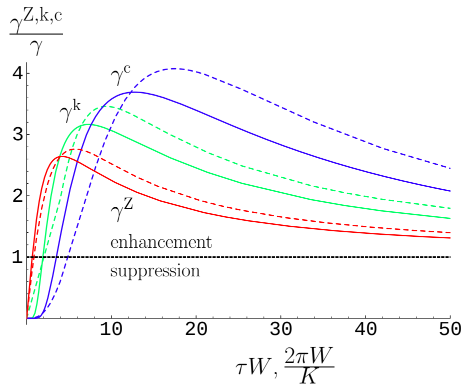

FIG. 5: Comparison among the three control methods. The full and dashed lines refer to the exponential and polynomial form factors, respectively. BB kicks and continuous coupling are more effective than *bona fide* measurements for combatting decoherence, as the regime of "suppression" is reached for larger values of τ and K^{-1}.

is suppressed, as expected. On the other hand, $\gamma^{\mathrm{c}}(K) \to \gamma$, when $K \to 0$ (uncontrolled evolution). Notice that the role of K in this subsection and the role of $1/\tau$ in the previous ones are equivalent. This yields a natural comparison [8] between different timescales (τ for measurements and kicks, $1/K$ for continuous coupling).

The ratio $\gamma^{\mathrm{c}}(K)/\gamma$ is shown in Fig. 5 as a function of $2\pi/K$. The transition between these two regimes takes now place at $K = K^*$ where K^* is defined by the equation

$$\gamma^{\mathrm{c}}(K^*) = \gamma. \tag{40}$$

6.4. Comparison among the three control strategies

There is a clear difference between projective measurements and the other two cases, BB kicks and continuous coupling. In the former case τ^* depends on the global features of the form factor (i.e., its integral). By contrast, in the other two cases τ^* "picks" some particular ("on-shell") value(s). This important difference is due to the different features of the evolution (non-unitary in the first case, unitary in the latter cases). The different features discussed above yield very different outputs, clearly apparent in Fig. 5, that can be important in practical applications: decoherence can be more easily halted by applying BB and/or continuous coupling strategies. These two methods yield values of τ^* (or K^*) that are easier to attain. However, this advantage has a price, because BB and continuous coupling yield a larger enhancement of decoherence for $\tau > \tau^*$, $K < K^*$.

7. SUMMARY AND OUTLOOK

We compared three control methods for controlling decoherence. The first is based on repeated quantum measurements (projection operators) and involves a description in terms of nonunitary processes. The second and third methods are both dynamical, as their description only involves unitary evolutions. In principle, decoherence can always be halted by very frequently measuring or very rapidly/strongly driving the system state. However, when the frequency is not high enough or the coupling not strong enough, in general the controls may accelerate the decoherence process and deteriorate the performance of the quantum state manipulation. The control of decoherence may be viewed as a manifestation of the quantum Zeno effect, by which frequent "measurements" (whatever loose meaning one attributes to this expression) suppress the evolution of a quantum system. Similarly, the acceleration of decoherence is analogous to the inverse Zeno effect, namely the acceleration of the decay of a quantum unstable state due to frequent measurements [18, 19]. Our analysis clarifies that the expression "large N" (frequent measurements or interruptions) and "large K" (large coupling) should not be taken lightheartedly, as they are directly related to the physically relevant timescales characterizing the evolution. This is the key issue to address, in view of possible applications.

Acknowledgments

We thank D. Lidar, H. Nakazato and S. Tasaki for discussions. This work is partly supported by the bilateral Italian-Japanese project 15C1 on "Quantum Information and Computation" of the Italian Ministry for Foreign Affairs.

[1] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and H.-D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996); M. Namiki, S. Pascazio, and H. Nakazato, *Decoherence and Quantum Measurements* (World Scientific, Singapore, 1997).

[2] A. Galindo and M.A. Martin-Delgado, Rev. Mod. Phys. **74**, 347 (2002); M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[3] J. von Neumann, *Mathematical Foundation of Quantum Mechanics* (Princeton University Press, Princeton, 1955); A. Beskow and J. Nilsson, Arkiv für Fysik **34**, 561 (1967); L.A.

Khalfin, JETP Letters **8**, 65 (1968).

[4] B. Misra and E.C.G. Sudarshan, J. Math. Phys. **18**, 756 (1977).

[5] P. Facchi and S. Pascazio, Progress in Optics, ed. E. Wolf (Elsevier, Amsterdam, 2001), vol. 42, Chapter 3, p.147.

[6] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1998); P. Zanardi, Phys. Lett. A **258**, 77 (1999); D. Vitali and P. Tombesi, Phys. Rev. A **59**, 4178 (1999); Phys. Rev. A **65**, 012305 (2001); C. Uchiyama and M. Aihara, Phys. Rev. A **66**, 032313 (2002); M.S. Byrd and D.A. Lidar, Quantum Information Processing **1**, 19 (2002); Phys. Rev. A **67**, 012324 (2003).

[7] M. Simonius, Phys. Rev. Lett. **40**, 980 (1978); R.A. Harris and L. Stodolsky, Phys. Lett. B **116**, 464 (1982); A. Peres, Am. J. Phys. **48**, 931 (1980); L.S. Schulman, Phys. Rev. A **57**, 1509 (1998); A. Luis and L.L. Sánchez–Soto, Phys. Rev. A **57**, 781 (1998); K. Thun and J. Peřina, Phys. Lett. A **249**, 363 (1998); A.D. Panov, Phys. Lett. A **260**, 441 (1999); J. Řeháček, J. Peřina, P. Facchi, S. Pascazio, and L. Mišta, Phys. Rev. A **62**, 013804 (2000); P. Facchi and S. Pascazio, Phys. Rev. A **62**, 023804 (2000); B. Militello, A. Messina, and A. Napoli, Phys. Lett. A **286**, 369 (2001); A. Luis, Phys. Rev. A **64**, 032104 (2001).

[8] P. Facchi, D.A. Lidar, and S. Pascazio, Phys. Rev. A **69**, 032314 (2004).

[9] P. Facchi, S. Tasaki, S. Pascazio, H. Nakazato, A. Tokuse, D.A. Lidar, Phys. Rev. A **71**, 022302 (2005); S. Tasaki, A. Tokuse, P. Facchi, and S. Pascazio, Int. J. Quant. Chem. **98**, 160 (2004).

[10] C. Monroe *et al*, Phys. Rev. Lett. **75**, 4714 (1995); R.J. Hughes *et al*, Fortschr. Phys. **46**, 32 (1998); D.G. Cory *et al*, Fortschr. Phys. **48**, 875 (2000); M. Lieven *et al*, Nature **414**, 883 (2001); L. Jacak, P. Hawrylak, and A. Wojs, *Quantum Dots* (Springer, Berlin, 1998); D. Steinbach *et al*, Phys. Rev. B **60**, 12079 (1999); Y. Makhlin, G. Schön and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001); E. Paladino, L. Faoro, G. Falci, and R. Fazio, Phys. Rev. Lett., **88**, 228304 (2002); T. Calarco, A. Datta, P. Fedichev, E. Pazy, and P. Zoller, Phys. Rev. A **68**, 012310 (2003).

[11] J. Schwinger, Proc. Nat. Acad. Sc. **45**, 1552 (1959); *Quantum kinematics and dynamics* (Perseus Publishing, New York, 1991).

[12] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, Dordrecht, 1998).

[13] P. Facchi and S. Pascazio, Phys. Rev. Lett. **89**, 080401 (2002).

[14] G.M. Palma, K.A. Suominen and A.K. Ekert, Proc. R. Soc. Lond. A **452**, 567 (1996); L.M. Duan and G.C. Guo, Phys. Rev. Lett. **79**, 1953 (1997); P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997); Mod. Phys. Lett. B **11**, 1085 (1997); D.A. Lidar, I.L. Chuang and K.B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998); L. Viola, E. Knill and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).

[15] B. Militello, A. Messina and A. Napoli, Fortschr. Phys. **49**, 1041 (2001).

[16] P. Facchi and S. Pascazio, Phys. Lett. A **241**, 139 (1998); I. Antoniou, E. Karpov, G. Pronko and E. Yarevsky: Phys. Rev. A **63**, 062110 (2001).

[17] G.S. Agarwal, M.O. Scully and H. Walther, Phys. Rev. Lett. **86**, 4271 (2001).

[18] A.M. Lane, Phys. Lett. A **99**, 359 (1983); W.C. Schieve, L.P. Horwitz and J. Levitan, Phys. Lett. A **136**, 264 (1989); B. Elattari and S.A. Gurvitz, Phys. Rev. A **62**, 032102 (2000); A.G. Kofman and G. Kurizki, Nature **405**, 546 (2000); K. Koshino and A. Shimizu, Phys. Rev. A **67**, 042101 (2003).

[19] P. Facchi, H. Nakazato and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001)

[20] A. Beige, D. Braun, B. Tregenna and P.L. Knight, Phys. Rev. Lett. **85**, 1762 (2000).

[21] J. Pachos and H. Walther, Phys. Rev. Lett. **89**, 187903 (2002); M.O. Scully, S.-Y. Zhu and M.S. Zubairy, Chaos Solitons and Fractals **16**, 403 (2003)

[22] C.W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, 2000).

[23] H. Spohn and J.L. Lebowitz, Adv. Chem. Physics **38**, 109 (1979).